Excitations.

In the Ising AF phase, excitations carry **fractional** spin. This can be illustrated with a simple cartoon.

Consider flipping a single spin in the ground state

$\uparrow \downarrow \uparrow \downarrow \; \dots \; \uparrow \downarrow \uparrow \downarrow$ = ground

$\uparrow \downarrow \uparrow \downarrow \downarrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$ = state with one spin flipped.

This state has $S^z_{tot} = -1$ relative to gd state.

Now allow the J term to exchange some spins so that this configuration evolves. After one such exchange, can get

$\uparrow \downarrow \downarrow \uparrow \downarrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow$

$\xrightarrow{\text{one more exch.}}$ $\uparrow \downarrow \downarrow \uparrow \downarrow \uparrow \downarrow \downarrow \uparrow \downarrow \uparrow \downarrow$

one more exchange

$\uparrow \downarrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \downarrow \uparrow \downarrow$

The single flipped spin has now split into 2 separate objects each one of which is ~~a~~ domain wall in the AF ordering pattern.

As the 2 separate domain walls are equivalent, each of them must carry $\frac{1}{2}$ of the flipped spin

$\Rightarrow$ Each domain wall carries $S_z = \frac{1}{2}$.

Thus the domain walls carry fractional spins!

(Exactly similar results apply in the fermion description as well).

Similar fractionalized spin-$\frac{1}{2}$ excitations exist even in the gapless isotropic AF spin $\frac{1}{2}$ chain & may ~~are~~ loosely be understood as "domain walls". ~~[illegible]~~

What about antiferromagnets in $d > 1$?

For $d \geq 2$ on a square or cubic lattice, the large-S (spin-wave) expansion is convergent.

$\Rightarrow$ Neel ordered ground state is stable.

[May also be seen from the $O(3)$ non-linear sigma model field theory (valid for smooth configurations of the Neel field)]

The nearest neighbour spin-S antiferromagnets

$$H = J \sum_{\langle rr' \rangle} \vec{S}_r \cdot \vec{S}_{r'}$$

on $d = 2, 3$ square/cubic lattices are known to be in the Neel ordered phase.

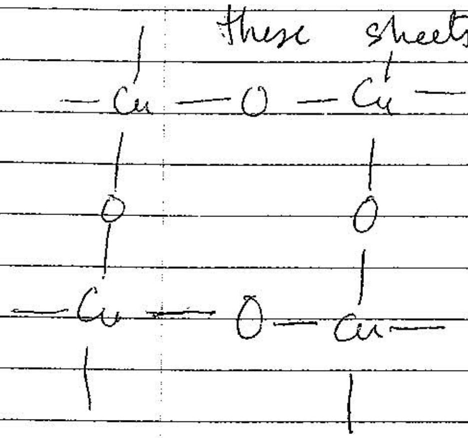
$S = 1/2$ model on $d = 2$ square lattice: Good model for

La_2CuO_4 = parent of hi T_c cuprates.

Brief introduction to hiTc problem

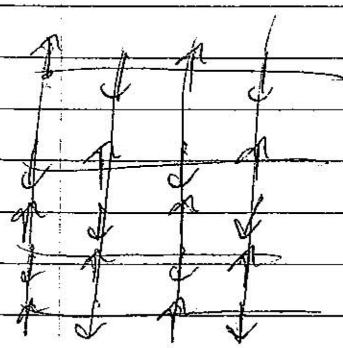
Materials: $La_{2-x}Sr_xCuO_4$, $YBa_2Cu_3O_{6+x}$, etc.

Layers of Cu-O sheets - important action is in these sheets.



Eg. La_2CuO_4 : 1 spin-$\frac{1}{2}$ hole per Cu (corresponding to $3d^9$ configuration).

Due to strong U, get Mott insulator with AF spin exchange $J \approx 1500\,K$ (very large as compared to most other Mott insulators).

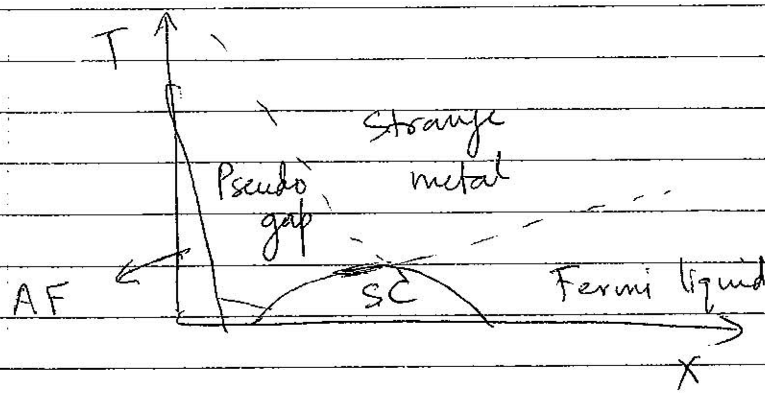


Sr doping removes electrons from the Cu-O layer $\Rightarrow$ remove spins to introduce holes

Very small hole concentration (2% or so) enough to destroy long range Néel order.

At somewhat higher doping, superconductivity appears.

Phase diagram



T_c is max for $x \simeq 15\%$ & then goes down.

Behaviour in the non-superconducting state above T_c is most interesting - there are 3 regimes which roughly depend on whether $x \lesssim x_{opt}$ ("underdoped"), $x \simeq x_{opt}$ ("optimal") or $x \gtrsim x_{opt}$ ("overdoped").

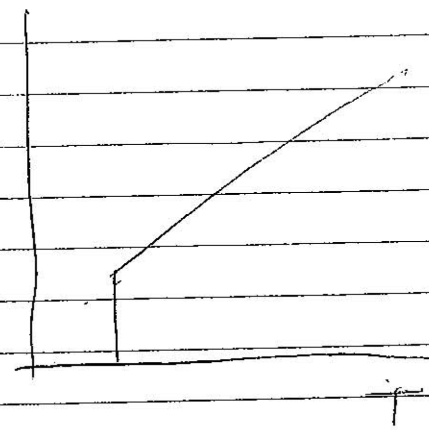
~~Overdo~~ In all 3 cases get metallic transport.

Overdoped: Not that well characterized but seems generally consistent with Fermi Liquid theory.

Optimally doped: Most interesting - metal unlike ~~anything so~~ any other conventional one.

("dubbed "strange metal")

Eg: Electrical resistivity $\rho(T)$



In some hiTc materials $\rho(T) \sim T$

for $10\,K \lesssim T \lesssim 700\,K$!

Simply stated law that has defied any sensible explanation for ≈ 17 years!

(In a Fermi liquid, $\rho(T) \simeq \rho_0 + AT^2$ at low T)

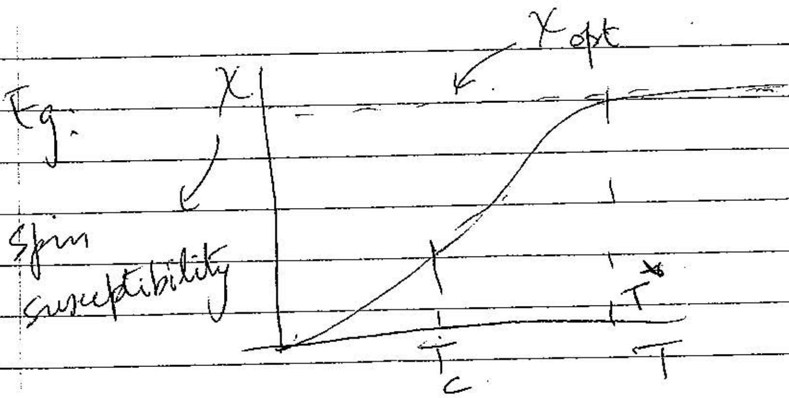
More detailed experiments (in particular "angle-resolved photoemission) show reasonably well-formed Fermi surface - but no sharply defined electron-like quasiparticles at this Fermi surface.

Theory of this metal - major challenge in many body physics.

Underdoped: Another strange metal known as the pseudogap state.

Many physical quantities are suppressed in this region as compared to their values at optimal doping.

Eg.



Suggests development of a gap in the spin excitation spectrum.

Similar suppression of specific heat, tunneling density of states, etc.

No fully accepted theory of pseudogap state exists but some basic picture seems clear - proximity to Mott insulator seems to play a big role.

[Most of the electrons are ~~[illegible]~~ localized most of the time - superexchange locks their spins into singlets]