Classical Mechanics III (8.09) Fall 2014
Assignment 1

Massachusetts Institute of Technology
Physics Department
September 6, 2014

Due September 15, 2014
6:00pm

Announcements

The first week is dedicated primarily to discussing and expanding upon material from Classical Mechanics II.

Reading Assignment for the first two weeks of the course

- Lagrangian and Hamiltonian classical mechanics, symmetries and conservation laws. Use of generalized coordinates to handle constraints to motion on surfaces or curves. Read the following parts of **Goldstein** to review and extend concepts you have already seen in Classical Mechanics II: pgs. 34-45, 55-56, 61-63, 334-342, then 343-347 and 353-356 (ie. specific parts of sections 2.1-2.3, 2.6, 2.7, 8.1, 8.2, 8.5.) If your feeling rusty with Clasical Mechanics II material you might want to review sections 1-10, and 40 of **Landau & Lifshitz**.

- On problem set #2 we will cover: D'Alembert's principle and systems with holonomic and non-holonomic constraints. Use of Lagrange multipliers to determine forces of constraint. If you want to read ahead the corresponding reading is **Goldstein**: sections 1.3-1.6, 2.4, 2.5, 8.5.

Problem Set 1

These five problems are intended to help you review and generalize results from Classical Mechanics II.

1. **Two Particles in a Gravitational Field** [12 points]

   Consider two masses m_1 and m_2 moving in three dimensions which are attracted to each other gravitationally, and are also acted on by a uniform gravitational field with acceleration strength g in the $-\hat{z}$ direction.

   (a) What is the Lagrangian? Use center of mass coordinates and the vector distance $\vec{r}$ between the two particles. This Lagrangian naturally splits itself into the sum of two parts, explain physically why this makes sense.

   (b) Consider the part of the Lagrangian depending on $\vec{r}$ and determine the Lagrangian and compute the Euler-Lagrange equations of motion using spherical coordinates.

   (c) Using the Lagrangian in (b), what is the corresponding Hamiltonian? What are the Hamilton equations of motion?

2. **Double Pendelum in a Plane with Gravity** [20 points]

   Since the double pendelum is our mascot on the course homepage, we'd best find the equations that govern its motion.
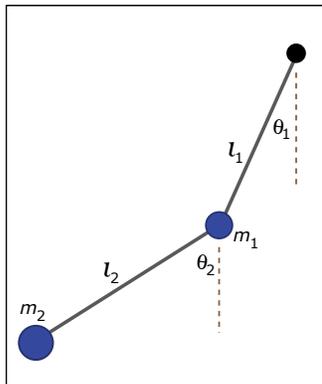
   Image by MIT OpenCourseWare.

   a) Obtain the Lagrangian and equations of motion for the double pendulum illustrated here. The point at the top is fixed and the oscillations take place in a two-dimensional plane. The length of the pendula are ℓ_1 and ℓ_2, and the rigid bars of the pendula can be taken to be massless. The balls on the ends have mass m_1 and m_2.

   b) Now take $m_1 = m_2$ and find the Hamiltonian for this system, and the Hamilton equations of motion. [If you wish you can make use of results derived in lecture.]

3. **Point Mass on a Hoop** [12 points]

   Do Goldstein Ch.2 Problem #18. Use spherical coordinates (r, θ, ϕ) with the origin at the center of the hoop.

4. **Spring System on a Plane** [10 points]

   A massless spring has an unstretched length b and spring constant k, and is used to connect two particles of mass m_1 and m_2. The system rests on a frictionless table and may oscillate and rotate.

   (a) What is the Lagrangian? Write it with two-dimensional cartesian coordinates.

   (b) Setup a suitable set of generalized coordinates to better account for the symmetries of this system. What are the Lagrangian and equations of motion in these variables?

   (c) Identify as many conserved generalized momenta as you can that are associated to cyclic coordinates in the Lagrangian from part b). If you think you are missing some, try to improve your answer to b). Show that there is a solution that rotates but does not oscillate, and discuss what happens to this solution for an increased rate of rotation. (A closed form solution is not necessary. A graphical solution will suffice.)

5. **Jerky Mechanics** [6 points]

   Consider an extension of classical mechanics where the equation of motion involves a triple time derivative, $\dddot{x} = f(x, \dot{x}, \ddot{x}, t)$. Lets use Hamilton's principle to derive the corresponding Euler-Lagrange equations. Start with a Lagrangian of the form $L(q_i, \dot{q}_i, \ddot{q}_i, t)$ for n generalized coordinates q_i, and make use of Hamilton's principle for paths $q_i(t)$ that have zero variation of both q_i and $\dot{q}_i$ at the end points. Show that

   $$\frac{d^2}{dt^2}\left(\frac{\partial L}{\partial \ddot{q}_i}\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) + \frac{\partial L}{\partial q_i} = 0$$

   for each $i = 1, \ldots, n$.

6.* **Extra Problem: Equivalent Lagrangians** [not for credit]

   Some of you have seen this problem already in Classical Mechanics II. If it does not sound familiar then I suggest you try it. (It will not be graded but solutions will be provided.)

   Let $L(q, \dot{q}, t)$ be the Lagrangian for a particle with coordinate q, which satisfies the Euler-Lagrange equations. Show that the Lagrangian

   $$L' = L + \frac{dF(q, t)}{dt}$$

   also satisfies the Euler-Lagrange equations where F is an arbitrary differentiable function.

8.09 Classical Mechanics III

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