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Let me begin by first asking whether there are any questions from last class, which was a week ago. Hope you had a good break. Any questions? OK. Before we begin today's topic-- question?

AUDIENCE: No.

ANDREW LO: No. Before we begin today's topic on arbitrage and the pricing of multiple fixed income securities, I want to just take a few moments to talk a bit about what's going on in financial markets, also to welcome the prospective students that we have sitting here in class today.

So for once, over the weekend, unprecedented things didn't occur. And so I'm glad to report that we're still here. Financial markets are still around. And as you know, the government has proposed some measures to deal with this current financial crisis. And at this point, it's still unclear as to what they're proposing. But we can actually see from the data what the market's reaction is.

Last time, remember, we looked at the yield curve and literally, it was just a week ago that the yield curve looked like this. Now remember that we focused on what happened at the very short end of the yield curve, which is three month Treasury bills. And last week when we looked at this graph, the yield was about three basis points for a three month Treasury bill. And we pointed out that that was telling us something about the market. In particular, it was telling us that the market is panicking. Yeah?

AUDIENCE: So aren't we sort of in this whole situation because looking at the market grossly mispriced things?

ANDREW LO: Well, I wouldn't say that it was looking at the market grossly mispriced things.

AUDIENCE: The market obviously did not officially price this risk, which looking at the market a year and a half ago, we would have been, oh, here's this much risk, this criteria [INAUDIBLE]. It turned out to be right, totally right.

ANDREW LO: Well, there are a number of things that are priced into a security. It's not just a risk, but it's also a reflection of supply and demand, right? So in other words, what's going on here-- the question that we want to answer from looking at the price is what do we know about what's

going on in the marketplace based on that price. What is it telling us? The cost of borrowing over a three month period-- when it goes down to three basis points, that's telling you that the price of that security, the price of Treasury bills is extraordinarily high relative to historical standards.

Now let's take a look at what happens more recently, in particular, today, if we go to any of these websites-- so in particular, let's go back to the Bloomberg site where we originally looked at this. First of all, this is now the yield curve. And it's hard to compare because I've got a different slide for last weeks. This is just yesterday's, the orange, not last week's. But the one thing you'll note is that at the very short end, now instead of three basis points, the three month Treasury is yielding 41 basis points. What does that tell you about the price?

Now, so Treasury securities, short-term Treasury securities, have declined in price over the last week. And that's one sign that perhaps markets are not as panicked as they were last week. There isn't this mad rush to get into Treasury securities in the short term. All right, so short term means have gone up. Yeah.

AUDIENCE: So would you say that this would be a lot about the psychology of [INAUDIBLE].

ANDREW LO: Yeah.

AUDIENCE: So what I was wondering is when the fact-- it seemed like, as you said, there's a flight in liquidity last week. Now why was there such a huge movement, or what led to the price dropping so much when, I guess, it could be reasonably expected that the price would come back up and so people would just wait it out and take a shot.

ANDREW LO: Which price are you talking about dropping?

AUDIENCE: I should probably say the yield dropping.

ANDREW LO: The yield dropping and the price going up. Well, so there are a number of factors at play, but the current perspective that most of us have in financial markets about last week-- and this is just perspective. Remember, last week is not that far away. What happened last week, by most accounts, is that there was a very significant rush to the exits by investors.

By rushing to the exits, I mean getting out of risky securities and into safer securities. And at this point, it doesn't seem like there's much of a safe haven other than US Treasury securities, and in particular, short-term securities because you know you can get the money out over a

relatively short period of time.

So that's what the yield curve told us last week, that three basis points means that basically people didn't care about the yield. All they wanted to do was to get into US treasuries at almost any price. OK.

This week it's different. In particular, not only has the short-term yield gone up, so now instead of three basis points, we're up to 41. But look at the long end of the yield curve. Before, the long end of the yield curve-- let me just go back and remind you what that looked like. At the 30 year, a week ago, the 30 year yield was 4%.

Let's take a look at what it is today. It's now, the 30 year yield, 4.37. That's another big movement. Why is that? Why would the yield for the long-term bond go up? What is the market thinking today?

AUDIENCE: They might be more worried about inflation. The government has promised \$700 billion.

ANDREW LO: OK, so inflation has now been incorporated, just over the last seven days. So question-- is this price correct or was last week's price correct? Getting to your point, I mean, what do we do? Is the short end of the yield curve appropriate today at 41 basis points, or was it really appropriate at three basis points?

There's no answer to that question because there is no right answer. These prices are a reflection of the current expectations of all the market participants. Right or wrong, it really reflects the combined either wisdom or fear or greed of the marketplace.

And so our approach is to try to understand what that is. We want to explicate the information that happens to be in prices, but you have to understand that these are the same imperfect kind of prices that we came up with on day one when you bid for that little package. And it turned out that you got lucky and got an iPod for whatever, \$45. But it could've gone the other way. And in fact, , in the second class it did go the other way. So we won't talk about that.

The prices reflect all aspects of the economy, the rational as well as the irrational. And so last week, was it irrational for people to pull their money out from all sorts of investments and put them into Treasury bills? Well this goes to the heart of why the Treasury acted so quickly, and why Chairman Bernanke has said that he wants to get a quick resolution.

Something very significant happened last week. And I don't know how many of you really got

wind of it. Certainly, the Treasury knew what was going on and the Fed did, but it wasn't really highlighted in the newspapers in the way that I would have thought it should have been, given the importance. Anybody know what I'm talking about? Yeah.

AUDIENCE: Stopping the short sell--

ANDREW LO: Well, that was one piece of news. The SEC mandated that for a period of time, to be determined, we are not allowed to short sell financial stocks because they wanted to stop the kind of run that there has been on these securities. I'm going to come back and talk about it in the end of this lecture because we're going to talk about short sales. But that's not what I was referring to. That's certainly a concern, but that's not the major concern that I think the market was responding to. Yeah.

AUDIENCE: Was it the \$8 billion of redemption in money funds?

ANDREW LO: That's right. Where was that coming from? What was going on with that? Why did that happen?

AUDIENCE: Because people are losing confidence in the short-term debt securities to put in money market funds.

ANDREW LO: And was there a reason for that loss of confidence? I mean, money funds, what does that have to do with mortgages and Lehman and Goldman? What's the connection?

AUDIENCE: Well, I just think more of it was a psychological reaction than it was an actual--

ANDREW LO: Absolutely, it was a psychological reaction, but did something trigger that? Is that psychological reaction completely unreasonable? If your grandmother asked you what she should do now with her money market fund, should you tell her, don't worry about it, just stay the course and see what happens?

Something happened last week that is related exactly to that issue. So you're on to something. What is that? Megan.

AUDIENCE: Well, one of the major money funds broke the buck.

ANDREW LO: Broke the buck, exactly. Which money fund? Do you remember?

AUDIENCE: Lehman, no, no.

ANDREW LO: The Reserve Fund.

AUDIENCE: It's the Reserve.

ANDREW LO: The Reserve Fund is one of the first, if not the first, money market funds. And what is a money market fund? Do we know what that is? You all know what that money market fund is? You all probably have money in a money market fund. Whether you know it or not.

Money market fund is a fund that contains relatively short-term and supposedly riskless securities, like CDs, treasuries, and other kinds of very, very safe assets. And what does it mean to break the buck?

AUDIENCE: That for every dollar that people invested in money market funds had invested what they would be able to redeem at that time.

ANDREW LO: Right.

AUDIENCE: --less than already spent.

ANDREW LO: Right. Breaking the buck means that when you put in \$1, money funds are supposed to be so safe that at the very least when you withdraw the money, you're going to get \$1 back. Breaking the buck means that if you withdraw, there is a possibility that what you withdraw is less than \$1.

Now that's scary because think about a bank-- when you put your money into a checking account for a bank, you expect to get that money out, maybe not with a lot of interest, maybe even with no interest if things don't go well, but you expect to get what you put in. You expect to get the principal back, right?

Well, money market funds are very much the same way. People use them as if they were checking accounts. In fact, there are money market funds where you can write checks on them, right? And so breaking the buck has been a major concern, not just among the money funds, but among regulators. Because if it turns out that retail investors, ordinary consumers are scared about what's going on with their money market accounts, they will do in mass what happened last week, which is pull out huge sums of money from these money market funds.

And as I mentioned earlier, no business can sustain a massive withdrawal of all capital all at once. It's just not possible for a business to be able to be conducted in that manner. If that

happens, we will see mass failures of financial institutions that will make the last four weeks look like the good old days. And that's what the Fed is concerned about. That's what the Treasury is concerned about.

And so the hope is that the measures that they put in place will calm the fears of the public. That's the first order of business. It's calming the psychological kinds of effects that these headlines have produced. And so the hope is that once they put these measures in place, that will take care of the concerns.

What they've done is to propose to guarantee money market funds the same way that the FDIC guarantees your banking accounts. And the way that the SIPC guarantees your brokerage accounts. And there are other measures that have been proposed. We won't have time to talk about them here, but over the next couple of weeks, the finance group at the Sloan School will be putting together some kind of a panel discussion that will focus exactly on these issues. So we'll let you know when that happens. And we'll take on these issues head on in that session.

OK, but it looks like for the moment, at least from the yield curves that we saw, that things are actually quieting down. We'll see on a day by day basis. So this is obviously last week. This week, we have yields going up a little bit, so that suggests that there isn't the same kind of pressure. But every day is another day and with another set of revelations. So by looking at these pieces of information, we can actually glean what the market is thinking.

Is it right? Of course not. All forecasts are by construction incorrect to some degree, but it's a window on exactly what's going on in the marketplace and what people are thinking. Yeah.

AUDIENCE: Sorry, what's the connection between this money market issue and the yield curve? Could you make the connection again? I missed it.

ANDREW LO: Yeah, sure. So the money market concern is that what people thought were safe, apparently is not as safe as people thought. And the Reserve Fund breaking the buck-- by the way, breaking the buck in that case meant that if you put in a dollar, when you withdrew, as of last week, you would have withdrawn \$0.97 so you'd lost \$0.03 to the dollar, which may not seem like much, but if you went to your Bank of America ATM and you did a withdrawal. And for every dollar you put in, you'd get \$0.97 back, you'd be pretty ticked off, right? So it's something that is of great concern to retail investors.

Anyway, so what happened last week was that-- actually the estimate, I think, is \$90 billion. \$90 billion of money came out of these funds in a week and were put into either cash in the mattress or were put into Treasury securities like the three month T-bills. That's what pushed the prices of those T-bills up and therefore depressed the yields.

And now we're back to a somewhat more reasonable level. I say reasonable because having this kind of a short-term yield of 40 basis points by historical standards is still pretty low. So there are still many nervous investors out there that are trying to figure out what's going on and are waiting for the Treasury to come up with something.

This is another reason why Chairman Bernanke said we have to act quickly because markets are not going to stand and wait for the Treasury or the Fed to do something. Markets will react. And if we wait too long, the fear of this breaking the buck could actually return. And then once you have a mass panic, it's very, very hard to stop that.

Anybody who's ever seen one of these old animal kingdom type of movies about a stampede-- if you've got water buffalo stampeding, it's pretty hard to try to just say, oh, calm down, stop it. Slow down. You can't easily do that once it begins. So you've got to stop it before it actually gets to that critical point. And that's exactly what the government is trying to do. Yes.

AUDIENCE: When you say that-- if you interpret these, like the prices went down so now there's less demand so it means people are more relaxed--

ANDREW LO: Yeah.

AUDIENCE: What if you say, like, if the interest rate went up, it means they are considering the Treasury bills to be more risky or riskier?

ANDREW LO: Well, remember that Treasury bills don't have any default risk, at least as far as we know. We have to be careful about stating. All these unprecedented things have happened. The reason that Treasury bills don't have any default risk is because what the Treasury security is an IOU from the government that says I owe you a certain number of US dollars. And because the Treasury owns the printing press, they can always print out more dollars to give it to you, as long as you're willing to take it.

And at least from this graph, it seems like a lot of people are willing to take it. They really want Treasury bills right now. And they're happy with that. Maybe they're not happy with it, but that's the smallest of all the evils that they can think of in terms of putting their money.

AUDIENCE: So decreasing the yield in Treasury bonds do not mean an increase in the default risk?

ANDREW LO: We hope not. I mean, I guess it could be possible that people are betting that the United States is going to default in 30 years. But my sense is that what's more likely, given that these are default free in the sense that their nominal bonds-- so these bonds are going to be paid off in the little certificates called US dollars that the printing presses can always come up with. There's no risk that they can't print up more dollars. The risk is that \$1 30 years from now isn't going to be worth as much as we thought it was going to be because of inflationary expectations. So as of today, that 30 year yield is not 4%, it's 4.37% and the 0.37 one could attribute to an inflationary expectation by the marketplace.

AUDIENCE: Do you think that the short-term change in the yield has anything to do with the recent devaluation of the dollar against foreign currencies?

ANDREW LO: It could be that because of that devaluation, dollars are cheaper and people are putting more money into US securities. That's also a possibility. But another way of putting that is that foreign investors are now finding treasuries more attractive for whatever reason. So yes, that's also part of that supply and demand story. OK, last question. Yeah.

AUDIENCE: Historically, what is a reasonable yield? You mentioned that before, like [INAUDIBLE] or before all this time?

ANDREW LO: Well, I'm glad you asked that question because we have a graph. These are the historical yields of the three month, six month, one year, two year, five year, 10 year, and 30 year from 1962 to I think it's 2004. And it depends on what flavor you're looking at. And it's kind of hard to read this graph because of the colors. But if you look at the dark blue line, they actually all move together pretty much. But at one point, that short-term yield was 4%. 4% for a three month Treasury bill.

Now these are all annualized remember, so 4% doesn't mean 4% over three months. It means 4% on an annualized basis, which is why 41 basis points-- that's an annualized yield-- for a three month loan just seems ridiculously small by historical standards.

But when people are scared about not getting paid, that kind of fear, that psychological pressure can be overwhelming. And do you believe in these prices? Does it make sense? Well, that I'm hoping to get you not to ask the question in that way, but rather to ask the

question, given market prices and what I know about it, what can I interpret from what's going on and how does that affect me in terms of the financial decisions that I want to make.

So if you are thinking about pricing other securities based upon these kinds of numbers, you need to ask yourself whether you believe the numbers make sense or are they just completely out of whack. And the only way to do that is to understand what the basis is for these numbers. So that's where we're going next in trying to understand how to measure the various different characteristics of these numbers to get a sense of what's reasonable and what's not. Question.

AUDIENCE:

I had a question. I was wondering, those sort of funds, Treasury bills, [INAUDIBLE], is that really individual people buying the Treasury bills or is it more hedge funds and that sort of thing that are systematically hedging the risk against other securities they have that are--

ANDREW LO:

Well, obviously, it's very difficult to tell because we don't see who's making the purchases and sales, but we can tell from certain mutual funds and other money market flows that it seems like most of the flows last week came not from hedge funds, but rather from retail investors that were taking their money out of these money market accounts and then putting them into certain mutual funds that buy only Treasury securities, as well as Treasury securities directly.

All of you can actually purchase Treasury securities directly. There is a website called TreasuryDirect.gov and you can give them your credit card and register as a user and actually participate in Treasury auctions and buy Treasury securities. So it was a combination of those. But it really seemed like it was the retail sector, not institutions, not sophisticated hedge funds that we're trying to do some kind of complex arbitrage.

It was just investors saying, I'm really scared, I want to put my money into something that's real and that will be there. And so short-term treasuries seemed like an answer. And as we saw from last week, gold was the other answer. It's not an answer that I would propose for the typical investor because gold prices are quite volatile. And so you have to be very careful when you make an investment in that security or in that particular asset. But it is something that reflects the state of panic of the marketplace.

And by the way, I'm telling you something that you probably already know in the sense that-- my guess is that deep down inside, all of you are feeling stressed out, right? I mean, you're probably stressed out about things like what does this mean for the job market, for career prospects, and so on. I would urge you all to take a deep breath and not get panicked about

that because, as I said, this is the kind of dislocation that, while very traumatic for market participants today and for those on the losing end, there are as many opportunities created as there are taken away.

And so my guess is that in a year's time, the job market is going to look extraordinarily attractive, particularly for those individuals that are trained in the science and art of financial analysis. So you're all going to be very well equipped for that, even though you may not feel that way right now because of what's going on in the marketplace. So I wouldn't panic certainly.

And by the way, you can see the opportunities that are already being created. Warren Buffett just spent \$5 billion purchasing a stake in Goldman Sachs. And we're going to talk about that in a couple of lectures when we do common stock because I want to use it as an example of getting a good deal in markets. I mean, first of all, Warren Buffett given the success he's enjoyed as an investor, you know that when he plunks down \$5 billion in cash, he's probably doing it for a good reason, not out of charity. And by the way, Goldman raised another \$5 billion from additional rights issues. That's \$10 billion of capital that they raised relatively quickly.

Also Nomura is in the negotiations to purchase certain assets of Lehman Brothers. Lehman has a terrific franchise and has some very significant operations in Asia, as well as in the US. And it's a very smart move on Nomura's part to take advantage of that. So these are the kind of opportunities I'm talking about.

And when Nomura buys Lehman, they're going to have to hire people in order to run the operations because you can bet that the whole dislocation ended up shaking loose a number of very talented professionals from those organizations. So they got to hire. It's going to mean the next two or three months, there may be some difficulties in getting the attention of these organizations because they're in the midst of trying to figure out exactly what kind of organization they are going to have when all the dust settles.

But there are plenty of opportunities that are being created today, including, by the way, the opportunity for the US government to take advantage of all of these distressed assets. So one of the things that you should be careful about when you read that there's a \$700 billion bailout, that is somewhat misleading in the sense that, first of all, we don't really know at this point exactly what the \$700 billion will be for, how it will be used, or how it's dispersed, or is it really

\$700 billion.

A lot of it depends upon how the money is spent and also what happens to housing markets. There is a scenario that I can imagine where the actual amount expended is either zero or negative. In other words, the government actually makes money from the current state of the markets because they can buy assets very cheaply, hold onto them forever until they pay off, and then gain the kind of profit that the original financial engineers were expecting but could not take advantage of because of this liquidity crunch. So we'll talk about that over the next few lectures because we're going to develop some techniques to be able to illustrate how these kind of arbitrage strategies work. Yeah.

AUDIENCE: Thinking about Nomura, is it more certain that it's going to happen or are there many plans that are trying to buy the [INAUDIBLE].

ANDREW LO: Well, certainly there are a number of banks. I wouldn't say many, simply because there aren't that many banks that are well capitalized enough to be able to take on a large unit of a business as big as Lehman Brothers. So Nomura is one of a handful of banks that are engaged, but they seem to be the front runner at this point.

AUDIENCE: Generally, what I've heard kind of from the Asian analyst in Lehman that I had spoken to is that it's quite likely that that might be not be true as well because we find a lot of [INAUDIBLE].

ANDREW LO: Oh, absolutely. There are a number of issues that will come up in any kind of a deal. And so you don't know whether or not something is going to go through until it actually goes through. The same thing could be said for what happened with Merrill Lynch, with AIG. All of these deals are sort of put together at the last minute. And either they get consummated or there's some hitch at the end that makes it difficult.

So yeah, I mean, with a grain of salt, you should take all of these news reports. And literally until the deal is signed, you will not know whether or not something's on. But the point I'm illustrating is that these assets are not completely worthless. What's happened is a very significant liquidity crunch and panic. When that happens, the pricing of all these assets becomes questionable.

OK. We're going to actually see an example of that. So if you wouldn't mind, let me put that off for a few minutes. And then if you have further questions about this, we can come back to it.

Let me start-- so this is lecture six. And what I want to do is to start where we ended last time,

which was a discussion of coupon bonds and how you price coupon bonds simply as a package or a portfolio of pure discount bonds. Underlying this approach to pricing coupon bonds is a very important principle. That's a principle that was given to you in the very first day of class where we talked about the six fundamental principles of financial markets. And it's the principle of the law of one price.

So what I want to do today is to focus on that and talk about the law of one price and what it means for things like arbitrage, leverage, short selling, and relative pricing. Those are the key concepts we're going to cover today. So let me talk about the law of one price and remind you all what it is. It's a very simple idea. It's so simple that you might think it's obvious, but it's got some very, very dramatic implications.

The law of one price says that two identical cash flows must have the same market price. OK. Let me repeat that. Two identical cash flows must have the same price. Now remember that when we think of an asset. We think of an asset as just the sequence of cash flows. That's what an asset is.

So all I'm saying is that when you have two identical assets, they have to have the same price. That's not a very controversial statement. And this principle is one of the most important ideas in all of modern finance because it leads to the pricing of all sorts of securities, including all the derivatives that have ever been priced on Wall Street. They use this idea of the law of one price, OK. Yeah.

AUDIENCE: But wouldn't you have to qualify that by saying it's at equilibrium?

ANDREW LO: No, no. I don't have to qualify that at all. First, because this is a free country and I don't have to do anything I don't want to do. But more importantly, it's because I don't want to restrict it to an equilibrium. By equilibrium, you mean when supply equals demand, right? I don't care about supply and demand. Supply may very well not equal demand. That's OK with me.

This principle of the law of one price, that two identical cash flows have to have the same market price, the only assumption that I need in order for that law to be true is that people prefer more money to less money. And it's not even people. I just need one person in the economy that prefers more money to less money. And I'm happy to volunteer for that position, OK?

Why is that? It's because if that law is violated, if you can show me two identical cash flows

that sell for different market prices, first of all, tell nobody but me about it. What I'm going to do is I'm going to buy the cheaper asset, I'm going to sell the more expensive asset. So as of today, I make money, right, because I bought the cheap, I sold the more expensive. That difference is positive for me.

And then I argue that from that point on, I have no further risk and, in fact, no further obligations. I can just forget about the deal and take my money and spend it. Why? Because I've bought and sold identical cash flows. So from that point on in the future, all the cash flows cancel out. So I'm done.

That's called an arbitrage, or more technically, a free lunch. I've been able to create money out of nothing. It doesn't assume supply equals demand. It doesn't assume any kind of mathematical formula for any kind of instrument. All it assumes is that more people prefer more money to less.

AUDIENCE: So that means so that day when you auctioned the iPhone, then the only difference between an open package and the concealed packaged was that in one case you basically did not give enough information.

ANDREW LO: Right.

AUDIENCE: And it wasn't reaching a fair value and so--

ANDREW LO: Well, wait, wait, wait, wait a minute. Well, when you say fair value, that's a loaded term. What do you mean by fair value? It was a fair value, given all the information that the market had.

AUDIENCE: But it's not because [INAUDIBLE] of that object. When the person buys objects that's packaged, once they open it, they'll know that the cash flow of that object is the same as if it were not packaged. And they're going to read the [INAUDIBLE], as we just mentioned.

ANDREW LO: That's right, but don't you think there's a difference between the package wrapped and the package unwrapped?

AUDIENCE: Yeah, one of them is-- I mean, it's just like a factory which will produce the same stuff but is valued at the market at a lower price than what it would be--

ANDREW LO: Right, but if one factory didn't tell you how it produced it and another factory did. You think that they would sell for the same price?

AUDIENCE: The factories wouldn't, but their future cash flow into revenue could be the same.

ANDREW LO: Only if it turns out that, as a matter of fact, it is identical. But you don't know that ahead of time. You can only price something with the information you have, OK.

So when I say two cash flows are identical, I'm saying that we acknowledge that the cash flows are, in fact, identical. And we know that they're identical. If I put two packages upfront in the room, one is an iPod and the other one is wrapped so it looks like an iPod. It's got the same shape, the same dimensions, but it's wrapped. Would you say that they're identical? You can't know that. If you knew that, then you would price it accordingly.

By the way, that gives you a very important piece of information. Suppose that I did that experiment, I had the iPod that was clearly unwrapped and it was an iPod. And then I had another package that was the identical dimension but it was wrapped, OK. And I auctioned them off and it became clear initially that the two were priced at about the same.

Well, if you saw that, you would then know that it was quite likely that what was in the wrapped package was the same as what was in the unwrapped package, right? But the only reason you would know that is because somebody in the audience apparently bid the same price. Now why would they do that? Either they're knuckleheads or they know something that you don't know. And now that you looked at the price, you actually learn something about what's in there. So what matters for pricing is what the entire market knows, not just what you know, but what the entire market knows.

Now going back to this arbitrage, let's not worry about informational asymmetries. So let's assume that we all know exactly what there is to know, which is that these two securities have the same cash flow. If they have the same cash flow, they've got to have the same price. And that's exactly what we saw last time with this example. Same cash flows and therefore, they have to have the same price.

If they don't have the same price, then instead of feeling upset and despondent that somehow finance theory is in question, the most exciting thing for a finance professor is to see that this theory breaks down, because then we actually can go transact in the marketplace and make money. So if the law of one price fails, instead of calling me up and complaining about it, you should call me up and tell me what it is so I can take advantage of it, all right? It's a great thing.

And here's the example. This price of a coupon bond is got to be equal to the prices of the discount bonds multiplied by the number of bonds you need in order to yield an identical cash flow. OK. Now if you have an identical cash flow, the left-hand side has to equal to the right-hand side. If it doesn't, if the price of the left-hand side is greater than the prices of the right-hand side, then you should feel very happy because what you're going to do is to make money.

Now how much money are you going to make? Well, let's see. What you're going to do is you're going to buy the right-hand side. You're going to buy those bonds and you're gonna sell the left-hand side bond, so what you make is that difference, right? Because you're buying, you're spending a certain amount of money. That's the right-hand amount of money. And the left-hand side is what you're going to get. And so you're actually going to be able to make that difference.

But here's the key. When you make that difference, it's not a risky investment. There is no risk. By the way, how much money did you have to spend? How much of your own personal wealth did you have to commit to this trade?

AUDIENCE: Just the risk.

ANDREW LO: Just the risk? What risk?

AUDIENCE: The difference between them the moment they can buy and sell.

ANDREW LO: Did you have to spend that?

AUDIENCE: You can guarantee it.

ANDREW LO: That's money that you get, not that you have to spend. Is there any money that comes out of your pocket to do this trade? No, because what you are buying is actually financed by what you sold. And then on top of that, you have a little extra leftover, OK?

So this is like one of these infomercials at two o'clock in the morning, real estate, how to make a million dollars in real estate with no money down, right? You've put no money down. You haven't spent any money because what you've done is you've sold the bond and you've gotten this amount of cash. With that cash, you can buy these bonds and how do you know you have money left over? By assumption, I'm assuming that this price is greater than that price.

So now you have money left over. You've put no money down. You have cash in the pocket. You have no risk. You have no obligations because all the future cash flows that you owe are financed completely by the bonds that you bought. One for one, dollar for dollar, it matches. So literally, as of today, you walk away from this transaction a richer individual. Yeah.

AUDIENCE: Professor, isn't, like, by the time that you recognize a an arbitrage opportunity, it corrects itself?

ANDREW LO: Well, let me get back to that. That's an interesting point. The question is, can these things really exist, because once they do, it's almost too late because they're gone.

AUDIENCE: And then follow-up, is it just my imagination or do arbitrage desks exist in financial firms today?

ANDREW LO: Arbitrage desks absolutely exist in financial firms. And what they do is look for this stuff all day long. That's what they do. And so is it true that by the time you identify, it's gone? Well, it's true if you're a finance academic, as well as a retail investor.

But you guys, you're going to go out in the job market and you get to be hired by some of these arbitrage desks. So you're going to be doing this. In fact, you're going to be doing something quite a bit more complicated. I'll get to that in one minute. Before I do that, I want to make sure that everybody is with me about this arbitrage. Yes.

AUDIENCE: Is there any assumption that there is no transaction cost?

ANDREW LO: Very good point. Yes, in this context, there's an assumption that you can buy and sell freely with no transactions cost. Obviously, in the real world, there are transactions cost. You've got to stick those in and figure out whether you can still make money with transactions cost.

Now one form of transactions cost is not a numerical cost, but it's a friction. What do you have to be able to do in order to do this transaction? What financial operation do you need in order to be able to get this deal done? Yeah.

AUDIENCE: Well, you have to be able to short sell.

ANDREW LO: Short sell, right. Remember we talked about short selling last time. That's selling something you don't own. So you have to borrow the bond from somebody and then sell it and get those proceeds. What if it were the case that somebody imposed a constraint that you can't short sell? Who would ever do that? Well, we've seen what happens in the marketplace that can

generate that kind of a constraint. What happens if you can't short sell? Yeah.

AUDIENCE: You need to upfront the cost.

ANDREW LO: What's that?

AUDIENCE: You need to put up the cost of the original security.

ANDREW LO: Well, in fact, yes, you need to put up the cost, but it really defeats the purpose because if you're going to buy the bond and then sell the bond, you've not really done anything, right? This arbitrage argument relies on the fact that you can sell something you don't own by borrowing and shorting it and getting the proceeds, and then giving it back to the person you borrowed whenever they want, OK?

If I don't allow you to short sell, this argument doesn't work anymore. And what that means is that this pricing relationship, this left-hand side has to equal the right-hand side, that relationship goes out the window. For the next several weeks and possibly several months, finance theory is going to be on vacation because the government has suspended short sales for certain securities. And so the kind of force of markets that drive prices towards not an equilibrium, but a pricing relationship that does not depend on equilibrium, but depends on the law of one price, that goes out the window if you can't short sell. Yeah, Megan.

AUDIENCE: I guess I'm just wondering why the Treasury, just looking for insight on this, why the Treasury wouldn't just make the short interest on those short sales so high that it was almost prohibitive, but people who really wanted to short sell would, these instruments to hedge portfolios, could do that if they wanted to pay that.

ANDREW LO: You know, that's a fantastic alternative. I think that would have been far better. You're absolutely right. Increase the cost of short selling by raising borrowing costs. By the way, when you short sell, as we said last time, it's not free. People aren't going to lend you the security for free. They're going to charge you for it. So what if instead of forbidding it altogether, why not just triple the cost or quadruple the cost? Therefore, only people who really, really need to do it will do it.

The problem with that is more of a political one. The political problem is that we want these evildoers, these short sellers that are driving the prices of financial securities to stop their bad activities. And so what we're going to do is to mandate by law that they can't do that. Now, that may be a reasonable thing from a political perspective, but it's not a reasonable thing from an

economic perspective, because what it does is it disrupts relationships like this, pricing relationships like this. Yeah.

AUDIENCE: Can you actually change the borrowing costs for a few types of security [INAUDIBLE] by selling the short [INAUDIBLE] and shorting [INAUDIBLE] the interest rate [INAUDIBLE].

ANDREW LO: Well, if you're the government and you're in a time of crisis, it seems like you can do anything you want. So I mean practically, yes, you can say, for all financial stocks that are on this list, we will now charge an extra high rate of interest for borrowing those stocks.

And, in fact, there exists a mechanism even before this rule was put in place that for certain stocks that are quote "hard-to-borrow"-- that's a technical term that Wall Street firms, brokerage firms use. Securities that are hard-to-borrow means that they're very actively traded and it's very hard to find a counterparty that's willing to let you borrow it from them. The hard-to-borrow stocks typically are lent out at a premium. So they are charged higher prices for those that are very, very popular.

And by simply eliminating short sells, you basically make those costs infinite. That's exactly what's going on. So would it have been better to make them finite, but bigger? That would have been better than what they did, which is making it infinite, but from the purely financial markets perspective, it would be best if there were no restrictions and no transactions cost at all. Obviously, transactions costs are inevitable.

So when you price these relationships, the equality is going to be up to transactions cost. There might be a little bit of a wedge between the left-hand side and the right-hand side, but the difference will have to be small enough that apart from transactions costs, they are not significantly different. Yeah.

AUDIENCE: Could it be that they also stopped the shorting because of some illegal transactions or inside information?

ANDREW LO: Well, if it were illegal transactions, the way they should have done it was to prosecute the illegal parties, as opposed to stopping short sales for everybody, right? That would be a far more effective way of dealing with illegal activities. That's not what they were concerned about primarily. The SEC has an enforcement division whose sole job it is to check on all the kind of transactions that may have occurred to see whether or not there's any kind of illegal activity going on.

It wasn't the illegal activity that prompted the short sales restriction. It was really the concern that financial firms were being pounded by short sellers that were betting on them failing. And the more pressure that they impose on the stock price going down, down, down, the more likely it is that people would lose confidence and then all of a sudden stop doing business with it.

In many ways, that's what happened with Bear Stearns. At least from the historical record, it seems like what happened was that there was a rumor that Bear Stearns was not going to be solvent, even though they didn't have any particular pressure to pay certain debts at that point in time. And somehow that rumor grew into general fear and the short sellers got in and started shorting the stock.

The stock price went down, people looked and said, oh my god, the stock is going down, I'd better take my business elsewhere. And everybody started doing that, and once everybody started doing that, the firm began having great difficulties in maintaining a business. Yeah.

AUDIENCE: Even the days that short selling is not allowed, then the investors who own the more expensive--

ANDREW LO: That's right.

AUDIENCE: --asset are going to sell them because they don't think if they don't sell this today, I'm buying the right-hand--

ANDREW LO: That's right, that's right.

AUDIENCE: So again, you do the [INAUDIBLE].

ANDREW LO: That's a very good point. Let me repeat it. The point is that even if you're not allowed to short sell, then the folks who own the left-hand side, who own the coupon bond, they can say, hey, my coupon bond is worth 110, but I can get the exact same payoff by buying a bunch of pure discount bonds. And it's only cost me 100 to buy them. I'm going to sell my coupon bought at 110 and I'm going to buy \$100 worth of these this discount bonds and I just made \$10.

Now that can happen as long as two things occur. One, the person who owns the bond knows about this. And two, they actually want to take the trouble to do the trade or they're able to do the trade. The problem is that most of the folks that bought the bond on the left-hand side are

pension funds that are not in the business of doing this kind of arbitrage transactions.

So they're trying to do that. All they want to do is they've got a bunch of pension plan participants. They need to pay out their benefits. They've got contributions. They just want to match assets to liabilities. They're not in the business of doing this kind of high speed transaction.

So what that means is that you're still right, that if they realize this relationship is there, and they are in a position to do the trades, they will sometimes. And then that will force prices closer, but it's not going to be the same kind of numerical identity that has to hold when you've got greedy people like myself trying to do the trades and being able to do the trades at a moment's notice.

AUDIENCE: I'm especially more interested about if these players are big enough in the financial markets, then it will correct and it could get out of that.

ANDREW LO: Exactly, right. There are all sorts of additional complexities of being able to do the trade that as a pension plan, you're not even allowed to do, never mind whether you want to do them or not. So these are the kind of restrictions that would present this gap.

But let's for now assume that there's no gap. We're going to assume that there's no frictions. We're going to assume that there's no short sales prohibitions. And when that happens, this pricing relationship has to hold.

Now I promised you-- I want to show you something more complicated that all of you may do when you get out of here, and that's this. Instead of looking at just one bond, what if we looked at a whole bunch of coupon bonds? Now, let's take a look using this pricing that's based on the law of one price and basic greed.

We've got n bonds here, one through n , and each has its own coupon, whatever that is. I'm just going to use notation to write down that each bond has its own particular coupon, right? So you've got a 3% 10 year bond. You've got a 4% 30 year bond. You may have 5 and 1/2% five year bond.

So some of these coupons may be zero because capital T I'm going to assume is the most extreme 30 year period, right? So a five year bond would have coupons for the first five years and then for all of the coupon payments after that, I'm going to have zero, zero, zero, zero, zero, right?

So this should hearken back to your high school algebra days, where you have multiple equations with multiple unknowns. Now what's unknown here in these equations? What are the unknowns? Well, you observe the prices, right, from the marketplace, so the left-hand side is not unknown.

What about the right-hand side? What are the unknowns? What do you observe, what are the knowns? How about that? Let's start with that. Yeah.

AUDIENCE: The coupons.

ANDREW LO: The coupons, right. If I tell you that I've got a 30 year 4 and 1/2% bond, you know what the coupon is going to be, right?

AUDIENCE: Because we don't know the yield.

ANDREW LO: What's that?

AUDIENCE: The yield.

ANDREW LO: The yield is what we don't know. All right. That we either get from the marketplace or we can try to solve it from these prices. But how many different yields do we need in order to price each bond? These are T year bonds. How many yields?

AUDIENCE: T.

ANDREW LO: Yeah, we need T yields, one for each year. We have T yields in these equations and they're the same across the different bonds, right? Because we're using the same pure discount bonds to replicate each of these coupon bonds.

I've got T unknowns. How many equations do I have?

AUDIENCE: n.

ANDREW LO: n bonds, right? Now, on any given day, you might have 200 to 300 bonds that are trading. But you've only got 30 unknowns. So let's think back to your high school algebra or college linear algebra days. If you've got 200 equations and 30 unknowns, how many solutions do you have?

Well, let's get simpler. Suppose you had two equations and two unknowns.

AUDIENCE: Just one solution.

ANDREW LO: You have one solution, assuming certain kind of conditions that hold like invertability, which we're going to talk about in a minute. Two equations, two unknowns. You have one solution. How about one equation and two unknowns. How many solutions? Infinite. OK, that's right, because you've got that extra degree of freedom, right? There's lots of different solutions.

What about three equations and two unknowns. Now how many solutions do you have? Yeah.

AUDIENCE: Because it's linear, you only have one.

ANDREW LO: It's linear. Are you guaranteed to have one?

AUDIENCE: You might have zero.

ANDREW LO: You might have zero. What are the conditions under which you would have one?

AUDIENCE: If the solution of two equations applied for the third one.

ANDREW LO: Right. There is a possibility you have one solution if the solution of the two equations actually applies to the third. Do you remember under what condition--

AUDIENCE: Exhausted.

ANDREW LO: --that, what?

AUDIENCE: Exhausted.

ANDREW LO: Exhausted? That's not quite the term that mathematicians use. There's something called linear dependence. That's a very complicated word that describes the fact that if you have these two equations and two unknowns, and you have one solution, you can just take an average of those two, a combination of those two, and you'll get the third one. You can actually replicate the third equation from those two.

What happens if you don't? What happens if you cannot take combinations of the first two equations to get the third? What does that mean?

AUDIENCE: One of the numbers is wrong.

ANDREW LO: One of the numbers is wrong. What do you mean by that?

AUDIENCE: It just couldn't be-- it can't be a solution.

ANDREW LO: It can't be a solution. OK, now this is getting really interesting because on the one hand, you're probably getting confused about what any of this has to do with money. In a minute, I'm going to tell you it has everything to do with money, as you might expect.

If we've got two equations and two unknowns, and we have one solution, that basically says that there are two yields, a one year yield and a two year yield that is able to price both bonds. And that better be the case because we're going to use these two yields to replicate the bonds. And so this kind of a relationship has to hold.

Now if we add a third bond, if we add a third bond, and those same two yields that worked for the first two bonds, they don't work for the third bond, something's wrong, right? It means that this third bond, its price does not satisfy the relationship between those two yields.

That's evidence of a mispricing. Something is wrong. But in this case, what's wrong is that there's something mispriced between those two bonds and the third. So when you run into a situation like that-- and we're going to give you an example in the problem set-- when you come across that, what that means is you should be extremely excited, as opposed to depressed in math class, you know, gee, there's no solution.

In finance, what no solution means is that there is a transaction. There exists a linear combination of the first two bonds and the third that A, costs you no money down; B, will generate cash flow today; C, will require no future payments of any sort, so it's riskless. It is a free lunch. It is an arbitrage.

Now that's with three equations and two unknowns. Anybody can do that, right? That's easy.

What if it were 200 equations and 30 unknowns? Now, not so easy. Now you actually have to know something about linear algebra. Now the whole notion of what an invertible matrix is, what the eigenvalues are, all the kind of infrastructure that you can build for understanding this becomes relevant as a quant trading on a proprietary trading desk.

In the 1970s, a number of MIT graduates were hired by Salomon Brothers. They knew very little about fixed income securities. They knew the basics, which is what was taught here, but they didn't know much about market realities in practice. And so when they went to solve their three equations and two unknowns, they observed inconsistencies. And they didn't do three

equations and two unknowns, they did 200 equations and 30 unknowns.

And in the 1970s, that was not easy to do because we didn't have PCs, we didn't have Excel. We didn't have a lot of the tools that we have today. And what they did was they took these simultaneous linear equations. This is high school algebra. Even back then it was high school algebra, OK? And they just cranked through and looked for mispricing, looked for no solutions. And they found a lot of cases with no solutions.

In one of the years during the 1970s or 80s, one of these MIT grads was paid an annual bonus of \$22 million for doing this, for solving simultaneous linear equations. So this is an extraor-- and if that was what he got paid, you imagine what he generated for Salomon Brothers. It was a lot more than \$22 million.

This activity is known as fixed income arbitrage. There are many other versions of it, but this is the plain vanilla version. And by the way, the plain vanilla version, it still works in the sense that occasionally, if you're quick enough and you have the right tools, you can identify mispricings and take advantage of them quickly.

It doesn't last long, so you're absolutely right. It doesn't last long. But the person who gets paid is the person who can do this the fastest and is able to understand the interrelationships among the securities. And you can understand now why retail investors have no chance of doing this kind of thing on their own. This is something you definitely don't want to try at home, OK?

I know you guys have MATLAB and you can do matrix inversions, but there are lots of other frictions, transactions costs, and imperfections that you have to build into this analysis. But once you do, all sorts of interesting things start popping out. All right, question, yeah.

AUDIENCE: Can you give a couple reason why this does still exist? Like, why hasn't the arms race between everyone narrowed it down to basic efficiency?

ANDREW LO: Well, it's actually much narrower now than it used to be. So in fact, the market has gotten much, much more competitive. But it's not down to zero precisely because there are frictions and other aspects. For example, some of these bonds, they have weird features, like they're callable. Or in some cases, they may have certain types of other requirements and market institutional imperfections that require you to build in those constraints when you do the analysis.

And so it's really about who has the better model. And frankly there's an arms race that's been going on for the last three decades as to who has the fastest computers. The first supercomputer that was ever installed on Wall Street, a Cray-2, was installed at Salomon Brothers doing simultaneous linear equations, among other things. And so this is where technology has played a really big role in the developments of market prices. Another question?

OK, so this very, very simple idea, and again, it is simple, has all sorts of important ramifications for the pricing of bonds and other securities. What that tells us is that market prices have all sorts of information that are incorporated into it. And one of the things that we want to understand is how to interpret that information.

In particular, the first thing I want to do is to understand the risks, all right, because we talked about the fact that some of these market prices may have missed the risks that were implicit in some of the trades. And so the question is why and how do we measure the risk of a bond portfolio.

So what I want to do is to now look at the market price of a bond in terms of a function that has inputs and the price is the output. And I want to ask the question, what kind of fluctuation in the input will yield fluctuation in the output? You know, we now know how to price these things. We now know how the relationships must work from a present value approach. And now what I want to do is to ask whether we can measure the sensitivity.

So one way to do it is to just graph the price of a bond as a function of its yield. And we know that there's an inverse relationship, right? Just like we saw with treasuries this week versus last week, right? Last week it was a big run on treasuries, lots of people wanted to get into them. Price goes up, yield goes down. And this week, less pressure, so we have yield going up and the prices coming down. That provides us with one kind of measure of risk.

So the kind of measure I'm talking about is the slope of this line. The instantaneous slope that tells us for a bit of a tweak in interest rates or yield, what does that do to the market price? Because obviously when you're an investor, you're focusing on the price, right? Yield is a convenient way of summarizing the properties of a bond, but ultimately what you care about in your portfolio is price.

And so the question is for a move in the interest rate, what does that do to the price? Well, it

turns out that there is one way to get at that that is somewhat more intuitive than just looking at this kind of fluctuation. It's called duration and it's named after a person Macaulay who first proposed it as a way of measuring how risky a bond is. What Macaulay noted was that the longer the maturity of a bond, the more sensitive is the bond price to the yield.

So for example, a 30 year bond, when you move the interest rate by one basis point, will have a much, much larger price fluctuation than a three month bond, right? Why is that? Anybody give me some intuition for why that should be? Why that makes sense? Why should a longer maturity bond be more sensitive to changes in yield? Yeah, Ken.

AUDIENCE: Because you're hanging on, because the cash is tied up longer.

ANDREW LO: Yeah, and?

AUDIENCE: And because the risk-- because the opportunity cost essentially of having that money tied up for that long means you can't spend it on all these other things over those 30 years.

ANDREW LO: That's right. You're investing for a longer period of time so, in fact, the opportunity costs, as measured by the foregone interest, is going to be much larger. In other words, the discount rate that you use, this one plus r , you've raised that to the 30th power, not to the 1/4 power, and so something that's raised to the 30th power in the denominator, it has a much larger impact when you perturb that denominator by a little bit, because you're actually increasing that power. You're increasing that quantity by that 30th power.

Well, so if that's the case, if it's the case that the longer the maturity, the more at risk you are per basis point of interest rate fluctuation, then why don't we just measure the average duration of the bond? By average duration, I mean how long does the bond last when you weighed it by the coupon payments that it pays you?

So for a pure discount bond, the duration is simple. It's just the maturity date, right? If you've got a 30 year strip that pays you \$1 in 30 years, the duration of that bond is just 30 years. But what if you had a coupon bond that paid you coupons all along the way? Well, then, you should take a weighted average of the 30 years and give some weight to the early years too, because the early years are years where you're going to receive cash. And therefore, interest rate fluctuations are going to affect the value of those cash flows.

So the weighted average term to maturity is simply equal to the sum of the date, 1, 2, 3, 4, 5, 6, 7, multiplied by a weight factor that sums to one. And let's just use, as a weight factor, the

proportion of present value of cash flow on that date. So if you take the weighted average where you take the weights as the present value of the coupon divided by the present value of the bond, those weights certainly sum to one. And then you're weighting that by the date, the year that you get paid, that number will give you what's called Macaulay duration.

So it turns out that the reason that Macaulay duration is interesting is that when you take a look at bond prices and you ask the question, for a certain percentage change, a certain basis point change in the interest rate, what does that do to the bond price as a percentage of its current price? So this is the sensitivity. It turns out that you can show that that's actually the negative of the Macaulay duration divided by one plus the bond yield.

So this is a long way of stating that the duration gives you a measure of how sensitive the bond price is to changes in yield. The longer the duration, the more sensitive the bond is to changes in yield. And duration now just means a weighted average of all of the payout dates that a bond will have. If a bond pays a lot of its cash up front and very little in later periods, is the duration high or low?

AUDIENCE: Low.

ANDREW LO: Low. And so if a duration is low what that says is that there's not a lot of sensitivity to changes in yield on the bond price itself. If on the other hand, all of your payment is out in the future, way out in the future, that's going to make it very interest rate sensitive. And the negative number here indicates that there's an inverse relationship between price and yield.

So when bond investors look at a particular portfolio of bonds, and this is key, they look at a portfolio of bonds because that's what they're going to be investing in. When you put your money in a money market fund or in a bond fund, medium term, long term, you're not putting it in one bond, you're putting it in a whole set of bonds. Your natural question is, how sensitive is that portfolio to changes in interest rates? And the answer is it's related to the duration. And so if I told you that this portfolio has a duration of five and 1/2 years, that will give you some intuition for how sensitive or how risky that portfolio is.

So duration is a measure that I've defined for a bond, but you can define it for a portfolio of bonds, simply by taking the cash flows at every period and then computing a weighted average where the weights are the present value of those cash flows as a function of the entire portfolio. OK, so here's an example where I've got a four year Treasury note with a face

value of \$100, 7% coupon selling at a \$103.50, which yields 6%. That's the yield, that's the why that makes this 103.5 equal to the present value of all of those coupon payments plus the return of principal.

And so here what I've done is to calculate for you the cash flows of the bond, the present value of those cash flows, and then the respective product of t times the present value of cash flows. So you can actually compute for yourself that duration number. And you can get a sense of exactly what that is.

So the duration is about 7.13 years. So duration is a measurement that is in units of years or half year units. Sorry, 7.13 half year units. And the modified duration is going to be given by 6.92. Price risk at a yield of 3% therefore is going to be given by just this expression right here.

What that says is that if the yield moves up by 1/10 of a percent or 10 basis points, the bond price is going to decrease by 68 basis points. That will give you a sense of how exposed you are. If you have very long duration portfolios, that means that you're going to be in for a wild ride as interest rates swing around a lot. And if you're willing to take on that risk, that's great, but you're going to be at least compensated for that.

Macaulay duration, you can compute it in this way. For intra-year coupons, that's a very straightforward calculation. So you want to make sure you know how to do that. You simply just use the usual discounting and dividing the yield by the appropriate payment periods.

And now the last concept that I want to cover today is convexity. Convexity is another measure of risk. It's the second derivative. What it measures is how the sensitivity itself changes. And it turns out that convexity, as a measure, gives you sort of a higher order approximation both to the price of the bond, as well as how the bond is going to move with respect to interest rates.

So let me just show you what the derivation is. It's pretty straightforward, but if you have any questions, I'm happy to discuss it. You take the price of a bond and you take the second derivative with respect to the yield. What you're going to get out of it is an expression that looks like this. Now that expression in and of itself has relatively little intuition. But let me just write down the percentage change in the first derivative as $v_{sub m}$, and then I'm going to show you an interesting relationship.

For those of you who remember your high school calculus or college calculus class, you'll remember there's something called a Taylor approximation, right? Taylor series. This is a

method of approximating nonlinear relationships using polynomials, powers of the variables in question.

If you took the bond price as a function of the yield, it's a nonlinear function, of course, because you've got that discounting going on. Then you can ask the question, how can I approximate the bond price as a function of the yield if I'm willing to take a couple of terms of the approximation? And when you do that, you get what may look like an awful expression, but actually it's quite beautiful in its own right.

What this says is that the price of the bond at a new interest rate, at a new yield, if the yield changes, then price of the bond at the new interest rate is going to be equal to the price of the bond at the old interest rate, multiplied by a factor of something or other. And that factor is going to be given by one minus a term that's a linear function of the yield, plus another term that is a quadratic function of the change in the yield.

So this is what I mean when I say this is a second order approximation to the pricing relationship. And this basically gives us a way to figure out when yields move, what does that do to my portfolio. Now you might be thinking, gee, this is an awful long way to go to try to figure that out. Can't we just use an Excel spreadsheet and then move the interest rate and then recalculate and see what that does?

Today, you can, but in the 1970s, you couldn't. You didn't have that ability. So much of this framework was developed in the 1970s because people wanted to have easy ways of not only pricing bonds, but figuring out what the risk of their positions were as bond traders. And to do that quickly is very difficult.

You remember the story I told you about when I got a mortgage 20 years ago and the bank loan officer just could not figure out what my mortgage payments were. She had to go look for a book and try to thumb through what those calculations are. Well, imagine if you're a bond trader trading literally every minute of the day, and there was bond trading going on in the 1970s, believe it or not. You have to figure out these pricing relationships. And it was not so easy.

So a number of mathematicians came up with these kind of relationships for bond prices. In fact, it's funny. Bond pricing is actually quite a bit more of a mathematical art than equity pricing simply because with a bond, there aren't that many moving parts. And so they aren't subject to mathematical analysis like this, whereas with stock prices, since there are so many

factors impinging on it, the actual tools that we use are quite a bit less complex, at least from a historical perspective.

So the purpose of this is really just to approximate the risk of a bond portfolio. You want to know when you change yields what that does to the portfolio. And you also want to know if the yield changes in volatility, what does that do to the portfolio. So this first term here, this term tells you about shifts in the interest rate. What this term does is tell you about fluctuations or volatility of interest rates. And it turns out that volatility has an impact on bond portfolios, as does changes in the level of interest rates.

Just looking at this, can anybody tell me off the top of their head what direction this goes? In other words, if you are holding a bond and interest rates rise, we know that bond prices will fall. But what if the volatility of interest rates rise, as they have over the last few weeks? Yields have bounced around a lot more lately than they have in the past.

What does that do to a bond? Does it make it more valuable and less valuable? Let's put it that way. How do you know? Less, more? We have some volatility here.

Take a look at that expression. v sub m is going to be the second derivative, and then you've got a term there that's going to be the change in the yield squared. If the change in the yield squared goes up, other things equal, and other things are never equal, but economists like to say other things equal, it actually makes bonds more valuable.

In this respect, owning a bond actually is sort of like owning an option. Now you don't know about options yet. We're going to come to in a few lectures. But it turns out that having an option when volatility goes up can be very valuable. And similarly for bonds, bonds have option-like characteristics. And we see that here with this approximation. That second term will actually yield some actual value when volatility goes up.

Now I do a numerical example here that I'd like you to look through on your own where I compute for you both the first and second terms of that approximation. And you can actually see how good the approximation is. So I'd like you to take a look at that, make sure you understand it, and next time, be happy to answer questions about this.

What we're going to cover on Monday is risky debt, which of course is directly related to what's going on in markets today. And so I'd like you to read up on that in the textbook. And when we come back on Monday, we're going to talk about debt ratings and possibly what went wrong

with all of these subprime securities.