8.871

Solutions to problem set #3

1.

a) In 11d, we have a single supergravity theory with 32 supercharges. In 10d we have two, Type IIA, which we get by dimensional reduction of the 11d theory, and Type IIB. Upon reduction on a torus, both of these theories give the same massless fields in all lower dimensions. This is clear in the context of string theory, since Type IIA and Type IIB are exchanged by T-duality.

In any spacetime dimension d ($d=3\ldots 9$) the bosonic massless fields consist of a metric ($g_{\mu\nu}$), a collection of antisymmetric n-forms ($B^{(n)}$) and a number of scalars (ϕ). Their fermionic superpartners will be a number of vector-spinor gravitinos (ψ_μ) and spinors (λ), which can be chiral depending on the dimension (actually, only in $d=6$ we encounter chiral fermions). These fields fall into representations of the little group $SO(d-2)$. The dimensionality of each of these irreps is:

$$g_{\mu\nu} : \frac{(d-2)(d-1)}{2} - 1$$

$$B^{(n)} : \binom{d-2}{n}$$

ϕ : 1

$$\left.\begin{array}{ll} \psi_\mu : & (d-3)\, 2^{[\frac{d}{2}-1]} \\ \lambda : & 2^{[\frac{d}{2}-1]} \end{array}\right\} \times \frac{1}{2} \text{ if chiral}$$

The bosonic fields decompose into $SO((d-1)-2)$ irreps simply as:

$$\begin{array}{lcl} SO(d-2) & \to & SO(d-3) \\ g_{\mu\nu} & \to & g_{\mu\nu} + B_\mu + \phi \\ B^{(n)} & \to & B^{(n)} + B^{(n-1)} \\ \phi & \to & \phi \end{array}$$

The decomposition of fermionic fields can be read out from group theory tables.

The resulting spectrum is:

9d : $g_{\mu\nu}$, $B^{(3)}$, $2B^{(2)}$, $3B^{(1)}$, 3ϕ, $2\psi_\mu$, 4λ

8d : $g_{\mu\nu}$, $B^{(3)}$, $3B^{(2)}$, $6B^{(1)}$, 7ϕ, $2\psi_\mu$, 6λ

7d : $g_{\mu\nu}$, $5B^{(2)}$, $10B^{(1)}$, 14ϕ, $4\psi_\mu$, 16λ

6d: $g_{\mu\nu}$, $5B^{(2)}$, $16B^{(1)}$, 25ϕ, $4\psi_{+\mu}$, $4\psi_{-\mu}$, $20\lambda_+$, $20\lambda_-$

5d: $g_{\mu\nu}$, $27B^{(1)}$, 42ϕ, $8\psi_\mu$, 48λ

4d: $g_{\mu\nu}$, $28B^{(1)}$, 70ϕ, $8\psi_\mu$, 56λ

3d: 128ϕ, 128λ

There is no potential for the scalar fields, so their number at each dimension gives the dimension of the moduli space of vacua.

b) The moduli space is a coset space G/H, with G the global symmetry group of the supergravity theory and H the unbroken symmetry at any point in the moduli space. More specifically, in d spacetime dimensions, with $n = 11-d$ $G = E_{n(n)}$ and H is its maximal compact subgroup.

d	G	H
9	$SL(2,\mathbb{R}) \times SO(1,1,\mathbb{R})$	$SO(2,\mathbb{R})$
8	$SL(2,\mathbb{R}) \times SL(3,\mathbb{R})$	$SO(2,\mathbb{R}) \times SO(3,\mathbb{R})$
7	$SL(5,\mathbb{R})$	$SO(5,\mathbb{R})$
6	$SO(5,5,\mathbb{R})$	$SO(5,\mathbb{R}) \times SO(5,\mathbb{R})$
5	$E_{6(6)}(\mathbb{R})$	$USp(8)$
4	$E_{7(7)}(\mathbb{R})$	$SU(8)$
3	$E_{8(8)}(\mathbb{R})$	$SO(16,\mathbb{R})$

c) In 4d, a string can couple magnetically to a scalar. There are 70 compact scalars in 4d, and an equal number of cosmic string types.