

Due 11:04 am Friday 2006 March 3

0. This problem appeared on last week's set. You were given the option of doing it with PS#2 or with this set. There is, unfortunately, no good treatment of the material in a text.

The radial velocity for a star in a binary orbit is given by

$$v_r = \frac{2\pi a}{P} \frac{\sin i}{\sqrt{1-e^2}} [e \cos \phi + \cos(\phi + \theta)]$$

where a is the semimajor axis of the star's orbit (with respect to the center of mass), P is the period, i is the inclination of the orbital plane to the line of sight, θ is the angle made by the star with the position of the star at periastron and ϕ is the angle made by the position at periastron with the line of nodes (where the plane of the orbit crosses the plane of the sky), measured from nodal line to periastron in the direction of motion. This gives you velocity as a function of θ . To compare this with observations, you'd like velocity as a function of time. You can get this using the parametric representation of an ellipse based on the eccentric anomaly, η :

$$t = \frac{P}{2\pi} (\eta - e \sin \eta) \quad ,$$

$$\tan \frac{\theta}{2} = \left(\frac{1+e}{1-e} \right)^{1/2} \tan \frac{\eta}{2} \quad .$$

Consider the case of the binary pulsar, PSR 1913+16, for the discovery of which Taylor and Hulse shared the Nobel Prize. The pulsar (a neutron star) emits pulse of radiation every 59 seconds. The frequency with which these pulses are received is doppler shifted by the star's motion. By analyzing the pulse arrival times, Taylor and collaborators have been able to measure a number of properties of the system with extraordinary accuracy. In particular:

$$P = 27906.980894s,$$

$$e = 0.6171304,$$

$$a_1 \sin i / c = 2.341761s \quad \text{and}$$

$$\sin i = 0.734,$$

where the uncertainties are all in the last decimal place quoted (pulsars may be better clocks than any on Earth). The product of the semimajor axis of the pulsar's orbit, a_1 , and $\sin i$ is better known than either individually, hence the odd way of presenting the numbers (drawn from Taylor and Weinberg, 1989, *Astrophysical Journal*, **345**, 434-450). The 8 hour period and masses of the pulsar ($1.44M_\odot$) and its companion ($1.38M_\odot$) strongly suggest that the companion is also a neutron star.

Due to general relativistic effects, the periastron of the orbit precesses 4.2° per year. Hence ϕ changes with time. Plot the pulsar's velocity through one orbit as a function of *time* for each of the following values of ϕ : a) 0° , b) 90° , c) 45° , and d) 135° . Feel free to use a computer if it makes this easier.

1. Optical observations of the faintest objects are often limited by the brightness of the sky. The faintest objects can therefore only be observed between the end of evening twilight and the beginning of morning twilight, which astronomers define as the time when the Sun is 18° below the horizon. How many “dark” hours are available for observing the faintest objects at an observatory with the latitude of Boston ($+42^\circ$) on the shortest night of the year. Remember that the earth's equator is inclined 23.5° to the ecliptic.
2. Consider two planets of negligible mass in orbit around the same star. The orbit of the first planet is circular. The second planet has the same period as the first, but its distance from the star at apastron is 9 times its distance at periastron.
 - a) Find the ratio of the semimajor axis of the second planet to the semimajor axis of the first planet.
 - b) Find the ratio of the angular momentum of the second planet to the angular momentum of the first planet.
 - c) Find the ratio of the semiminor axis of the second plane to the semiminor axis of the first planet.
3. Consider a test particle in an elliptical orbit of period P with eccentricity e and semimajor axis a around a point mass M . Let r be the separation between the point mass to the particle.
 - a) What is dr/dt at pericenter and apocenter?
 - b) What is the maximum value of dr/dt ?