# SOLUTIONS TO HOMEWORK SET #5

1.

   a.   Game analysis

      i. *Nash Equilibrium*

      A pair of actions is a Nash equilibrium when each firm does the best it can, given the other firm's action. This game (as described in the matrix) has a unique Nash equilibrium with KL setting a price of 1400 and LAL a price of 1300. It can be seen from the matrix that with $p_{KL}$=1400, a price of 1300 is indeed LAL's best response. The analogous argument holds for KL. For any other set of prices at least one firm would be better off changing its price.

      ii. *Dominant Strategy*

      A firm has a dominant strategy if its profits are maximized by one and the same price level, regardless of the other firm's price. In this case, neither firm has such strategy available: there is no price level that LAL could choose thereby maximizing its profits, regardless of KL's reaction and vice-versa.

      iii. *Maximin Strategy*

      A firm's maximin strategy is the price that maximizes its minimum possible profit, i.e. that maximizes its profit assuming that the other firm does precisely the worst thing from your perspective. This means that KL would choose to price its product at 1100, because this level maximizes its minimum profit (which is in this case 254). Similarly, LAL would price at 1000, thereby maximizing its minimum payoff at 124. The maximin set of strategies would therefore be 1100, 1000 and the profits 423 for KL and 310 for LAL. Their risk aversion has forced them to far from optimal pricing.

      iv. *Contractual Collusion*

      If the two firms can write a contract that specifies both their prices and a transfer, then any optimal contract must specify those prices that maximize the sum of their profits (since otherwise they could improve the contract by switching to the prices that maximize joint profits and choose the transfer so as to split the gain in profit equally). The transfer will then

determine how that joint profit is actually split up. This will be determined by bargaining and is difficult to predict.

If the two firms cannot include a transfer in their contract, then it is not necessarily the case that they will maximize joint profits. They might agree to sacrifice some profits to make the profit split more equitable. (Note that before issues of equity could always be dealt with by the transfer.) We can, however, seriously restrict the number of possible contracts in the following way. A set of prices cannot be optimal if there exists an alternative set of prices that gives both firms higher profits (i.e. in 'econ-speak' if there exists a set of prices that Pareto dominates). So the optimal contract must be one of the following :

| $p_{LAL}$ | $p_{KL}$ |
| --- | --- |
| 1900 | 1700 |
| 1900 | 1800 |
| 1900 | 1900 |
| 1800 | 1900 |
| 1700 | 1900 |
| 1600 | 1900 |

v. **Commitment**

LAL would indeed like to be able to commit to a price. In particular, if it could commit to setting a price of 1500, it would be better off than under the Nash equilibrium: the best response of KL is to also set a price of 1500, which gives LAL a (gross) profit of 1028, which is higher than its profit in Nash equilibrium. (Note, however, how its profit would be still higher if KL were the one who could commit. KL would then commit to 1700, to which LAL would respond with 1500 for a LAL profit of 1542).

If it could, LAL would instead prefer to commit to always match KL prices. In that case, KL's optimal price is 1900, which gives a LAL profit of 1652).

vi. **What should LAL do?**

There are several options available for LAL. Probably the best solutions would be to try to collude (by sending the signal that if KL decides to sell at 1900, LAL will match the price). This could be supported by a promise to match the best price offer of any competitor.

In case this solution turns out to be unsustainable – because KL cheats or LAL cannot resist the temptation to cheat itself – the best alternative is to offer the Nash equilibrium pricing, as this is the only sustainable behavior for LAL.

b. Double market size

If the market doubles, the whole payoff matrix will change linearly, as the contributions are calculated using the following two formulas:

Contributions $_{LAL}$ = ($P_{LAL}$ - \$841) (Market Size) (Market Share $_{LAL}$)

Contributions $_{KL}$ = ($P_{KL}$ - \$883) (Market Size) (Market Share $_{KL}$)

While total payoffs change, the optimal pricing strategies and therefore the Nash Equilibrium do not change.

2.

a. In order to solve this exercise we have to calculate the reaction curves that each firm faces while setting the optimal quantities produced in the Airline industry.

i. When the cost functions are the same, the solution is as follows:
Demand curve: $P = 100 - Q$
Total quantity produced: $Q = Q_{AA} + Q_{TA}$
Marginal Costs: $MC_{AA} = MC_{TA} = 40$

The problem for American Airlines therefore is to maximize profits, when:

$$\text{Revenues} \quad = R = P * Q_{AA} = (100 - Q)\, Q_{AA}$$
$$= (100 - Q_{AA} - Q_{TA})\, Q_{AA}$$
$$= 100\, Q_{AA} - Q_{AA}{}^2 - Q_{TA}\, Q_{AA}$$

Marginal Revenues therefore are:
$$dR / dQ_{AA} = MR_{AA} = 100 - 2\, Q_{AA} - Q_{TA}$$

The optimal quantity is reached when MR = MC. For American Airlines we have:

$100 - 2\, Q_{AA} - Q_{TA} = 40$, or
$\underline{Q_{AA} = 30 - \tfrac{1}{2}\, Q_{TA}}$

Considering that both companies face the same demand curve and that they have identical cost curves, the Reaction curve for Texas Airlines will be identical to the one of American Airlines:

$\underline{Q_{TA} = 30 - \tfrac{1}{2}\, Q_{AA}}$

The Cournot-Nash equilibrium requires that both firm produce at their reaction curves, which means that the optimal quantities $Q_{AA}$ and $Q_{TA}$ will be obtained by setting the two reaction curves equal to each other:

$Q_{AA} \quad = 30 - \tfrac{1}{2}\, Q_{TA} = 30 - \tfrac{1}{2}\, (30 - \tfrac{1}{2}\, Q_{AA})$
$= 30 - 15 + \tfrac{1}{4}\, Q_{AA} \Leftrightarrow .75\, Q_{AA} = 15$
$\underline{\mathbf{Q_{AA} = 20}}$

And: $\underline{Q_{TA} = 30 - \frac{1}{2} * 20 = 20}$
Therefore:     $Q = 20 + 20 = \mathbf{40}$
    $P = 100 - 40 = \mathbf{60}$
    $\mathbf{\Pi_{AA}} = P*Q_{AA} - TC(Q_{AA}) =$
    $\mathbf{\Pi_{TA}} = P*Q_{TA} - TC(Q_{TA}) = 60 * 20 - 40 * 20 = \mathbf{400}$

ii. If Texas Airlines now has the benefit of a lower marginal cost (and American Airlines knows about it) the problem changes as Texas Airlines will have a new reaction curve, while American will still calculate its optimal output using the same equation ($\mathbf{Q_{AA} = 30 - \frac{1}{2} Q_{TA}}$)

Texas Airlines' problem is to set the Marginal Revenues equal to the new Marginal Costs. In numbers:
    $MR_{TA} = MC_{TA} \Leftrightarrow 100 - 2 Q_{TA} - Q_{AA} = 25 \Leftrightarrow$ The new reaction curve is : $\mathbf{Q_{TA} = 37.5 - \frac{1}{2} Q_{AA}}$

The new Cournot-Nash equilibrium is therefore given by:
    $Q_{TA}$  $= 37.5 - \frac{1}{2} Q_{AA} = 37.5 - \frac{1}{2} (30 - \frac{1}{2} Q_{TA})$
        $= 37.5 - 15 + \frac{1}{4} Q_{TA}$
        $.75 Q_{TA} = 22.5$
    $\mathbf{Q_{TA} = 30}$ and $\mathbf{Q_{AA}} = 30 - \frac{1}{2} * 30 = \mathbf{15}$

Total output will be:   $Q = 30 + 15 = \mathbf{45}$
            $P = 100 - 45 = \mathbf{55}$


    $\mathbf{\Pi_{AA}} = P*Q_{AA} - TC(Q_{AA}) = 55 * 15 - 40 * 15 = \mathbf{225}$
    $\mathbf{\Pi_{TA}} = P*Q_{TA} - TC(Q_{TA}) = 55 * 30 - 25 * 30 = \mathbf{900}$

iii. If TA licenses the process to AA, both firms have the above reaction curve $Q_x = 37.5 - Q_y/2$. The new Cournot-Nash equilibrium is therefore given by:
    $Q_{TA}$  $= 37.5 - \frac{1}{2} Q_{AA} = 37.5 - \frac{1}{2} (37.5 - \frac{1}{2} Q_{TA})$
        $= 37.5 - 18.75 + \frac{1}{4} Q_{TA}$
        $.75 Q_{TA} = 18.75$
    $\mathbf{Q_{TA} = 25}$ and $\mathbf{Q_{AA}} = 30 - \frac{1}{2} * 30 = \mathbf{25}$

Total output will be:   $Q = 25 + 25 = \mathbf{50}$
            $P = 100 - 50 = \mathbf{50}$


    $\mathbf{\Pi_{AA}} = P*Q_{AA} - TC(Q_{AA}) = (50-25)*25 = \mathbf{625}$
    $\mathbf{\Pi_{TA}} = P*Q_{TA} - TC(Q_{TA}) = (50-25)*25 = \mathbf{625}$


Texas Airline loses 275 in profits. That is the minimum price it is willing to accept to share its technology. American gains 400 in profits. That is the maximum price it is willing to pay. So the firms should agree on a license for a price somewhere between 275 and 400. (Note that it is not always the case

that firms are willing to share technology. It is easy to see that with perfect Bertrand competition, a firm would never want to share its technology, although it would be beneficial to society as a whole.)

b.

   i. The gross profit equations are:

$$\Pi_{LAL} = 3.9 \, (p_{LAL} - 841) \, (400 + p_{KL} - p_{LAL})$$
$$\Pi_{KL} = 3.9 \, (p_{KL} - 883) \, (600 + p_{LAL} - p_{KL})$$

   ii. The first order conditions are:

$(400 + p_{KL} - p_{LAL}) - (p_{LAL} - 841) = 0$     or     $p_{LAL} = (400 + 841 + p_{KL})/2$
$(600 + p_{LAL} - p_{KL}) - (p_{KL} - 883) = 0$     or     $p_{KL} = (600 + 883 + p_{LAL})/2$

Combining the two equations gives:
$p_{LAL} = 1321.667$        and     $p_{KL} = 1402.333$

3.

a. Here is the payoff matrix (in millions of dollars):

|  | Steele | |
| --- | --- | --- |
| Magna | Prompt Delivery | Slow Delivery |
| Buy all | 2,3 | -5,4 |
| Buy part | 1,1 | -1,2 |

b. Dominant strategies:

- **Magna:** If Steele offers prompt delivery, Magma prefers buying all. If Steele does not offer prompt deliver, Magma prefers buying part. Thus, *Magma does not have a dominant strategy*

- **Steele:** If Magma orders all from Steele, then Steele prefers slow delivery. The same holds if Steele orders only part of the aluminum. Thus, *Steele has a dominant strategy: to provide slow delivery.*

c. Given that Steele has a dominant strategy to provide slow delivery, Magna will only buy part of its aluminum from Steele. The Nash equilibrium will be (Buy Part, Slow Delivery), with a payoff of (-1,2).
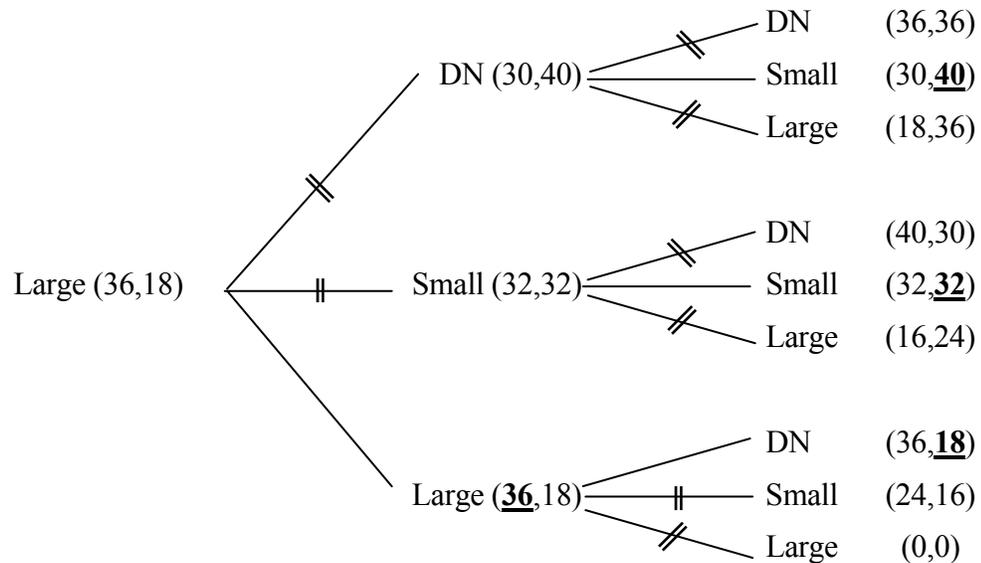
4.

   a.

| **Dale** | DN | Small | Large |
|---|---|---|---|
| DN | $36m, $36m | $30m, **$40m** | **$18m**, $36m |
| Small | **$40m**, $30m | **$32m**, **$32m** | $16m, $24m |
| Large | $36m, **$18m** | $24m, $16m | $0m, $0m |

(with heading "Chip" spanning the Chip columns)

The Nash equilibrium in this game is that both Chip and Dale will conduct a small expansion.

   b.  If Dale moves first, this is the resulting tree:



Dale will decide a large expansion and then Chip will choose not to expand. The resulting payoff will be (36,18).

   c.  If Chip moves first, the result will be exactly the opposite (because the payoff matrix is symmetric). Therefore, Chip will decide a large expansion and Dale will choose not to expand.