

So let's begin modeling this problem as a linear optimization problem.

In terms of the framework that we have seen in the lectures, our objective is to maximize the revenue to Google.

Our decision is for each advertiser and each query to decide the number of times that advertiser's ad will be displayed for that query.

Our constraints are of two types.

Our first constraint is that the average amount paid by each advertiser, based on the number of times Google displays their ad for each query, cannot exceed the budget of that advertiser.

Our second constraint is that the total number of ads we display for a query cannot exceed our estimate of the number of requests that we expect to see for that query.

Let's quickly review our problem data.

We have the average price per display for each advertiser and each query.

We have the budget of each advertiser.

And we have estimates of the number of requests for each query.

So let's now think about how to model all the pieces of our problem.

First, how should we define our decision variables?

Well, we should define them for each advertiser and each query.

So for example, one of our decision variables will be x_{A1} .

x_{A1} is the number of times we will display AT&T's ad for query 1.

Similarly, we can define x_{A2} and x_{A3} .

These are the numbers of times that we will display AT&T's ad for queries 2 and 3, respectively.

Now, of course, we have three different advertisers, so we will have variables x_{T1} , x_{T2} , and x_{T3} .

These variables represent the number of times that we display T-Mobile's ad for queries 1, 2, and 3, respectively.

And, of course, our last advertiser's Verizon.

So we will have variables x_{V1} , x_{V2} , and x_{V3} , which are the numbers of times that we display Verizon's ad for queries 1, 2, and 3, respectively.

Now, how do we compute the revenue to Google, which is our objective?

Well, we know the average price per display, and our decision variables tell us exactly how many times we'll display each ad for each query.

So all we need to do is we need to multiply each decision variable with its corresponding average price per display and add them all up.

If we did this, we would have $0.5 \cdot x_{A1} + 0.5 \cdot x_{A2} + 1.6 \cdot x_{A3} + 1 \cdot x_{T1}$.

And we can continue this.

And the last term in our sum will be $5 \cdot x_{V3}$.

To get the constraints, we use a similar process.

For instance, to get how much AT&T pays, we multiply the AT&T variables by their average prices per display and add them up.

So we'd get $0.5 \cdot x_{A1} + 0.5 \cdot x_{A2} + 1.6 \cdot x_{A3}$.

Now, one of our constraints, as you will recall, is that this cannot exceed AT&T's budget, which is \$170.

So our model would have the constraint $0.5 \cdot x_{A1} + 0.5 \cdot x_{A2} + 1.6 \cdot x_{A3}$, is less than 170.

We can repeat this to get the same kind of budget constraint for T-Mobile and Verizon.

Now, to get the number of times query 2 is used, we add up the decision variables corresponding to query 2.

So we have x_{A2} , the number of times AT&T is paired with query 2.

We have x_{T2} , which is the number of times T-Mobile is paired with query 2, and x_{V2} , which is the number of times Verizon is paired with query 2.

Now, one of our constraints is that this cannot exceed the estimated number of requests for query 2, which is 80.

So our model would have the constraint $x_{A2} + x_{T2} + x_{V2}$ all less than 80.

We can do this for the other queries.

So we can do this for query 1 and query 3.

And this is all that we really need.

So now, let's take this problem to LibreOffice and actually solve it.