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Problem 1 – Box Carbon Model

This problem will start with you being given a simple model of the Earth's carbon system as a single box (the Carbon in the atmosphere). The reason to consider such a model is to look at some of the impacts that anthropogenic emissions and natural sinks have on the amount of Carbon in the atmosphere. As we discussed in HW #1, this is important because as the amount of carbon in the atmosphere (e.g. CO₂) increase, so does its associated impact on the radiative balance of the Earth.

To start this problem off, we assume that the amount of Carbon in the atmosphere can be modeled using the following simple equation, which only considers the anthropogenic emissions of Carbon and a natural sink for Carbon to the land and oceans.

Equation 1a

$$\frac{dM}{dt} = E - \frac{M}{\tau(M)}$$

where M: total stock of Carbon in the atmosphere [PgC]
 t : time [yr]
 E : amount of anthropogenic Carbon emission [PgC/yr]
 τ: lifetime of Carbon in the atmosphere due to the natural sinks [yr]

Notice that the loss of carbon from the atmosphere is a function of the total amount of the carbon in the atmosphere. Over time we expect the ability of the land and ocean sinks to take up carbon is not constant over time.

Equation 1b

$$\tau(M) = \tau_0 + \tau_1 \left(\frac{M - M_0}{M_0} \right)^2$$

In equation 1b, we are modeling this effect by stating that the residence time of the carbon in the atmosphere will increase as the total amount of carbon in the atmosphere increases. In specific, we are looking at two effects, a constant lifetime of carbon in the atmosphere, based on today's amount of carbon in the atmosphere, and an increasing component, based on the amount of increase of the carbon in the atmosphere from today. Note that this model only works to look at the lifetime of the carbon uptake in a world in which the total carbon in the atmosphere is increasing.

Q1.1) In your own words, first explain what equation **1a** tells us, and then give two examples of how you could make the equation (i.e., a global atmospheric mass balance of carbon) more realistic. Next, explain in your own words, and give at least one reason why we would expect both the land and ocean sinks to be less effective in the future.

Take $E \approx 9 \text{ PgC/yr}$ as a rough estimate. Additionally, a current estimate of the stock of carbon in the atmosphere can be found by starting with the current atmospheric concentration of CO_2 , which is about 382ppm, globally averaged over the surface of the Earth (see figure 1)

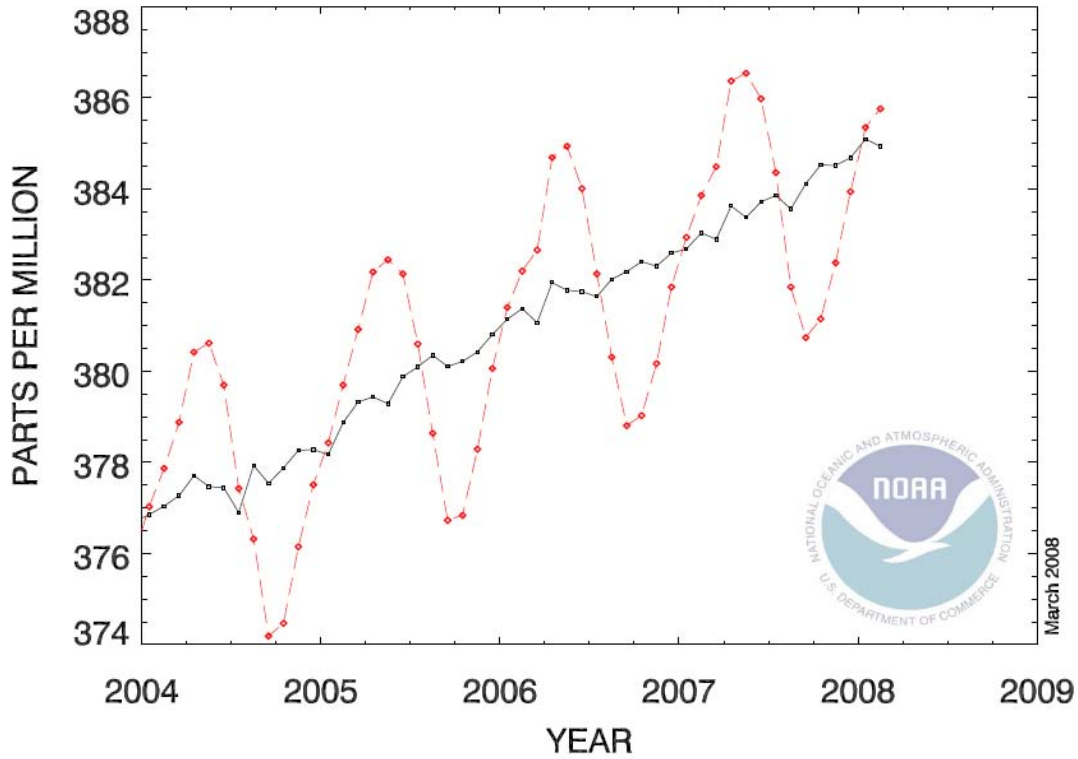


Figure 1: Recent monthly mean CO₂ at Mauna Loa

Source: Dr. Pieter Tans, NOAA/ESRL, www.esrl.noaa.gov/gmd/ccgg/trends

We can then multiply this number by the total number of moles of the atmosphere to come up with the number of moles of CO_2 , and then multiplying this by the molecular weight of carbon, which gives a stock of carbon in the atmosphere. This process gives us a rough answer as $M_o \approx 8.4 \times 10^2 \text{ PgC}$.

Q1.2) Looking at equations **1a** and **1b**, set up an equation for τ_o . Next, using the data from figure 1 over the past few years, approximate the derivative, dM / dt (e.g., use the slope of the line). Finally, solve this equation for τ_o . Now, suppose that for a doubling of atmospheric carbon, that the total lifetime of carbon is expected to increase by 20%, based on the current level of emissions and atmospheric carbon stock. Using this information, setup and solve an equation for τ_1 .

The ultimate goal of climate policy will be to at least stabilize the atmospheric concentration (and hence stock) of carbon. Let us use this simple model to investigate this idea more closely. The first way to look at this is to say that the atmospheric stock of carbon has stopped changing in time. In this model, we can calculate for simple stabilization scenarios by assuming the model has come to steady state (i.e., constant M).

Q1.3) Assume that the model has come to describe a system which has stabilized atmospheric carbon. Explain why a steady state assumption is a valid way to represent this situation. Assuming that the atmospheric stock of carbon is double today's present value, what is the new lifetime of carbon in the atmosphere, and what is the maximum level of emissions? Also, compute these values for the case with three times, and four times of today's present level. Can you comment on what is happening to the lifetime of carbon in the atmosphere as its total stock increases? What do you think will have to happen to the emission of carbon as the concentration in the atmosphere continue to increase, if we are to keep the stock in steady state (for this final example, you can use a mathematical abstract of the lifetime going to infinity, if it helps your thinking)? In reality, the lifetime of carbon in the atmosphere will decrease faster, since the ability of the atmosphere and the ocean to uptake carbon will eventually stop, if the stock continues to remain at a very high level. How does the answer to what is required of emissions for stabilization based on this past statement compare with the answer of what is required of emissions for stabilization based on your previous question?

Problem 2 – Modeling Growth, Temperature, and Climate Damage

In this assignment we extend the spreadsheet model from HW #2 with 100-year projections of temperature change and climate damages, and use it to carry out studies of temperature change and cost-benefit analyses. A worksheet entitled “Damage”, added to the Excel file used in HW #2, is used in this problem. The worksheet is extended to include columns for radiative forcing, temperature change and % reduction in GDP. Also the time horizon, and the period of optimization, is extended to 2150. These additional years are added to avoid end effects; in the problems below please focus on the period to 2100. The revised worksheet is available on the course website. Analyses to be conducted are described in Section 2.

1. INCLUDING TEMPERATURE CHANGE AND CLIMATE DAMAGE

1.1 Radiative Forcing

Column R contains the equations for radiative forcing $F(t)$. The increase in the strength of radiative forcing from additional CO_2 is proportional to the natural log of the ratio of atmospheric carbon to its preindustrial levels.

$$F(t) = 4.1\{\ln[M(t)/590]/\ln(2)\} \quad (1)$$

1.2 Temperature Change

Columns S and T contain equations for a two-box model of the climate system, temperature of the atmosphere and mixed-layer of the ocean $T(t)$ and the temperature of the deep ocean $T^*(t)$. The temperature of the upper layer has the radiative forcing as a source term, as well as a feedback term to reflect the climate sensitivity.

$$T(t) = T(t-1) + c_1\{F(t) - LAM * T(t-1) - c_3[T(t-1) - T^*(t-1)]\} \quad (2)$$

The feedback term LAM can be determined from the desired climate sensitivity CS as $LAM = 4.1/CS$. The coefficient c_1 represents the heat capacity of the atmosphere and the c_3 is the coefficient of heat diffusion between the upper and lower layers (also corrected for atmospheric heat capacity).

The temperature of the deep ocean is calculated as

$$T^*(t) = T^*(t-1) + c_4[T(t-1) - T^*(t-1)] \quad (3)$$

Where c_4 is the coefficient of heat diffusion between the upper and lower layers.

1.3 Climate Damage

To capture the economic impacts of climate damages, we add the equation in column U for the percentage loss of GDP $D(t)$:

$$D(t) = a_1T(t) + a_2T^2(t)$$

1.4 Parameter Values

The additional parameter values for the new equations are:

$$LAM = 1.41 \quad (\text{feedback on temperature change})$$

$c_1 = 0.226$	(atmospheric heat capacity)
$c_3 = 0.44$	(heat diffusion coefficient-atmos)
$c_4 = 0.02$	(heat diffusion coefficient-ocean)
$a_1 = 0.006$	(linear damage term)
$a_2 = 0.0015$	(quadratic damage term)

The initial-year values of the variables in the model are the following:

$T_0 = 0.76$	(degrees C)
$T^*_0 = 0.10$	(degrees C)

2. TASKS WITH THE MODEL

You are to prepare calculations with the “Damage” version of the model, which is a dynamic or forward-looking formulation. For each solution, please hand in

1. A copy of each worksheet where appropriate.
2. Plots of the time paths of consumption, carbon emissions, and atmospheric concentrations. Feel free to plot any other variables that you find of interest.
3. Comments on the solutions as requested.

Once you have set up the worksheet, you may want to explore the effect on the results of alternative values of key parameters, e.g., γ , η , σ_{KLE} . Note that the model may not solve if you depart far from the reference parameters specified above.

Problem 1: Reference Projection Ignoring Climate Damage. Set the damage coefficients a_1 (A47) and a_2 (A48) to zero, and remove any constraints on carbon emissions, concentrations or temperature change. What is the projection of global mean surface temperature (i.e., “Atmos Temp”) change by the year 2100.

Problem 2: Reference Projection Considering Climate Damage. Now assume the damage function has the parameter values (a_1 , a_2) given above. How does would the economic system as represented by this model respond, in terms of consumption and energy use, as compared to the behavior if damage is not considered?

Problem 3. Role of the Discount Rate. Applying the model with the damage function, analyze and discuss the effect on energy use, consumption and temperature change of lowering the discount rate Θ from 0.03 to 0.001 (closer to the value suggested by the Stern Review).

Problem 4. Temperature Target. Applying the model with the damage function, analyze and discuss the effect on energy use, temperature and welfare of the imposition of a 2°-temperature target. Also, what is the % welfare loss in 2100 (ignoring climate change benefits) that is attributable to this target?

3. HINTS TO AVOID GIVING PROBLEMS TO SOLVER

- (1) In solver, include a constraint that energy (col. F) is greater than zero to 2150.
- (2) Before each solution with solver, “Reset” the energy column to 600 throughout.