Prof. Wolfgang Ketterle

Assignment #9

Due: Wednesday, May 1, 2013

Consider the following questions for Na. Try to plug in numbers after obtaining an analytical expression.

Na oven temperature $T = 600K$

natural line width $\gamma = 2\pi\times$ 10 MHz

Zeeman splitting 1.4 MHz/gauss

wavelength $\lambda = 2\pi/k = 589$ nm

1. **A Zeeman Slower**

If you want to slow an atomic beam efficiently, you have to compensate for the changing Doppler shift ($\vec{k} \cdot \vec{v}$) during the deceleration. This can be done by sweeping the frequency of the slowing beam, i.e., chirping (as discussed in class). A method of producing a continuous beam of cold atoms is Zeeman slowing.

A well collimated beam of atoms is originating from an oven with a temperature T. The beam propagates along a distance L with a longitudinal magnetic field $B(x)$ ($0 < x < L$). A laser beam of intensity I is counter propagating. Its frequency is detuned by δ ($\delta \equiv \omega - \omega_0$) from the transition frequency at $B = 0$.

a) Calculate the maximum deceleration a_{max} you can achieve. Assume you could choose arbitrarily large laser intensities.

Assume you want to slow down atoms with speeds lower than the peak (most probable) velocity v_{peak} of the thermal distribution to a stand still using the constant deceleration fa_{max}. ($0 < f < 1$) The Zeeman effect shifts the resonance $\omega_0 \to \omega_0 + g\mu_B B(x)$ (where $g\mu_B = (2\pi)1.4$ MHz/gauss, in this case).

b) Calculate the spatial dependence of the magnetic field $B(x)$ and the length L of the slower as a function of f.

c) Assume three different models for spontaneous emission: (i) Photons are emitted along the $+/-\hat{x}$ direction, (ii) isotropic emission, or (iii) emission in a dipole pattern. What is the momentum diffusion constant $\mathcal{D}$ for the (longitudinal) x-component of the momentum in the three cases, at a given photon scattering rate Γ_s?

2. **Slowing an atom with off-resonant light**

   Assume you want to slow an atom of velocity v_{peak} with a counter propagating laser beam that is on resonance with the atom at rest. (Use the same v_{peak} as in Problem 1, and $I = 5I_{sat}$.)

   a) How long would it take?

   b) How far would the atom travel?

   (Hint: Think about integrating the equation of motion)

3. **Density Limit in a MOT**. In a 3-D magneto-optical trap, the density of the trapped atoms is limited by a net outward radiation pressure which opposes the trapping force. We can divide the density-dependent photon-pressure force into two parts. First there is a repulsive 'radiation trapping force' due to atoms absorbing photons scattered from other atoms in the trap. Also, there is an attractive 'attenuation force' which is caused by atoms at the side of the cloud attenuating the laser beams, thus creating an intensity imbalance which leads to an inward force.

   (a) Show that the radiation trapping force obeys the equation

   $$\nabla \cdot \mathbf{F}_R = \frac{6\sigma_L \sigma_R I n}{c},$$

   where $\mathbf{I}$ is the intensity of one of the trapping laser beams, n is the number density of atoms in the cloud, σ_L is the cross-section for absorption of the laser beam, and σ_R is the cross-section for absorption of the scattered light. (**Hint**: Find the magnitude of the force between two atoms separated by a distance d, where one atom re-radiates a laser photon and the second atom absorbs it. Now, since this is an inverse-square force, you can use Gauss' law to find $\nabla \cdot \mathbf{F}_R$. Assume that photons are only scattered twice.)

   (b) The attenuation force may be obtained simply by replacing σ_R with $-\sigma_L$, so that

   $$\nabla \cdot \mathbf{F}_A = -\frac{6\sigma_L^2 I n}{c}.$$

   Explain why this is so.

   (c) The total force is the sum of $\mathbf{F}_R, \mathbf{F}_A$, and the trapping force $\mathbf{F}_T = -\kappa \mathbf{r}$, where κ is the spring constant of the trap. For stability we require that the total force is attractive, $\nabla \cdot \mathbf{F}_{total} < 0$. Find the maximum density of the trapped cloud at a given κ from the condition $\nabla \cdot \mathbf{F}_{total} = 0$.

   (d) Give a qualitative argument for whether we expect $\sigma_R = \sigma_L, \sigma_R > \sigma_L$, or $\sigma_R < \sigma_L$. (**Hint**: sketch the absorption and emission spectra for an atom in a strong laser field with a red detuning.)

   (e) Suppose that some of the atoms can be put into a 'dark state', so that only a fraction f of the atoms absorb the trapping light. How do F_R, F_A and F_T vary with f? What happens to the limiting density n_{max}? This is the concept of the Dark SPOT trap (PRL **70**, 2253 (1993)).

8.422 Atomic and Optical Physics II
Spring 2013