4. Hint on Problem 4 of Pset 3:

We ask you to show that

$$\Gamma^{\alpha'}{}_{\beta'\gamma'} = \frac{\partial x^{\alpha'}}{\partial x^{\alpha}}\frac{\partial x^{\beta}}{\partial x^{\beta'}}\frac{\partial x^{\gamma}}{\partial x^{\gamma'}}\Gamma^{\alpha}{}_{\beta\gamma} - \frac{\partial^2 x^{\alpha'}}{\partial x^{\beta}\partial x^{\gamma}}\frac{\partial x^{\beta}}{\partial x^{\beta'}}\frac{\partial x^{\gamma}}{\partial x^{\gamma'}}$$

$$= L^{\alpha'}{}_{\alpha}L^{\beta}{}_{\beta'}L^{\gamma}{}_{\gamma'}\Gamma^{\alpha}{}_{\beta\gamma} - L^{\beta}{}_{\beta'}L^{\gamma}{}_{\gamma'}\partial_{\beta}L^{\alpha'}{}_{\gamma}$$

You may find at the end of your calculation that you have instead derived a rule that looks like

$$\Gamma^{\alpha'}{}_{\beta'\gamma'} = L^{\alpha'}{}_{\alpha}L^{\beta}{}_{\beta'}L^{\gamma}{}_{\gamma'}\Gamma^{\alpha}{}_{\beta\gamma} + L^{\beta}{}_{\beta'}L^{\alpha'}{}_{\gamma}\partial_{\beta}L^{\gamma}{}_{\gamma'}$$

This may look totally wrong — the sign on the final term is incorrect. However, by inspecting this closely, you'll see that the matrix being differentiated in the second term is not the same in the two versions — the primed and unprimed indices are in opposite locations.

By noting that $L^{\gamma}{}_{\gamma'}L^{\alpha'}{}_{\gamma} = \delta^{\alpha'}{}_{\gamma'}$, you should be able to show that these two formulas are equivalent.

8.962 General Relativity
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