Problem set #10

1) Dual Coxeter numbers
a) Find the commutant of SU(2) in a group G for G being each of the simple gauge groups:
A_n, B_n, C_n, D_n, E_n, G_2, F_4.
The commutant of a group H in a larger group G which has H as its subgroup is the maximal subgroup of G which commutes with H. For example: The commutant of SU(2) in SU(n) is SU(n-2)*U(1).
b) Compute the dimension d of the coset space G/[SU(2)*C] with C the commutant of SU(2) in G that was computed in a). Compute it for all the groups which appear in a). Show that d is always divisible by 4.
c) For each of the values computed in b) add 8 (4 translation modes, 1 size, 3 SU(2) orientations) to get the dimension of the moduli space of one instanton on R^4 for gauge group G.
d) For each of the values of c) divide the result by 4 to get the dual coxeter number of the gauge group G. Compare this result with tables of group theory.
e) What is the dual coxeter number for E_8? Compare this to the gravitational anomaly coefficient of the tensor multiplet in 6 dimensions. What is the dual coxeter number for SO(32)?

2) Matter content for D*p* branes on O(*p*+4) planes.
a) Find the matter content living on a D*p*‐brane which probes (in other words, in the presence of) an O(*p*+4)^+ plane.
b) Analyze the Coulomb branch and Higgs branch of this theory.
b) Add more D*p* branes to this configuration to a total of k D*p* branes and write down the matter content.
c) Find the Coulomb & Higgs branches for this theory.
d) Add n D(*p*+4) branes. What is the gauge theory now? What is the global symmetry?
Write down the two branches of the moduli space of vacua.
e) Write a quiver representation for this theory.
f) Compute the dimension of the Higgs branch for a general k and n.
g) Compare your result of f) to the results you found in problem 1).

3) Moduli space of instantons ADHM construction
a) For G a simple group of each of the following sequence: G=A_n, B_n, C_n, D_n find a specific gauge theory with 8 supercharges that has the property that its Higgs branch is the moduli space of G instantons on R^4.
b) Write down the (ADHM) equations for the moduli space of instantons for the group G.
c) These equations are complex equations in some higher dimensional space. After imposing these equations we get the moduli space of instantons. Verify that the dimension of this complex space is equal to the dimension of the Higgs branch of the corresponding gauge theory.