8.851 Homework 4

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Problem 1) Chiral Perturbation Theory and Decay Constants

a) Work out the tree level Feynman rules for the four meson interactions with $SU(3)$ chiral perturbation theory and the leading order Lagrangian. (Use the octet basis $M = \pi^a \lambda^a / \sqrt{2}$ rather than the charged particle basis.)

b) Work out the tree level Feynman rules in chiral perturbation theory for the left-handed SU(3) octet current $(\bar{\psi} \gamma^\mu P_L \lambda^a \psi)$ with one and three external mesons.

c) Write down the loop diagrams and terms in the chiral Lagrangian that are needed to give the decays constants at order p^4 (where $p \sim m_\pi \sim m_K$ so this also means order $m_\pi^4 \sim m_q^2$ etc.).

d) Determine the explicit contribution of the order p^4 Lagrangian terms to f_π and f_K taking $m_u = m_d = \hat{m}$, but $\hat{m} \neq m_s$.

e) Calculate the loops in part c) using dimensional regularization. Combine your result with d) to derive the full expression for f_π and f_K/f_π at this order (still taking the isospin limit $m_u = m_d$).

f) Extract a value for the relevant low energy contants, L_i, using the data $f_K/f_\pi = 1.23 \pm .02$, and discuss whether your result agrees with naive dimensional analysis.