12.3 Problem Set 3 Solutions

Feynman rules:

$$-i(g\mu_0 - \alpha\frac{p_\mu p_0}{p^2})\frac{\delta_{ab}}{p^2} \tag{12.99}$$

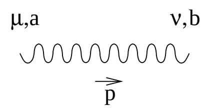

Figure 12.1: Feynman Rules (Equation 12.99).

Fermion:

$$\frac{i\not{p}}{p^2} \tag{12.100}$$

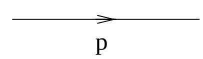

Figure 12.2: Fermion (Equation 12.100).

Ghost:

$$\frac{i\delta_{ab}}{p^2} \tag{12.101}$$

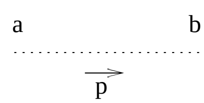

Figure 12.3: Ghost (Equation 12.101).

Scalar:

$$\frac{i}{p^2}\delta_{ik} \tag{12.102}$$

$$gf^{abc}[(k-p)^\rho g^{\mu_0} + (p-q)^\mu g^{\nu\rho} + (q-k)^\nu g^{\rho\mu}] \tag{12.103}$$

$$\begin{aligned}&-ig^2[f^{abc}f^{cde}(g^{\mu\nu}g^{\nu\sigma} - g^{\mu\nu}g^{\nu\rho}) + \\ &f^{ace}f^{bde}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\sigma}g^{\nu\rho}) + \\ &f^{cde}f^{bce}(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma})]\end{aligned} \tag{12.104}$$

$$ig\gamma^\mu tr^a \tag{12.105}$$

$$gf^{abc}p^\mu \tag{12.106}$$

$$ig(p+p')^\mu (tr^a)_{ij} \tag{12.107}$$

$$ig^2(tr^a tr^b + tr^b tr^a)_{ij} \tag{12.108}$$

1.

$$\xrightarrow{\log part} -g^2 C_2(\Gamma_F)\delta_{ab}\frac{4}{3}(g_{\mu\nu}p^2 - p_\mu p_\nu)\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.109}$$

where Λ is cut-off scale, μ is renormalization scale, and Ω_3 is 3D surface area.

$$\xrightarrow{\log div.part} -g^2 C_2(\Gamma_s)\delta_{ab}\frac{1}{4}(g_{\mu\nu}p^2 - p_\mu p_\nu)\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.110}$$

$$\xrightarrow{\log part} 0 \tag{12.111}$$

Similarly all tadpoles have no log div. part.

$$\xrightarrow{\log div.} -g^2 C_2(G)\delta_{ab}\frac{1}{6}(\frac{1}{2}g_{\mu\nu} - p_\mu p_\nu)\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.112}$$

$$\xrightarrow{\log div.} -g^2 C_2(G)\delta_{ab}\frac{1}{6}(p^2 g_{\mu\nu}(\frac{19}{12}+\frac{\alpha}{2}) - p_\mu p_\nu(\frac{11}{6}+\frac{\alpha}{2}))\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.113}$$

$$\xrightarrow{\log div.} 0 \tag{12.114}$$

Add up all contributions to get:

$$\begin{aligned} & \frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 g^2(p^2 g_{\mu\nu} - p_\mu p_\nu) \\ & \{(\frac{5}{3}+\frac{\alpha}{2})C_2(G) - \frac{4}{3}C_2(\Gamma_F) - \frac{1}{2}C_2(\Gamma_s)\} \qquad (12.115) \\ \Rightarrow \quad & -\delta_3|_{\mu^2} \qquad (12.116) \end{aligned}$$

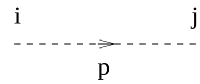

Figure 12.4: Scalar (Equation 12.102).

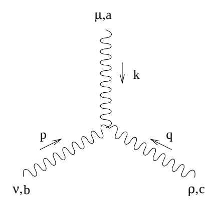

Figure 12.5: Equation 12.103.

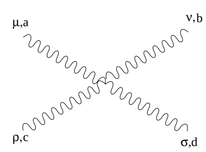

Figure 12.6: Equation 12.104.

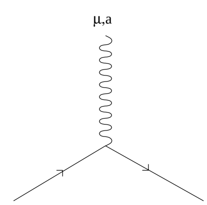

Figure 12.7: Equation 12.105.

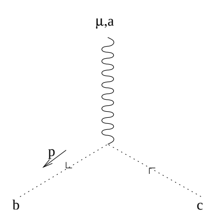

Figure 12.8: Equation 12.106.

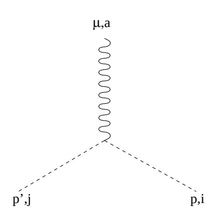

Figure 12.9: Equation 12.107.

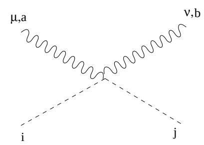

Figure 12.10: Equation 12.108.

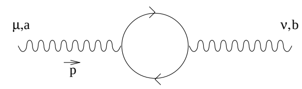

Figure 12.11: Equation 12.109.

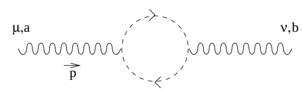

Figure 12.12: Equation 12.110.

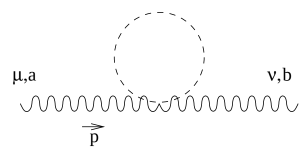

Figure 12.13: Equation 12.111.

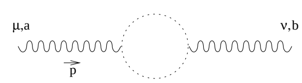

Figure 12.14: Equation 12.112.

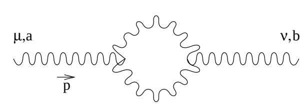

Figure 12.15: Equation 12.113.

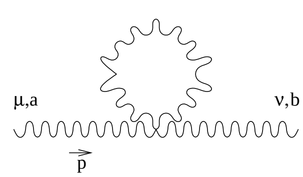

Figure 12.16: Equation 12.114.

- Note that inclusion of ghost contribution was crucial to get right tenser – structure $(p^2 g_{\mu\nu} - p_\mu p_\nu)$.
- We only evaluate log div. parts, since linear and quadratic divergence of the term $a\Lambda + b\lambda^2$ will not yield and contribution to β–function as β gets contributions from $\frac{\partial}{\partial\mu}\delta_1$ and $\frac{\partial}{\partial\mu}\delta_3$.

2. (a)

$$\xrightarrow{\log div.part} \quad -ig^3 f^{abc}[(k-p)^\rho g^{\mu_0} + (p-q)^\mu g^{\nu\rho} + (q-k)^\nu g^{\rho\mu}] \frac{1}{8}(13-9\alpha)C_2(G)\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.117}$$

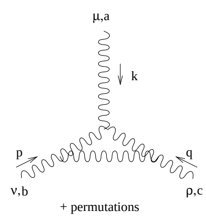

Figure 12.17: Equation 12.117.

$$\xrightarrow{\log part} \quad ig^3 f^{abc}[(k-p)^\rho g^{\mu_0} + (p-q)^\mu g^{\nu\rho} + (q-k)^\nu g^{\rho\mu}] \frac{1}{4}(9-\frac{3}{2}\alpha)C_2(G)\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.118}$$

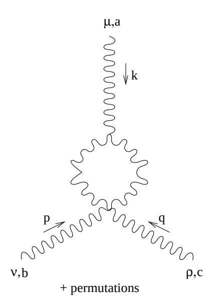

Figure 12.18: Equation 12.118.

$$\xrightarrow{\log div.part} \quad ig^3 f^{abc}[(k-p)^\rho g^{\mu_0} + (p-q)^\mu g^{\nu\rho} + (q-k)^\nu g^{\rho\mu}] \frac{1}{24}C_2(G)\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.119}$$

$$\overset{\log div.part}{\longrightarrow} \quad -ig^3 f^{abc}[(k-p)^\rho g^{\mu_0} + (p-q)^\mu g^{\nu\rho} + (q-k)^\nu g^{\rho\mu}] \\ \frac{4}{3} C_2(\Gamma_F) \frac{i}{2} \frac{\Omega_3}{(2\pi)^4} \log(\frac{\Lambda}{\mu})^2 \tag{12.120}$$

$$\overset{\log part}{\longrightarrow} \quad -ig^3 f^{abc}[(k-p)^\rho g^{\mu_0} + (p-q)^\mu g^{\nu\rho} + (q-k)^\nu g^{\rho\mu}] \\ \frac{1}{3} C_2(\Gamma_s) \frac{i}{2} \frac{\Omega_3}{(2\pi)^4} \log(\frac{\Lambda}{\mu})^2 \tag{12.121}$$

$$\overset{\log part}{\longrightarrow} 0 \tag{12.122}$$

Adding all up we get

$$-ig^3 f^{abc}\{C_2(G)(\frac{2}{3} + \frac{3}{4}\alpha) - \frac{4}{3} C_2(\Gamma_F) - \frac{1}{3} C_2(\Gamma_s)\} \\ [(k-p)^\rho g^{\mu_0} + (p-q)^\mu g^{\nu\rho} + (q-k)^\nu g^{\rho\mu}] \\ \frac{i}{2} \frac{\Omega_3}{(2\pi)^4} \log(\frac{\Lambda}{\mu})^2 \Rightarrow \frac{\delta_1}{\mu^2} \tag{12.123}$$

Therefore,

$$\beta = \mu \frac{\partial}{\partial \mu}(-\delta_1 + \frac{3}{2}\delta_3) \tag{12.124}$$

Therefore,

$$\beta_{3g} = \frac{\rho^3}{(4\pi)^2}[C_2(G)(-\frac{11}{3}) + C_2(\Gamma_F)\frac{4}{3} + C_2(\Gamma_s)\frac{1}{3}] \tag{12.125}$$

Note that α–dependence has cancelled out.

(b) Courtesy of Guide Festucchia

4 Gluons vertex: We have to evaluate the following diagrams

Using mathematica we obtain:

$$g^4 f^{fal} f^{ebg} f^{gch} f^{hdf} \int \frac{d^4K}{(2\pi)^4} \frac{1}{K^4} (\frac{K^\mu K^\nu K^\rho K^\sigma}{K^4}(34 - 12\alpha + 2\alpha^2) + \\ \frac{K^\mu K^\nu K^\rho K^\sigma + K^\mu K^\sigma g^{\rho\nu} + K^\rho K^\sigma g^{\nu\mu} + K^\nu K^\rho g^{\mu\sigma}}{K^2}(3 + 2\alpha - \alpha^2) +$$

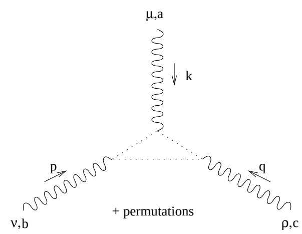

Figure 12.19: Equation 12.119.

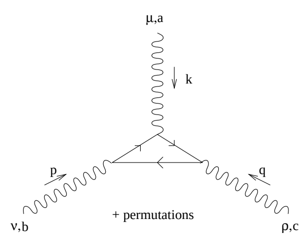

Figure 12.20: Equation 12.120.

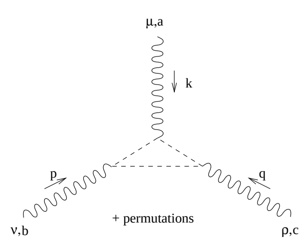

Figure 12.21: Equation 12.121.

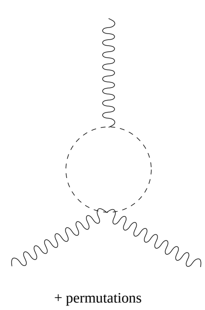

Figure 12.22: Equation 12.122.

$$
\begin{aligned}
& (g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho})(1 + 2\alpha + \alpha^2)) \\
= \quad & g^4 tr(t^a_G t^b_G t^c_G t^d_G)\frac{1}{24} \\
& ((g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho})(94 + 60\alpha + 14\alpha^2) + g^{\mu\rho}g^{\sigma\nu}(34 - 12\alpha + 2\alpha^2)) \\
& \int \frac{d^4K}{(2\pi)^4}\frac{1}{K^4} \qquad (12.126)
\end{aligned}
$$

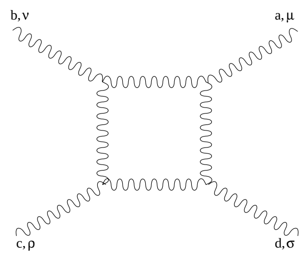

Figure 12.23: 4 Gluons Vertex (Equation 12.126).

The different permutations of the gluon vertices will be accounted for later.

$$
\begin{aligned}
& -g^4 tr(t^a_G t^b_G t^c_G t^d_G)\int \frac{d^4K}{(2\pi)^4}\frac{1}{K^4}(\frac{K^\mu K^\nu K^\rho K^\sigma}{K^4}(-3\alpha + \alpha^2) + \\
& K^\mu K^\nu g^{\rho\sigma}(-1 - 2\alpha - \alpha^2) + (K^\nu K^\rho g^{\mu\sigma} + K^\mu K^\sigma g^{\nu\rho})(3 + 3\alpha) + \\
& K^\rho K^\sigma g^{\mu\nu}(6 - 3\alpha - \alpha^2) + g^{\mu\nu}g^{\rho\sigma}(2 + 3\alpha + \alpha^2) + \\
& g^{\mu\rho}g^{\nu\sigma}(-2 - 2\alpha) + g^{\mu\sigma}g^{\nu\rho}(1 + \alpha))(c.\rho \leftrightarrow d, \sigma) \\
= \quad & -g^4 tr(t^a_G t^b_G t^c_G t^d_G)\frac{1}{24}((g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho})) \qquad (12.127) \\
= \quad & -\frac{1}{3}g^4(1 + 3\alpha)[(f^{ade}f^{bce} + f^{ace}f^{bde})g_{\mu\nu}g_{\rho\sigma} + \\
& (f^{ade}f^{cbe} + f^{abe}f^{cde})g_{\mu\rho}g_{\nu\sigma} + (f^{abe}f^{dce} + f^{ace}f^{dbe})g_{\mu\sigma}g_{\nu\rho}] \\
& \int \frac{d^4K}{(2\pi)^4}\frac{1}{K^4} \qquad (12.128) \\
= \quad & -\frac{1}{3}g^4(1 + 3\alpha)[f^{abe}f^{cde}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) + \\
& f^{ace}f^{bde}(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho}) + f^{ade}f^{bce}(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho})] \\
& \int \frac{d^4K}{(2\pi)^4}\frac{1}{K^4} \qquad (12.129) \\
& \qquad (12.130)
\end{aligned}
$$

This computation has been done with $\alpha \rightarrow -\alpha$

$$-\frac{1}{3}g^4(1-3\alpha)[f^{abe}f^{cde}(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho})+$$
$$f^{ace}f^{bde}(g_{\mu\nu}g_{\rho\sigma}-g_{\mu\sigma}g_{\nu\rho})+f^{ade}f^{bce}(g_{\mu\nu}g_{\rho\sigma}-g_{\mu\sigma}g_{\nu\rho})]$$
$$\int\frac{d^4K}{(2\pi)^4}\frac{1}{K^4} \tag{12.131}$$
$$\tag{12.132}$$

$$-g^4tr(t^at^bt^ct^d)\int\frac{d^4K}{(2\pi)^4}\frac{K_\alpha K_\beta K_\gamma K_\lambda}{K^4}tr[\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\gamma^\delta\gamma^\rho\gamma^\lambda\gamma^\sigma]+$$
$$(color\ trace\ reversed)$$
$$= -g^4tr(t^at^bt^ct^d)\frac{1}{24}(tr[\gamma^\alpha\gamma^\mu\gamma_\alpha\gamma^\nu\gamma^\beta\gamma^\rho\gamma_\beta\gamma^\sigma]+tr[\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\gamma_\alpha\gamma^\rho\gamma_\beta\gamma^\sigma]+$$
$$tr[\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\gamma_\beta\gamma^\rho\gamma_\alpha\gamma^\sigma])\int\frac{d^4K}{(2\pi)^4}\frac{1}{K^4} \tag{12.133}$$
$$= -\frac{8}{3}g^4tr(t^at^bt^ct^d)(g^{\mu\gamma}g^{\rho\sigma}-2g^{\sigma\gamma}g^{\mu\rho}+g^{\mu\sigma}g^{\rho\gamma})+$$
$$(\mu,a\leftrightarrow\gamma,b)+(\mu,a\leftrightarrow\sigma,d)+(color\ trace\ reversed) \tag{12.134}$$

This has exactly the same structure as before so the result is

$$-\frac{4}{3}g^4[f^{abe}f^{cde}(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho})+$$
$$f^{ace}f^{bde}(g_{\mu\nu}g_{\rho\sigma}-g_{\mu\sigma}g_{\nu\rho})+f^{ade}f^{bce}(g_{\mu\nu}g_{\rho\sigma}-g_{\mu\sigma}g_{\nu\rho})]$$
$$\int\frac{d^4K}{(2\pi)^4}C_2(\Gamma_F) \tag{12.135}$$
$$\tag{12.136}$$

$$= 32g^4tr(t^at^bt^ct^d)\int\frac{d^4K}{(2\pi)^4}\frac{K_\mu K_\nu K_\rho K_\sigma}{K^8}$$
$$= \frac{4}{3}g^4tr(t^at^bt^ct^d)(g^{\mu\nu}g^{\rho\sigma}+g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho})$$
$$\int\frac{d^4K}{(2\pi)^4}\frac{1}{K^4}+permutations \tag{12.137}$$

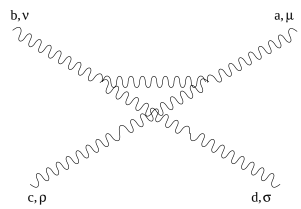

Figure 12.24: Equation 12.127.

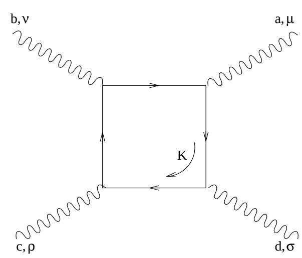

Figure 12.25: Equation 12.133.

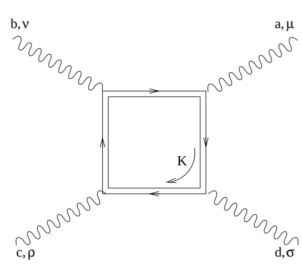

Figure 12.26: Equation 12.137.

$$= -8g^4 tr(\{t^a t^b\} t^c t^d) g^{\mu\nu} \int \frac{d^4K}{(2\pi)^4} \frac{K_\rho K_\sigma}{K^6}$$
$$= -2g^4 tr(\{t^a t^b\} t^c t^d) g^{\mu\nu} g^{\rho\sigma} \int \frac{d^4K}{(2\pi)^4} \frac{1}{K^4} + permutations \quad (12.138)$$

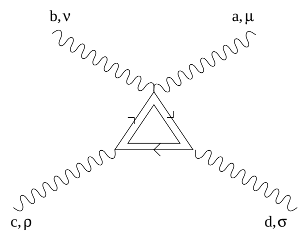

Figure 12.27: Equation 12.138.

$$= g^4 tr(\{t^a t^b\}\{t^c t^d\}) g^{\mu\nu} g^{\rho\sigma} \int \frac{d^4K}{(2\pi)^4} \frac{1}{K^4} + permutations \quad (12.139)$$

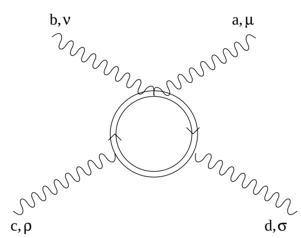

Figure 12.28: Equation 12.139.

Summing the last three graphs together and permuting the vertices we obtain:

$$\frac{1}{3} g^4 [g^{\mu\nu} g^{\rho\sigma} tr(-2t^a t^b t^c t^d + 4t^d t^b t^c t^a - 2t^b t^a t^c t^d) +$$
$$g^{\mu\rho} g^{\nu\sigma} tr(-2t^a t^c t^b t^d + 4t^d t^c t^b t^a - 2t^c t^a t^b t^d) +$$
$$g^{\mu\sigma} g^{\nu\rho} tr(-2t^a t^b t^d t^c + 4t^c t^b t^d t^a - 2t^b t^a t^d t^c)] \int \frac{d^4K}{(2\pi)^4} \frac{1}{K^4} \quad (12.140)$$

Now using

$$\begin{aligned} & tr(-2t^a t^b t^c t^d + 4t^d t^b t^c t^a - 2t^b t^a t^c t^d) \\ = \; & tr([a,d][b,c] + [a,c][b,d]) \qquad (12.141) \\ = \; & -C_2(G)[f^{ade} f^{bce} + f^{ace} f^{bde}] \qquad (12.142) \end{aligned}$$

and similar relocations we obtain:

$$-\frac{1}{3} g^4 [f^{abe} f^{cde} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) + f^{ace} f^{bde} (g_{\mu\nu} g_{\rho\sigma} - g_{\mu\sigma} g_{\nu\rho}) + f^{ade} f^{bce} (g_{\mu\nu} g_{\rho\sigma} - g_{\mu\sigma} g_{\nu\rho})] \int \frac{d^4K}{(2\pi)^4} \frac{1}{K^4} C_2(\Gamma_B) \qquad (12.143)$$

All the calculations sum to:

$$\begin{aligned} & g^4 [f^{abe} f^{cde} (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho}) + f^{ace} f^{bde} (g_{\mu\nu} g_{\rho\sigma} - g_{\mu\sigma} g_{\nu\rho}) \\ & + f^{ade} f^{bce} (g_{\mu\nu} g_{\rho\sigma} - g_{\mu\sigma} g_{\nu\rho})] \int \frac{d^4K}{(2\pi)^4} \frac{1}{K^4} \\ & \{-\frac{1}{3}(1-3\alpha) C_2(G) - \frac{4}{3} C_2(\Gamma_F) - \frac{1}{3}(\Gamma_B)\} \qquad (12.144) \end{aligned}$$

$$\begin{aligned} \beta \; = \; & (\frac{1}{2})^* \frac{2g^3}{(4\pi)^2} \\ & \{[-\frac{1}{3}(1-3\alpha) - \frac{4}{2}(\frac{5}{3} + \frac{\alpha}{2})] C_2(G) + \frac{4}{3} C_2(\Gamma_F)(-1+2) + \\ & \frac{1}{3} C_2(\Gamma_B)(-1+2)\} \qquad (12.145) \\ = \; & (-\frac{11}{3} C_2(G) + \frac{4}{3} C_2(\Gamma_F) + \frac{1}{3} C_2(\Gamma_B)) \frac{2g^3}{(4\pi)^2} \qquad (12.146) \end{aligned}$$

* $\frac{1}{2}$ comes from the fact that the vertex appears at color g^2 2 from the log div. of $\ln(\frac{1}{\pi^2})$ with respect to π.

3. (a)

$$\xrightarrow{\log} g^3 \gamma^\mu t^a (1-\alpha)[(2(\Gamma_F) - \frac{1}{2} C_2(G))] \frac{i}{2} \frac{\Omega_3}{(2\pi)^4} \log(\frac{\Lambda}{\mu})^2 \qquad (12.147)$$

$$\xrightarrow{\log} g^3 C_2(G) \gamma^\mu t^a (\frac{3}{2} - \frac{3}{4}\alpha)[(2(\Gamma_F) - \frac{1}{2} C_2(G))] \frac{i}{2} \frac{\Omega_3}{(2\pi)^4} \log(\frac{\Lambda}{\mu})^2 \qquad (12.148)$$

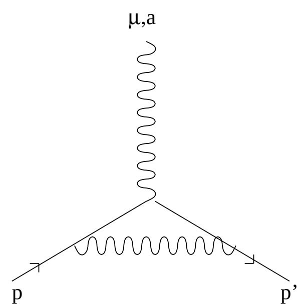

Figure 12.29: Equation 12.147.

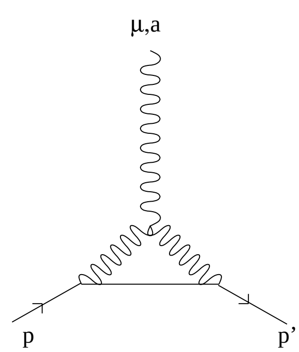

Figure 12.30: Equation 12.148.

From these read off δ_1^f

$$\xrightarrow{\log} C_2(\Gamma_F)g^3 \not{p}(1-\alpha)[(2(\Gamma_F) - \frac{1}{2}C_2(G))]\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.149}$$

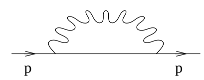

Figure 12.31: Equation 12.149.

Read off δ_2^f

$$\beta_F = \mu\frac{\partial}{\partial\mu}(-\delta_1^f + \delta_2^f + \frac{\delta_3}{2}) \tag{12.150}$$

Same as β_{3g}.

(b)

$$\xrightarrow{\log} -g^2 C_2(\Gamma_F)(2+\alpha)p^2\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.151}$$

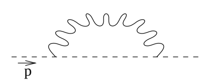

Figure 12.32: Equation 12.151.

$$\xrightarrow{\log} 0 \tag{12.152}$$

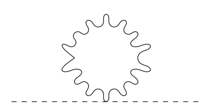

Figure 12.33: Equation 12.152.

Read off δ_2^s

$$\xrightarrow{\log} g^3 t^a (C_2(\Gamma_s) - \frac{1}{2}C_2(G))(p+p')^\mu(1-\alpha)\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.153}$$

$$\xrightarrow{\log} g^3 t^a C_2(G)\frac{3}{4}(1-\alpha)(p+p')^\mu\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.154}$$

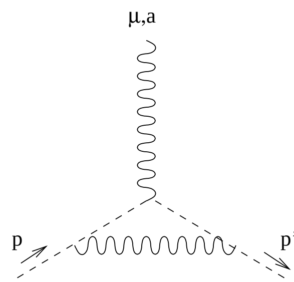

Figure 12.34: Equation 12.153.

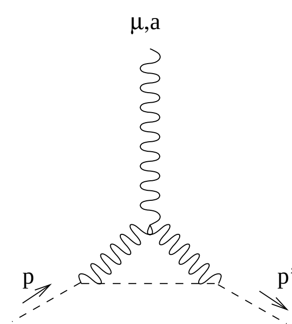

Figure 12.35: Equation 12.154.

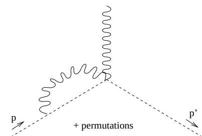

Figure 12.36: Equation 12.155.

$$\xrightarrow{\log} -g^3 t^a (p+p')^\mu \frac{3}{2}(2C_2(\Gamma_s) - \frac{1}{2}C_2(G))\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.155}$$

From last 4 graphs read off δ_1^s

$$\xrightarrow{\log} 0 \tag{12.156}$$

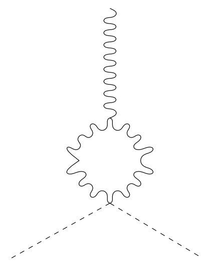

Figure 12.37: Equation 12.156.

Therefore,

$$\beta_s = \mu\frac{\partial}{\partial\mu}(-\delta_1^s + \delta_2^s + \frac{\delta_3}{2}) \tag{12.157}$$

Same as β_{3g}.

(c)

$$\xrightarrow{\log} ig^2\frac{1}{8}C_2(G)(1-\alpha)(p+p')^\mu f^{abc}\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.158}$$

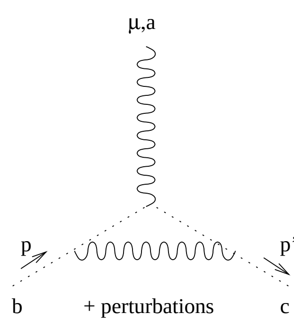

Figure 12.38: Equation 12.158.

$$ig^3\frac{3}{8}C_2(G)(1-\alpha)(p+p')^\mu f^{abc}\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.159}$$

Read off δ_1^g from there.

$$-g^2(\frac{1}{2}+\frac{\alpha}{4})C_2(G)\delta_{bc}p^2\frac{i}{2}\frac{\Omega_3}{(2\pi)^4}\log(\frac{\Lambda}{\mu})^2 \tag{12.160}$$

Read off δ_2^g from this.
Therefore,

$$\beta_g = \mu\frac{\partial}{\partial\mu}(-\delta_1^g+\delta_2^g+\frac{\delta_3}{2}) \tag{12.161}$$

Same as β_{3g}.
Looking back we see that

$$\delta_1^s-\delta_2^s=\delta_1^f-\delta_2^f=\delta_1^g-\delta_2^g==\delta_1^{3g}-\delta_3=\frac{1}{2}(\delta_1^{4g}-\delta_3) \tag{12.162}$$

These relations show the universality at the β–function coefficients which is a direct consequence of gauge invariance. One could have proven these relations directly from the renormalized Lagrangian.

4. In light of problem 3, the easiest way to compute the β–function is

   (a) Set $\alpha=0$ (since β is independent of α).
   (b) Compute $\delta_1^g-\delta_2^g$ (the ghost contribution).
   (c) Compute δ_3.

   Therefore,

$$\beta = \frac{\partial}{\partial\mu}(-\delta_1^g+\delta_2^g+\frac{\delta_3}{2}) \tag{12.163}$$

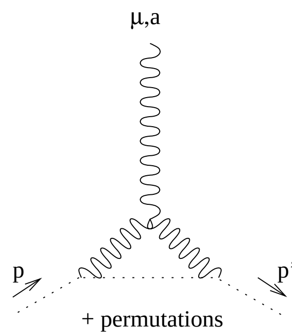

Figure 12.39: Equation 12.159.

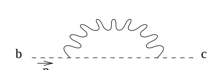

Figure 12.40: Equation 12.160.