8.321 Quantum Theory-I Fall 2017

Prob Set 1

1. (a) Show that the set of $N \times N$ matrices form a vector space of dimension N^2.

   (b) Show that $Tr(A^\dagger B)$ defines an inner product on this vector space.

   (c) Show that any 2×2 matrix A may be written as

   $$A = a_0 + i\vec{\sigma} \cdot \vec{a} \tag{1}$$

   $i.e$ the four 2×2 matrices $1, \vec{\sigma}$ provide a basis for the $N = 2$ case. Here $\vec{\sigma}$ are the three Pauli matrices. How can these matrices be used to form an orthonormal basis?

   (d) Express $a_0, a_i (i = 1, 2, 3)$ in terms of $Tr(A), Tr(\sigma_i A)$. Hence obtain a_0, a_i in terms of the matrix elements of A.

2. Prove the Schwarz's inequality $|\langle\psi|\phi\rangle|^2 \leq \langle\psi|\psi\rangle\langle\phi|\phi\rangle$ and the triangle inequality $||\,(\psi + \phi)\,|| \leq ||\psi|| + ||\phi||$ from the definition of the inner product. Here the norm of a vector is defined by $||\psi|| = \langle\psi|\psi\rangle^{\frac{1}{2}}$.

3. **Sakurai Prob 1.5**

4. **Sakurai Prob 1.10**

5. Prove that the trace of an operator $Tr(A) = \sum_i \langle\phi_i|A|\phi_i\rangle$ is independent of the particular orthonormal basis ϕ_i that is chosen for its evaluation.

6. Propose a low cost method for measuring Planck's constant $\hbar$ with precision better than 10 percent. The goal of this problem is to review key experimental facts that lead to $\hbar$; in answering the question please outline your method and provide a rough cost estimate.