

2 Introduction to SCET

2.1 What is SCET?

The Soft-Collinear Effective Theory is an effective theory describing the interactions of soft and collinear degrees of freedom in the presence of a hard interaction. We will refer to the momentum scale of the hard interaction as Q . For QCD another important scale is Λ_{QCD} , the scale of hadronization and nonperturbative physics, and we will always take $Q \gg \Lambda_{\text{QCD}}$.

Soft degrees of freedom will have momenta p_{soft} , where $Q \gg p_{\text{soft}}$. They have no preferred direction, so each component of p_{soft}^μ for $\mu = 0, 1, 2, 3$ has an identical scaling. Sometimes we will have $p_{\text{soft}} \sim \Lambda_{\text{QCD}}$ so that the soft modes are nonperturbative (as in HQET for B or D meson bound states) and sometimes we will have $p_{\text{soft}} \gg \Lambda_{\text{QCD}}$ so that the soft modes have components that we can calculate perturbatively.

Collinear degrees of freedom describe energetic particles moving preferentially in some direction (here motion collinear to a direction means motion near to but not exactly along that direction). In various situations the collinear degrees of freedom may be the constituents for one or more of

- energetic hadrons with $E_H \simeq Q \gg \Lambda_{\text{QCD}} \sim m_H$,
- energetic jets with $E_J \simeq Q \gg m_J = \sqrt{p_J^2} \gg \Lambda_{\text{QCD}}$.

Both the soft and collinear particles live in the infrared, and hence are modes that are described by fields in SCET. Here we characterize infrared physics in the standard way, by looking at the allowed values of invariant mass p^2 and noting that all offshell fluctuations described by SCET degrees of freedom have $p^2 \ll Q^2$. Thus SCET is an EFT which describes QCD in the infrared, but allows for both soft homogeneous and collinear inhomogeneous momenta for the particles, which can have different dominant interactions. The main power of SCET comes from the simple language it gives for describing interactions between hard \leftrightarrow soft \leftrightarrow collinear particles.

Phenomenologically SCET is useful because our main probe of short distance physics at Q is hard collisions: $e^+e^- \rightarrow \text{stuff}$, $e^-p \rightarrow \text{stuff}$, or $pp \rightarrow \text{stuff}$. To probe physics at Q we must disentangle the physics of QCD that occurs at other scales like Λ_{QCD} , as well as at the intermediate scales like m_J that are associated with jet production. This process is made simpler by a separation of scales, and the natural language for this purpose is effective field theory. Generically in QCD a separation of scales is important for determining what parts of a process are perturbative with $\alpha_s \ll 1$, and what parts are nonperturbative with $\alpha_s \sim 1$. For some examples this is fairly straightforward, there are only two relevant momentum regions, one which is perturbative and the other nonperturbative, and we can separate them with a fairly standard operator expansion. But many of the most interesting hard scattering processes are not so simple, they involve either multiple perturbative momentum regions, or multiple nonperturbative momentum regions, or both. In most cases where we apply SCET we will be interested in two or more modes in the effective theory, such as soft and collinear, and often even more modes, such as soft modes together with two distinct types of collinear modes.

Part of the power of SCET is the plethora of processes that it can be used to describe. Indeed, it is not really feasible to generate a complete list. New processes are continuously being analyzed on a regular basis. Some example processes where SCET simplifies the physics include

- inclusive hard scattering processes: $e^-p \rightarrow e^-X$ (DIS), $p\bar{p} \rightarrow Xl^+l^-$ (Drell-Yan), $pp \rightarrow HX$, ... (either for the full inclusive process or for threshold resummation in the same process)

- exclusive jet processes: dijet event shapes in $e^+e^- \rightarrow$ jets, $pp \rightarrow H + 0$ -jets, $pp \rightarrow W + 1$ -jet, $e^-p \rightarrow e^- + 1$ -jet, $pp \rightarrow$ dijets, ...
- exclusive hard scattering processes: $\gamma^*\gamma \rightarrow \pi^0$, $\gamma^*p \rightarrow \gamma^{(*)}p'$ (Deeply Virtual Compton), ...
- inclusive B-decays: $B \rightarrow X_s\gamma$, $B \rightarrow X_u\ell\bar{\nu}$, $B \rightarrow X_s\ell^+\ell^-$
- exclusive B-decays: $B \rightarrow D\pi$, $B \rightarrow \pi\ell\bar{\nu}_\ell$, $B \rightarrow K^*\gamma$, $B \rightarrow \pi\pi$, $B \rightarrow K^*K$, $B \rightarrow J/\psi K$, ...
- Charmonium production: $e^+e^- \rightarrow J/\psi X$, ...
- Jets in a Medium in heavy-ion collisions

Some of these examples combine SCET with other effective theories, such as HQET for the B -meson, or NRQCD for the J/ψ .

Before we dig in, it is useful to stop and ask **What makes SCET different from other EFT's?** Put another way, what are some of the things that make it more complicated than more traditional EFTs? Or another way, for the field theory aficionado, what are some of the interesting new techniques I can learn by studying this EFT? A brief list includes:

- We will integrate off-shell modes, but not entire degrees of freedom. (This is analogous to HQET where low energy fluctuations of the heavy quark remain in the EFT.)
- Having multiple fields that are defined for the same particle

$$\xi_n = \text{collinear quark field}, \quad q_s = \text{soft quark field}$$

which are required by power counting and to cleanly separate momentum scales.

- In traditional EFT we sum over operators with the same power counting and quantum numbers. In SCET some of these sums are replaced by convolutions, $\sum_i C_i \mathcal{O}_i \rightarrow \int d\omega C(\omega) \mathcal{O}(\omega)$.
- λ , the power counting parameter of SCET, is not related to the mass dimensions of fields
- Various Wilson Lines, which are path-ordered line integrals of gauge fields, $P \exp[ig \int ds n \cdot A(ns)]$, play an important role in SCET. Some appear from integrating out offshell modes, others from dynamics in the EFT, and all are related to the interesting gauge symmetry structure of the effective theory.
- There are $1/\epsilon^2$ divergences at 1-loop which require UV counterterms. This leads to explicit $\ln(\mu)$ dependence in anomalous dimensions related to the so-called cusp anomalous dimensions, and to renormalization group equations whose solutions sum up infinite series of Sudakov double logarithms, $\sum_k a_k [\alpha_s \ln^2(p/Q)]^k$.

2.2 Light-Cone Coordinates

Before we get into concepts, which should decide on convenient coordinates. To motivate our choice, consider the decay process $B \rightarrow D\pi$ in the rest frame of the B meson. This decay occurs through the exchange of a W boson mediating $b \rightarrow c\bar{u}d$, along with a valence spectator quark that starts in the B and ends in the D meson. We are concerned here with the kinematics. Aligning the π with the $-\hat{z}$ axis it is easy to work out the pion's four momentum for this two-body decay,

$$p_\pi^\mu = (2.310 \text{ GeV}, 0, 0, -2.306 \text{ GeV}) \simeq Qn^\mu, \quad (2.1)$$

where $n^\mu = (1, 0, 0, -1)$ in a $0, 1, 2, 3$ basis for the four vector. Here $n^2 = 0$ is a light-like vector and $Q \gg \Lambda_{\text{QCD}}$. This pion has large energy and has a four-momentum that is close to the light-cone. With a slight abuse of language we will often say that the pion is moving in the direction n (even though we really mean the direction specified by the $1, 2, 3$ components of n^μ). The natural coordinates for particles whose energy is much larger than their mass are light-cone coordinates.

We would like to be able to decompose any four vector p^μ using n^μ as a basis vector. But unlike cartesian coordinates the component along n will not be $n \cdot p$, since $n^2 = 0$. If we want to describe the components (we do) then we will need another auxillary light-like vector \bar{n} . The vector n has a physical interpretation, we want to describe particles moving in the n direction, whereas \bar{n} is simply a devise we introduce to have a simple notation for components.

Thus we start with light-cone basis vectors n and \bar{n} which satisfy the properties

$$n^2 = 0, \quad \bar{n}^2 = 0, \quad n \cdot \bar{n} = 2, \quad (2.2)$$

where the last equation is our normalization convention. A standard choice, and the one we will most often use, is to simply take \bar{n} in the opposite direction to n . So for example we might have

$$n^\mu = (1, 0, 0, 1), \quad \bar{n}^\mu = (1, 0, 0, -1) \quad (2.3)$$

Other choices for the auxillary vector work just as well, e.g. $n^\mu = (1, 0, 0, 1)$ with $\bar{n}^\mu = (3, 2, 2, 1)$, and later on this freedom in defining \bar{n} will be codified in a reparameterization invariance symmetry. For now we stick with the choice in Eq. (2.3).

It is now simple to represent standard 4-vectors in the light-cone basis

$$p^\mu = \frac{n^\mu}{2} \bar{n} \cdot p + \frac{\bar{n}^\mu}{2} n \cdot p + p_\perp^\mu \quad (2.4)$$

where the \perp components are orthogonal to both n and \bar{n} . With the choice in Eq. (2.3), $p_\perp^\mu = (0, p^1, p^2, 0)$. It is customary to represent a momentum in these coordinates by

$$p^\mu = (p^+, p^-, \vec{p}_\perp) \quad (2.5)$$

where the last entry is two-dimensional, and the minkowski p_\perp^2 is the negative of the euclidean \vec{p}_\perp^2 (ie. in our notation $p_\perp^2 = -\vec{p}_\perp^2$). Here we have also defined

$$p^+ = p_+ \equiv n \cdot p, \quad p^- = p_- \equiv \bar{n} \cdot p. \quad (2.6)$$

As indicated the upper or lower \pm indices mean the same thing.

Using the standard $(+ - - -)$ metric, the four-momentum squared is

$$p^2 = p^+ p^- + p_\perp^2 = p^+ p^- - \vec{p}_\perp^2. \quad (2.7)$$

We can also decompose the metric in this basis

$$g^{\mu\nu} = \frac{n^\mu \bar{n}^\nu}{2} + \frac{\bar{n}^\mu n^\nu}{2} + g_\perp^{\mu\nu}. \quad (2.8)$$

Finally we can define an antisymmetric tensor in the \perp space by $\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} \bar{n}_\alpha n_\beta / 2$.

2.3 Momentum Regions: SCET I and SCET II

Lets continue with our exploration of the $B \rightarrow D\pi$ decay with the goal of identifying the relevant quark and gluon degrees of freedom (d.o.f.) for designing an EFT to describe this process. We'll then do the same for a process with jets.

There are different ways of finding the relevant infrared degrees of freedom. We could characterize all possible regions giving rise to infrared singularities at any order in perturbation theory using techniques like the Landau equations, and then determine the corresponding momentum regions. We could carry out QCD loop calculations using a technique known as the method of regions, where the full result is obtained by a sum of terms that enter from different momentum regions. Then by examining these regions we could hypothesize that there should be corresponding EFT degrees of freedom for those regions that appear to correspond to infrared modes that should be in the EFT. (Either of these approaches may be useful, but note that when using them we must be careful that the degrees of freedom are appropriate to our true physical situation, and do not contain artifacts related to our choice of perturbative infrared regulators that are not present in the true nonperturbative QCD situation.) Instead, our approach in this section will be based solely on physical insight of what the relevant d.o.f. are, from thinking through what is happening in the hard scattering process we want to study. More mathematical checks that one has the right d.o.f. are also desirable, and we will talk about some examples of how to do this later on. This falls under the rubric of not fully trusting a physics argument without the math that backs it up, and visa versa.

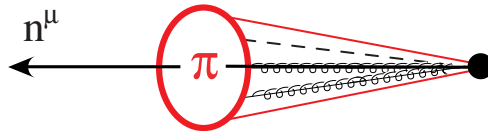
For $B \rightarrow D\pi$ in the rest frame of the B , the constituents of the B meson are the nearly static heavy b quark, and the soft quarks and gluons with momenta $\sim \Lambda_{\text{QCD}}$, ie. just the standard degrees of freedom of HQET. Since $|\vec{p}_D| = 2.31 \text{ GeV} \sim m_D = 1.87 \text{ GeV}$ the constituents of the D meson are also soft and described by HQET. The pion on the other hand is highly boosted. We can derive the momentum scaling of the pion constituents by starting with the $(+, -, \perp)$ scaling of

$$p^\mu \sim (\Lambda_{\text{QCD}}, \Lambda_{\text{QCD}}, \Lambda_{\text{QCD}}) \quad \text{for constituents in the pion rest frame,}$$

and then by boosting along $-\hat{z}$ by an amount $\kappa = Q/\Lambda_{\text{QCD}}$. The boost is very simple with light cone coordinates, taking $p^- \rightarrow \kappa p^-$ and $p^+ \rightarrow p^+/\kappa$. Thus

$$p_c^\mu \sim \left(\frac{\Lambda_{\text{QCD}}^2}{Q}, Q, \Lambda_{\text{QCD}} \right) \quad (2.9)$$

for the energetic pions constituents in the B rest frame. This scaling describes the typical momenta of the quarks and gluons that bind into the pion moving with large momentum $p_\pi^\mu = (0, Q, 0) + \mathcal{O}(m_\pi^2/Q)$, as in



The important fact about Eq. (2.9) is that

$$p_c^- \gg p_c^\perp \gg p_c^+ . \quad (2.10)$$

Whenever the components of p_c^μ obey this hierarchy we say it has a **collinear** scaling. Its convenient to describe this collinear scaling with a dimensionless parameter by writing

$$p_c^\mu \sim Q(\lambda^2, 1, \lambda) \quad (2.11)$$

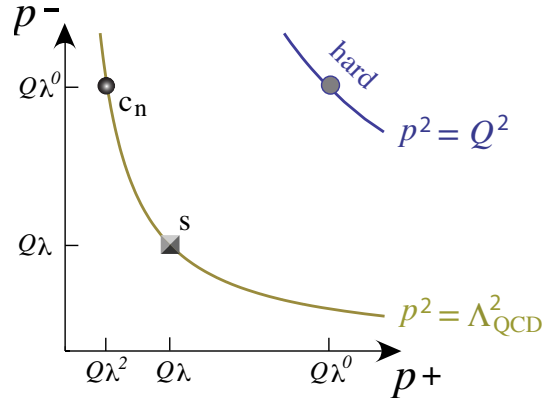


Figure 1: SCET_{II} example. Relevant degrees of freedom for $B \rightarrow D\pi$ with an energetic pion in the B rest frame.

where $\lambda \ll 1$ is a small parameter. This result is generic. For our $B \rightarrow D\pi$ example we have $\lambda = \Lambda_{\text{QCD}}/Q$.¹ This λ will be the power counting parameter of SCET. With this notation we can also say how the soft momenta of constituents in the B and D meson scale,

$$p_s^\mu \sim Q(\lambda, \lambda, \lambda). \quad (2.12)$$

Thus we see that we need both soft and collinear degrees of freedom for the $B \rightarrow D\pi$ decay.

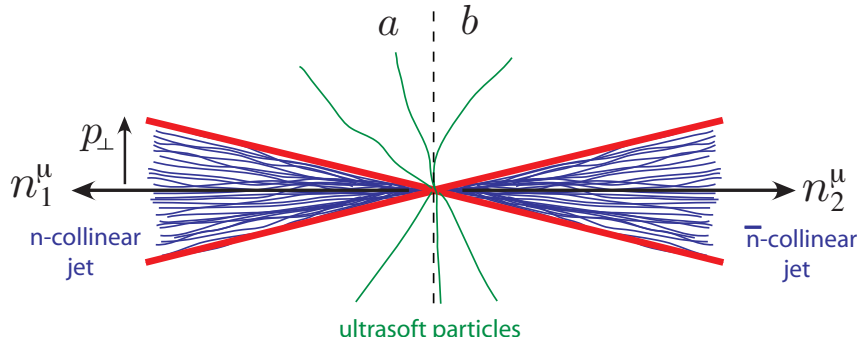
It is convenient to represent the degrees of freedom with a picture, as in Fig. 1. This picture has some interesting features. Unlike simpler effective theories SCET requires at least two variables to describe the d.o.f. The choice of p^- and p^+ as the axis here suffices since the \perp -momentum satisfies $p_\perp^2 \sim p^+p^-$ and hence does not provide additional information. The hyperbolas in the figures are lines of constant $p^2 = p^+p^-$. The labelled spots indicate the relevant momentum regions. We have included a hyperbola and a spot for the hard region where $p^2 \sim Q^2$, but these are the modes that are actually integrated out when constructing SCET. (For $B \rightarrow D\pi$ they are fluctuations of order the heavy quark masses.) On the $p^2 \sim \Lambda_{\text{QCD}}^2$ hyperbola in Fig. 1 we have two types of nonperturbative modes, collinear modes c_n for the pion constituents, and soft modes s for the B and D meson constituents. Since these modes live at the same typical invariant mass p^2 we need another variable, namely p^-/p^+ , to distinguish them. This variable is related to the rapidity, Y , since $e^{2Y} = p^-/p^+$. Put another way, we need both of the variables p^+ and p^- to define the modes for the EFT.

The example in Fig. 1 is what is known as an SCET_{II} type theory. Its defining characteristic is that the soft and collinear modes in the theory have the same scaling for p^2 , they live on the same hyperbola. This type of theory turns out to be appropriate for a wide variety of different processes and hence we give it the generic name SCET_{II}. Essentially this version of SCET is the appropriate one for hard processes which produce energetic identified hadrons, what we earlier called exclusive hard scattering and exclusive B-decays.

¹Please do not be confused into thinking that you need to assign a precise definition to λ . It is only used as a scaling parameter to decide what operators we keep and what terms we drop in the effective field theory, so any definition which is equivalent by scaling is equally good. In the end any predictions we make for observables do not depend on the numerical value of λ . The only time we need a number for λ is when making a numerical estimate for the size of the terms that are higher order in the power expansion which we've dropped.

When looking at Fig. 1 we should interpret the collinear degrees of freedom as living mostly in a region about the c_n spot and the soft degrees of freedom as living mostly in a region about the s spot. An obvious question is what determines the boundary between these degrees of freedom. In a Wilsonian EFT the answer would be easy, there would be hard cutoffs that carve out the regions defined by these modes. But hard cutoffs break symmetries. For SCET the cutoffs must be “softer regulators” so as to not to break symmetries like Lorentz invariance and gauge invariance. Dimensional regularization is one regulator that can be used for this purpose. If we were only trying to distinguish modes with the invariant mass p^2 then the dim.reg. scale parameter μ would suffice for the cutoff between UV and IR modes, and we would be set to go. But in SCET we also need to distinguish modes in another dimension, μ does not suffice to separate or distinguish the s and c_n modes of Fig. 1. We will see how to do this later on without spoiling any symmetries. In general it will require a combination of subtractions that localize the modes in the regions shown in the figure, as well as additional cutoff parameters. The bottom line is that the physical picture in Fig. 1 for where the modes live is the correct one to think about for the purpose of power counting. But when integrating over loop momenta in a virtual diagram involving one of these modes we integrate over all values with a soft regulator to avoid breaking symmetries.

Lets consider a second example involving QCD jets. Jets are collimated sprays of hadrons produced by the showering process of an energetic quark or gluon as it undergoes multiple splittings. The splitting is enhanced in the forward direction by the presence of collinear singularities. The simplest process is $e^+e^- \rightarrow$ dijets, which at lowest order is the process $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$ with each of the light quarks q and \bar{q} forming a jet. Let q^μ be the momentum of the γ^* , then in the center-of-momentum frame (CM frame) $q^\mu = (Q, 0, 0, 0)$ and sets the hard scale. If there are only two jets in the final state then by momentum conservation they will be back-to-back along the horizontal \hat{z} axis:



The $x - y$ plane defines two hemispheres a and b , and we consider a process with one jet in each of them. The energy in each hemisphere is $Q/2$ and is predominantly carried by the collimated particles in the jets. To describe the degrees of freedom we need two collinear directions. We align n_1^μ with the direction of the first jet and n_2^μ with the second. (These directions can be defined by using a jet algorithm to determine the particles inside a jet, or indirectly from the process of calculating a jet event shape like thrust.)

Lets first consider the energetic constituents of the n_1 -jet. Since these constituents are collimated they have a \perp -momentum that is parametrically smaller than their large minus momentum, $p_\perp \sim \Delta \ll p^- \sim Q$. In order that we have a jet of hadrons and not a single hadron or small number of hadrons we must have $\Delta \gg \Lambda_{\text{QCD}}$. Thus the jets constituents have $(+, -, \perp)$ momenta with respect to the axes $n_1 = (1, -\hat{z})$ and $\bar{n}_1 = (1, \hat{z})$ that have a collinear scaling

$$p_{n_1}^\mu \sim \left(\frac{\Delta^2}{Q}, Q, \Delta \right) = Q(\lambda^2, 1, \lambda). \quad (2.13)$$

As usual the scaling of the $+$ -momentum is determined by noting that we are considering fluctuations about $p^2 = 0$, so $p^+ \sim p_\perp^2/p^-$. Here the power counting parameter is $\lambda = \Delta/Q \ll 1$. Note that the jet

constituents have the same scaling as the constituents of a collinear pion, but carry larger offshellness p^2 . If we make Δ so large that $\Delta \sim Q$ then we no longer have a dijet configuration, and if we make Δ so small that $\Delta \sim \Lambda_{\text{QCD}}$ then the constituents will bind into one (or more) individual hadrons rather than the large collection of hadrons that make up the jet. Another way to characterize the presence of the jet is through the jet-mass m_J^2 , since a jet will have $Q^2 \gg m_J^2 \gg \Lambda_{\text{QCD}}^2$. For our example here we can make use of the a -hemisphere jet-mass,

$$m_{J_a}^2 \equiv \left(\sum_{i \in a} p_i^\mu \right)^2 \sim p_{n_1}^+ p_{n_1}^- \sim \Delta^2 \ll Q^2. \quad (2.14)$$

For the constituents of the n_2 -jet we simply repeat the discussion above, but with particles collimated about the direction, $n_2 = \bar{n}_1 = (1, \hat{z})$. A choice that makes this simple is $\bar{n}_2 = n_1 = (1, -\hat{z})$, since then we can simply take the n_1 -jet analysis results with $+ \leftrightarrow -$. Using the same $(+, -, \perp)$ components as for the n_1 -jet we then have

$$p_{n_2}^\mu \sim \left(Q, \frac{\Delta^2}{Q}, \Delta \right) = Q(1, \lambda^2, \lambda). \quad (2.15)$$

Again a measurement of the b -hemisphere jet-mass can be used to ensure that there is only one jet in that region jet-mass,

$$m_{J_b}^2 \equiv \left(\sum_{i \in b} p_i^\mu \right)^2 \sim p_{n_2}^+ p_{n_2}^- \sim \Delta^2 \ll Q^2. \quad (2.16)$$

Finally in jet processes there are also soft homogeneous modes that account for soft hadrons that appear between the collimated jet radiation (as well as within it). The precise momentum of these degrees of freedom depends on the observable being studied, and the restrictions it imposes on this radiation. In our $e^+e^- \rightarrow$ dijets example we can consider measuring that $m_{J_a}^2$ and $m_{J_b}^2$ are both $\sim \Delta^2$. In this case the homogeneous modes are ‘‘ultrasoft’’ with momentum scaling as

$$p_{us}^\mu \sim \left(\frac{\Delta^2}{Q}, \frac{\Delta^2}{Q}, \frac{\Delta^2}{Q} \right) = Q(\lambda^2, \lambda^2, \lambda^2). \quad (2.17)$$

To derive this we consider the restrictions that $m_{J_a}^2 \sim \Delta^2$ imposes on the observed particles, noting in particular that with a collinear and ultrasoft particle in the a -hemisphere we have

$$(p_{n_1} + p_{us})^2 = p_{n_1}^2 + 2p_{n_1} \cdot p_{us} + p_{us}^2 \sim \Delta^2. \quad (2.18)$$

The term $2p_{n_1}^- \cdot p_{us} = p_{n_1}^- p_{us}^+$ plus higher order terms, so $p_{us}^+ \sim \Delta^2/p_{n_1}^- \sim \Delta^2/Q$, which is the ultrasoft momentum scale given in Eq. (2.17). Any larger momentum for p_{us}^+ is forbidden by the hemisphere mass measurement. The scaling of the other ultrasoft momentum components then follows from homogeneity.

If we draw the degrees of freedom, then for the double hemisphere mass distribution measurement of $e^+e^- \rightarrow$ dijets in the p^+p^- plane we find Fig. 2. Again we have labelled hard modes with momenta $p^2 \sim Q^2$ that are integrated out in constructing the EFT (here they correspond to virtual corrections at the jet production scale). In the low energy effective theory we have two types of collinear modes c_n and $c_{\bar{n}}$, one for each jet, which live on the $p^2 \sim \Delta^2$ hyperbola. Finally the ultrasoft modes live on a different hyperbola with $p^2 \sim \Delta^4/Q^2$. The collinear and ultrasoft modes all have $p^2 \lesssim Q^2\lambda^2$ and are degrees of freedom in SCET, while modes with $p^2 \gg Q^2\lambda^2$ are integrated out. When we are in a situation like this one, where the collinear and homogeneous modes live on hyperbolas with parametrically different scaling for p^2 , then the resulting SCET is known as an SCET_I type theory. Note that the c_n and us modes have

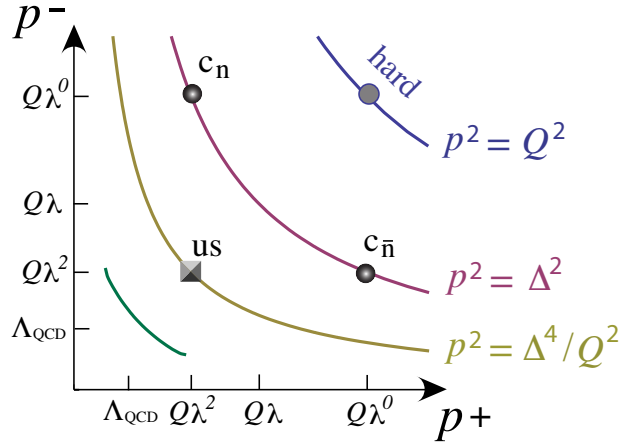


Figure 2: SCET_I example. Relevant degrees of freedom for dijet production $e^+e^- \rightarrow$ dijets with measured hemisphere invariant masses $m_{J_a}^2$ and $m_{J_b}^2$.

p^+ momenta of the same size, whereas the $c_{\bar{n}}$ and us modes have p^- momenta of the same size. The names collinear and ultrasoft denote the fact that these modes live on different hyperbolas.² Once again these degrees of freedom capture regions of momentum space, which are centered around the spots indicated and each of them extend into the infrared.

It is important to note in this dijet example that $\Delta^4/Q^2 \gtrsim \Lambda_{\text{QCD}}^2$, so in general the nonperturbative ultrasoft modes can live on an even smaller hyperbola $p^2 \sim \Lambda_{\text{QCD}}^2$ than the perturbative contributions from ultrasoft modes that have $p^2 \sim \Delta^4/Q^2$. An additional $p^2 \sim \Lambda_{\text{QCD}}^2$ hyperbola is shown in green in Fig. 2. If $\Delta^4/Q^2 \sim \Lambda_{\text{QCD}}^2$ then the yellow and green hyperbolas are not distinguishable by power counting, and hence are equivalent. If on the other hand we are in a situation where $\Delta^4/Q^2 \gg \Lambda_{\text{QCD}}^2$ then when we setup the SCET_I theory both the perturbative ultrasoft modes with $p^2 \sim \Delta^4/Q^2$ and the nonperturbative ultrasoft modes with $p^2 \sim \Lambda_{\text{QCD}}^2$ will be part of our single ultrasoft degree of freedom. This is convenient because we can first formulate the $\Delta/Q \ll 1$ expansion with the c_n , $c_{\bar{n}}$ and us d.o.f., and only later worry about making another expansion in $Q\Lambda_{\text{QCD}}/\Delta^2 \ll 1$ to separate the two types of ultrasoft modes that would live on the yellow and green hyperbolas.

If we compare Fig. 1 and Fig. 2 we see that it is the relative behaviour of the collinear and soft/ultrasoft modes that determine whether we are in an SCET_I or SCET_{II} type situation. (There are also SCET_{II} examples which involve jets with \perp measurements rather than jet masses, and we will meet these later on in Section 11.3 and 11.4.) Much of our discussion will be devoted to studying these two examples of SCET, since they are already quite rich and cover a wide variety of processes. In general however one should be aware that a more complicated process or set of measurements may well require a more sophisticated pattern of degrees of freedom. For example, we could have soft or collinear modes on more than one hyperbola, or might require modes with a new type of scaling. Indeed, this is not even uncommon, the collider physics example of $pp \rightarrow$ dijets in the CM frame requires both SCET_{II} type collinear modes for the incoming protons, and SCET_I type collinear modes for the jets. Nevertheless, after having studied both SCET_I and SCET_{II} we will see that often these more complicated processes do not really require additional formalism, but rather simply require careful use of the tools we have already developed in studying SCET_I

²In certain situations in the literature to use the names hard-collinear and soft to denote the same thing, and we will find occasion to explain why when discussing how SCET_I can be used to construct SCET_{II}.

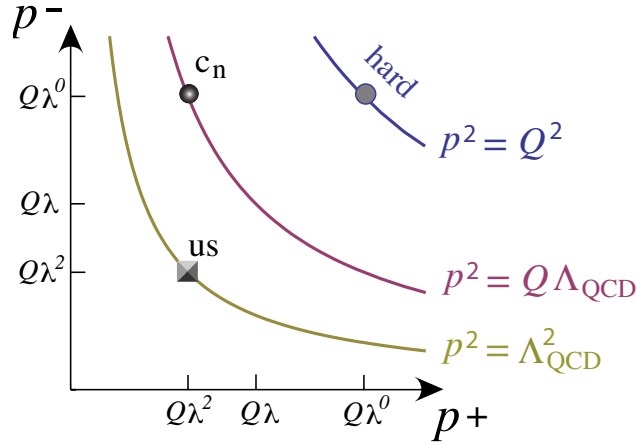


Figure 3: Another SCET_I example. Relevant degrees of freedom for $B \rightarrow X_s \gamma$ in the endpoint region.

and SCET_{II}.

A comment is also in order about the frame dependence of our degrees of freedom. In both of our examples we found it convenient to discuss the degrees of freedom in a particular frame (the B rest frame, or e^+e^- CM frame). Typically there is a natural reference frame to think about the analysis of a process, but of course the final result describing the dynamics of a process will actually not be frame dependent. Thus it is natural to ask what the d.o.f. and corresponding momentum regions would look like in a different frame. A simple example to discuss is a boost of the entire process along the \hat{z} axis. All the modes then slide along their hyperbolas (since p^2 is unchanged). The important point is that the relative size of momenta of different d.o.f. is unchanged by this procedure: the p^+ momenta of collinear and ultrasoft modes in SCET_I will be the same size even after the boost, and the p^+ momentum of a soft particle will always be larger than the p^+ momentum of a collinear particle in SCET_{II}. In $B \rightarrow D\pi$ such a boost can take us to the pion rest frame, where its constituents are now soft, and the constituents of the B and D are now boosted. Some components of the SCET analysis may look a bit different if we use different frames, but the final EFT results for decay rates and cross sections will obey the expected overall boost relations. In general it is only the relative scaling of the momenta of various degrees of freedom that enter into expansions and the final physical result. The relative placement of the spots for our d.o.f. in SCET_I and SCET_{II} is not affected by the \hat{z} boost.

Before finishing our discussion of d.o.f. we consider one final example. For the purpose of studying SCET_I it is useful to have an example with one jet rather than two, so the d.o.f. become simply c_n and us . This can occur for the process $B \rightarrow X_s \gamma$ or for $B \rightarrow X_u e \bar{\nu}$. The underlying processes here are the flavor changing neutral current process $b \rightarrow s \gamma$ or the semileptonic decay $b \rightarrow u e \bar{\nu}$. For these inclusive decays we sum over any collection of hadronic states X_s or X_u that can be produced from the s or u quark. In the B rest frame, the total energy of the γ or $(e \bar{\nu})$ is $E = (m_B^2 - m_X^2)/(2m_B)$ and ranges from 0 to $(m_B^2 - m_{H_{\min}}^2)/(2m_B)$ where $m_{H_{\min}}$ is the smallest appropriate hadron mass, either $m_{H_{\min}} = m_{K^*}$ or m_π for X_s or X_u respectively. An interesting region to consider for the application of SCET is

$$\Lambda_{\text{QCD}}^2 \ll m_X^2 \ll Q^2 = m_B^2 \quad (2.19)$$

where the photon or $(e \bar{\nu})$ recoils against a jet of hadrons which are the constituents of X . For $B \rightarrow X_s \gamma$ the picture is (double line being the b -quark, yellow lines are soft particles, and red lines are collinear particles):

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