Discount rates

Class 12
Financial Management, 15.414

Today

Discount rates

- Using the CAPM
- Estimating beta and the cost of capital

Reading

- Brealey and Myers, Chapter 9
- Graham and Harvey (2000, p. 1 – 10)

Review

The CAPM

- **Measuring risk**
  A stock's systematic risk is measured by beta, the slope when the stock return is regressed on the market:

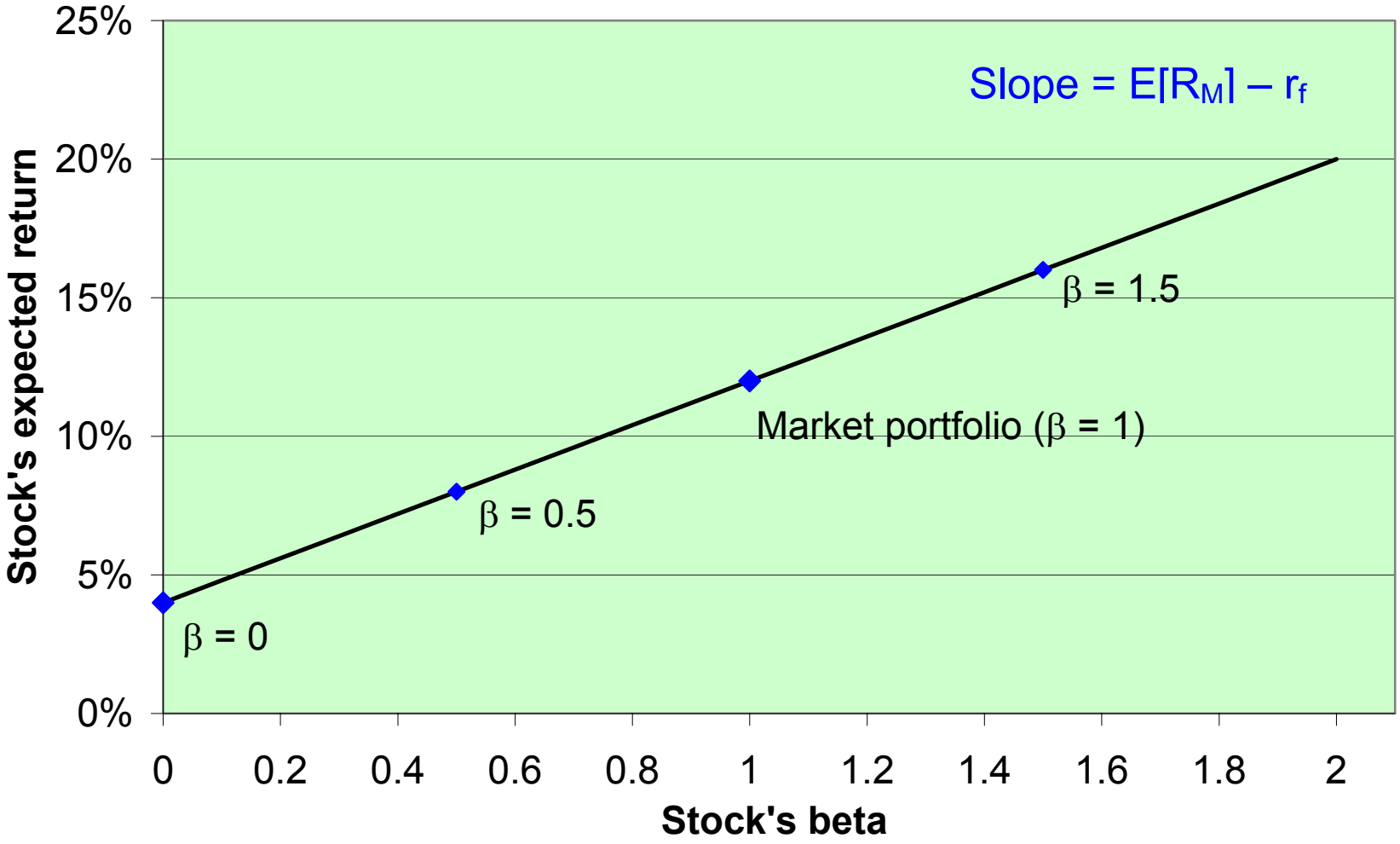
  $R_i = \alpha + \beta R_M + \varepsilon$

- **Required returns**
  Investors should be compensated for bearing non-diversifiable, beta risk. The required return on a stock is:

  $E[R_i] = r_f + \beta_i \underbrace{E[R_M - r_f]}$

  Market risk premium

The risk-return trade-off



Using the CAPM

Valuation

$$NPV = CF_0 + \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} + \ldots$$

Discount rate

The rate of return that investors demand on investments with the same level of risk.

CAPM

- Risk = the project's beta
- Discount rate = $r_f + \beta_{project}\, E[R_M - r_f]$

Using the CAPM

Practical issues

1: How can we estimate the project's beta?

2: What is the riskfree rate and the market risk premium?

3: How does debt affect risk and the cost of capital?

4: Additional risk factors?

Example

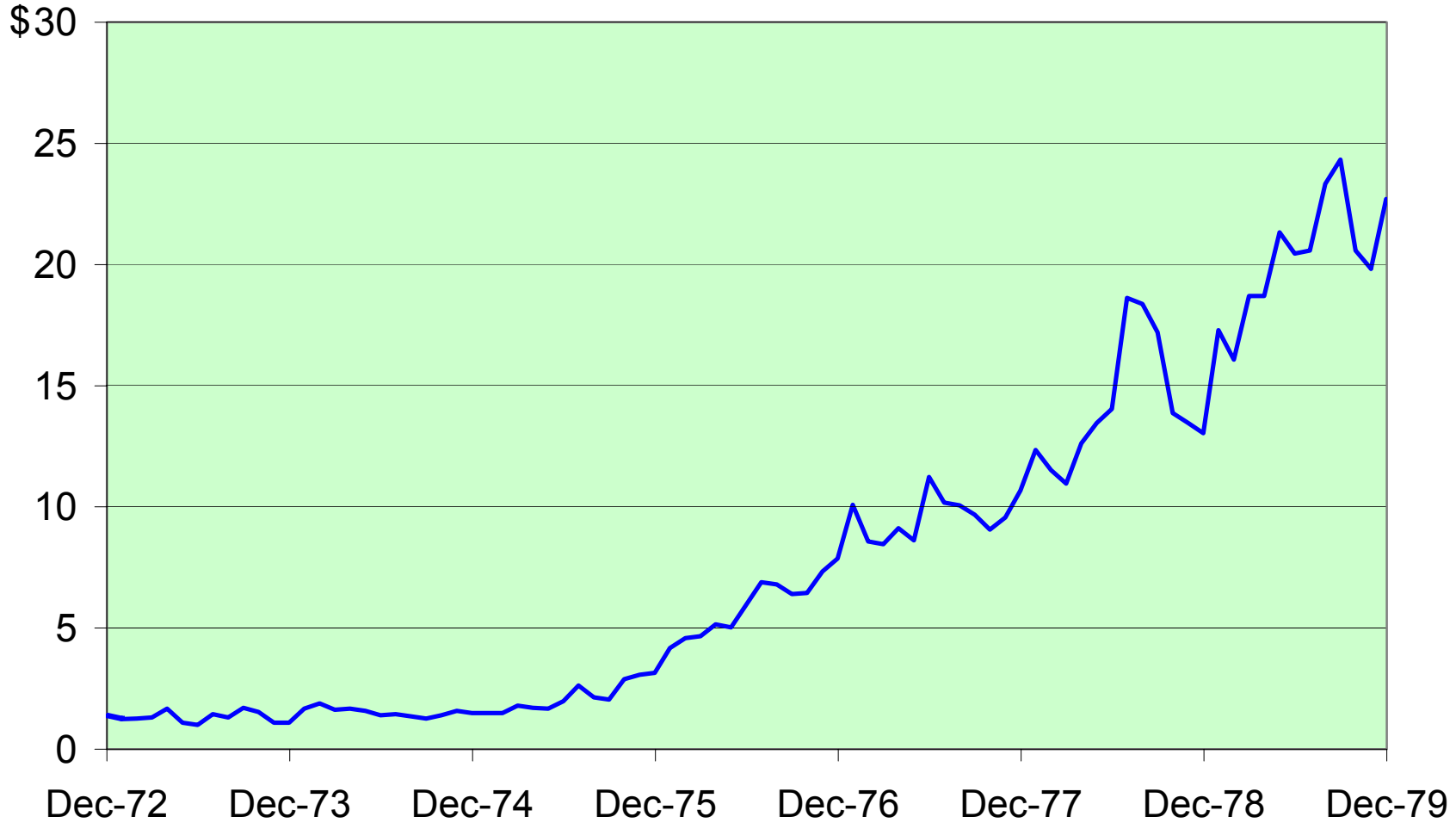
It's 1979. Southwest Airlines, a growing start-up, has been profitable as the low-cost airline in the Texas market. Southwest is thinking about expanding to other U.S. cities. Management forecasts that the expansion will cost $100 million over the next few years but will lead to strong future growth ($ millions):

Year	1977	1978	1979	1980	1981	1982
Sales	49.0	81.1	136.1	213.1	270.4	331.2
NI	7.5	17.0	16.7	28.4	34.2	34.0
NWC	5.1	9.7	10.7	12.4	11.1	19.3
CAPX	41.5	45.1	54.5	56.7	79.4	140.2

Growth is expected to slow to 10% annually after 1982.

What cost of capital should Southwest use to evaluate the proposed expansion?

Southwest stock price, 1970 – 1979



Issue 1

How can we estimate the project's beta?

What factors are important?

- **Two approaches**
  Estimate the firm's beta
  Estimate the industry's beta (comparables)

- **How much data?**
  5 – 10 years of monthly data

Estimating beta

1: Estimate the firm's beta

- **Advantage**

  If the project has the same risks as the firm (an expansion), this approach measures exactly what we want

- **Disadvantages**

  Generally not very precise (high standard error)

  Firm's beta might change over time

  Can't be used for projects in a new line of business or for diversified firms

Southwest

Is this approach useful for SW?

- Is the risk (beta) of the expansion likely to be the same as the beta of the firm?
- Is Southwest's past beta likely to be a useful guide for the future beta of the project?

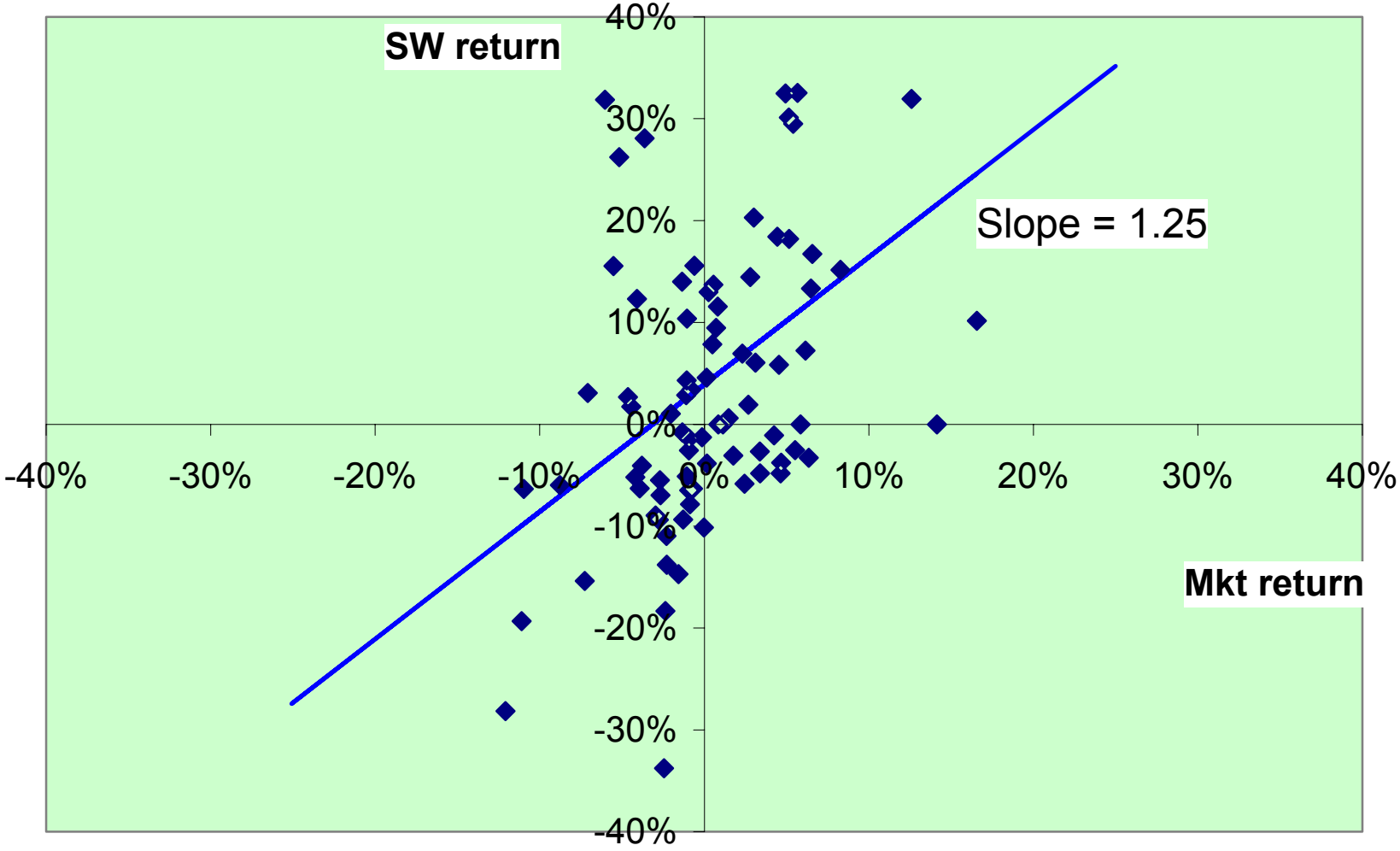
Southwest, 1973 – 1979 (84 months)

$R_{SW} = \alpha + \beta_{SW} R_M + \varepsilon_i$

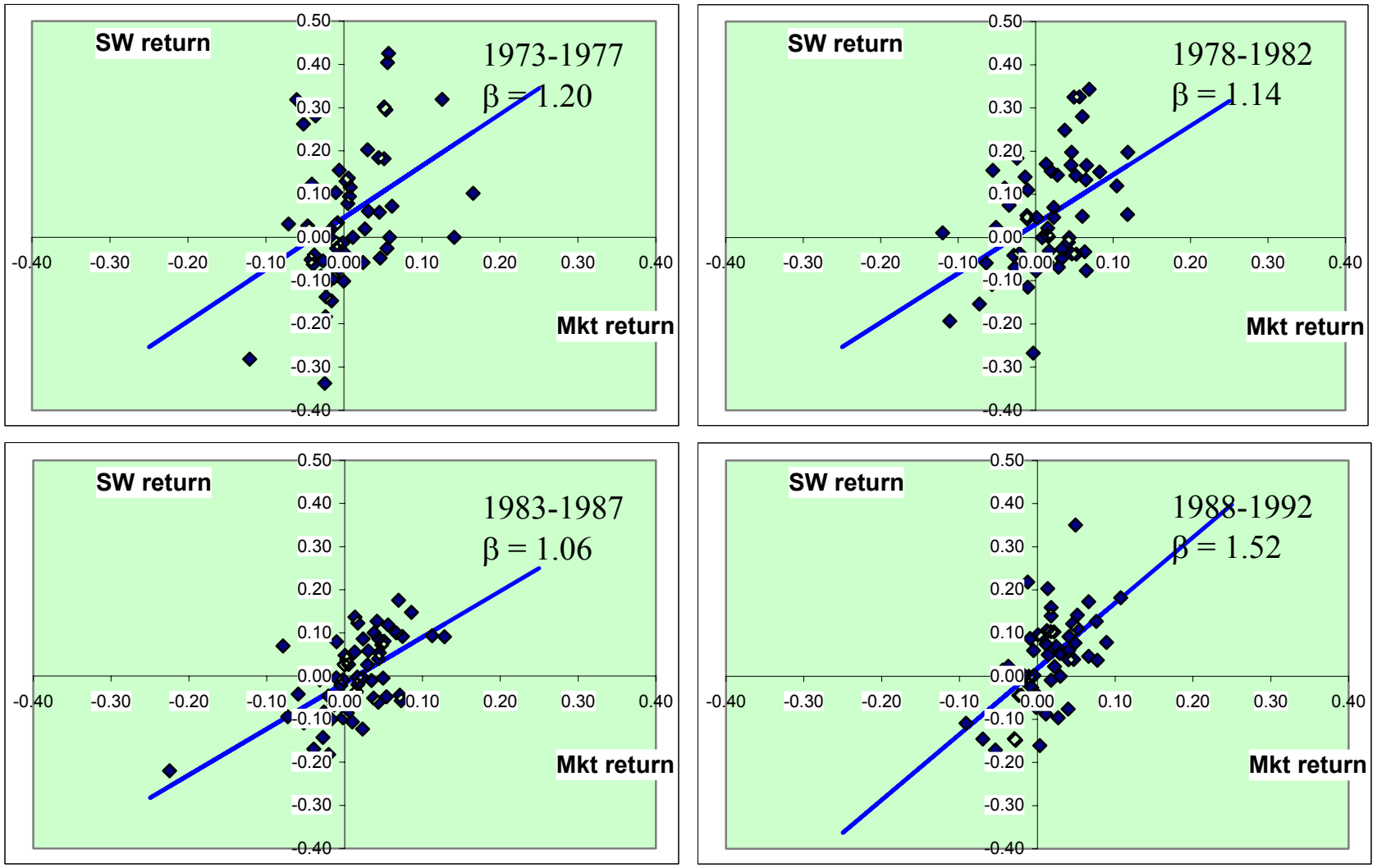
Estimate: β_{SW} **= 1.25** (std error = 0.31); $\mathbf{R^2 = 0.16}$

[R_M = return on a market index, like S&P 500]

Southwest vs. Total U.S. market return



Southwest's beta over time



Estimating beta

2: Estimate the industry's beta*

- **Advantages**

  Beta estimated more precisely.

  Appropriate if the project is in a new line of business.

- **Disadvantages**

  Do the firm's really have the same risk as the project?

  Do they serve different markets? Do they have more debt? Do they have the same cost stucture?

* Estimate the betas of individual firms and then average, or estimate the beta of an industry portfolio.

Southwest

Is this approach useful for SW?

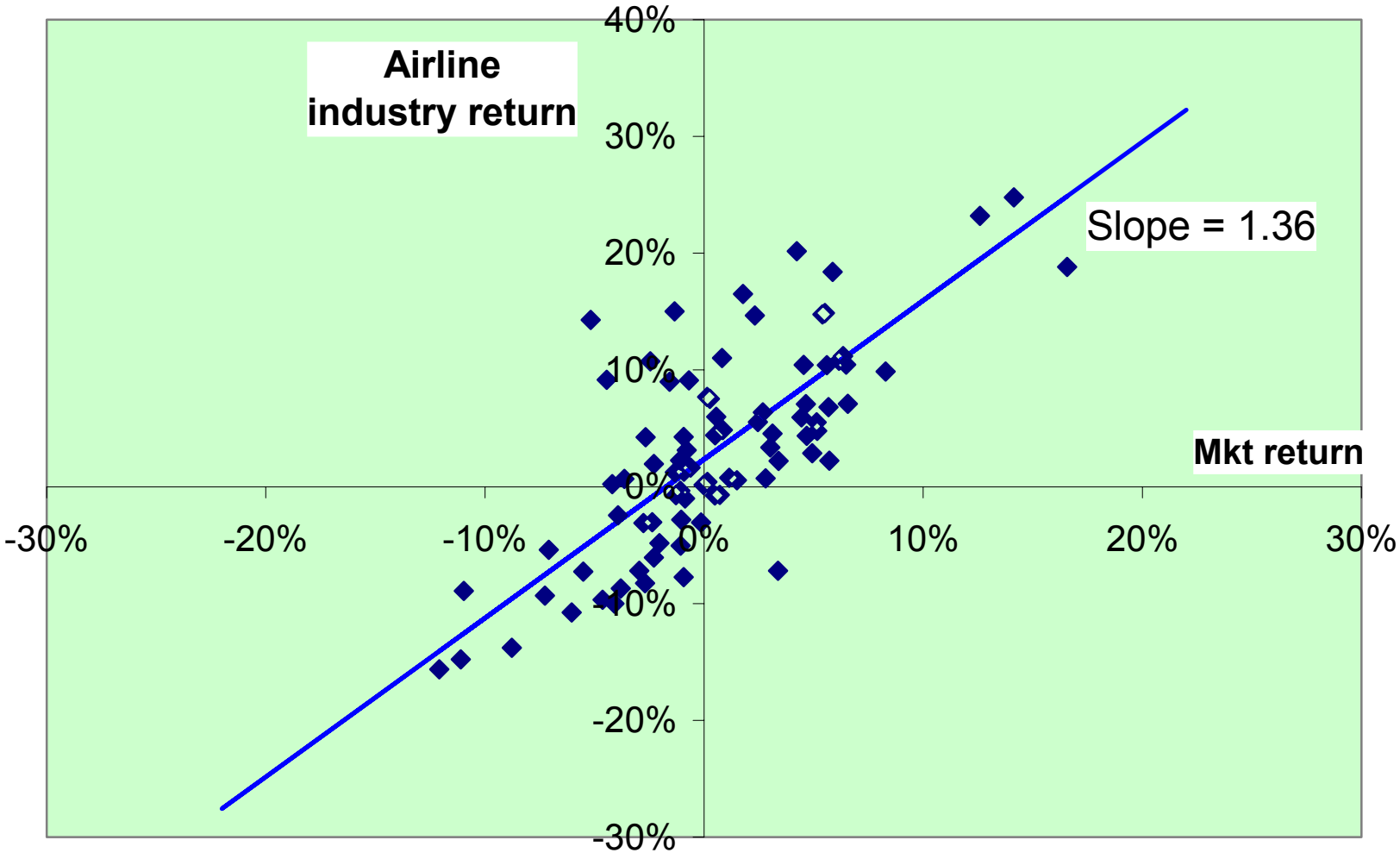
- Is the risk (beta) of the expansion likely to be the same as the beta of other airlines?

Airline betas, 1973 – 1979

Airline	β	Airline	β
American	1.42	Northwest	1.35
Continental	1.18	United	1.55
Delta	1.30	USAir	1.37

Average = 1.36, standard error of 0.13

Airline industry vs. Total U.S. market return



Issue 2

Riskfree rate?

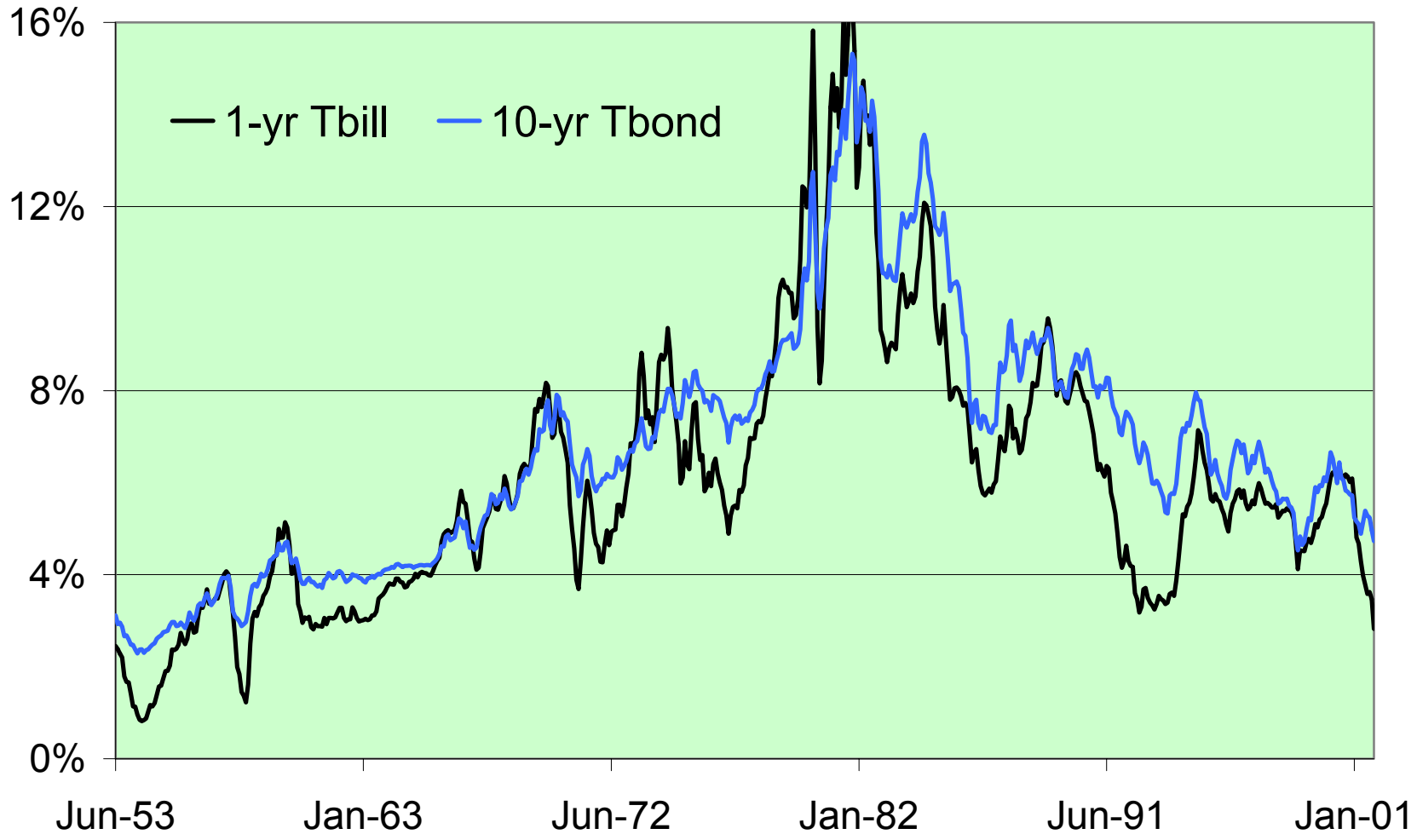
$r_{project} = r_f + \beta_{project} E[R_M - r_f]$

Should the riskfree rate be the short-term Tbill rate or the long-term Tbond rate?

Match horizons
If short-lived project, use Tbill rate
If long-lived project, use Tbond rate (say, 10-year)

Riskfree rate changes a lot over time
1979: Tbill rate = 9.65%, Tbond rate = 10.39%
2003: Tbill rate = 0.93%, Tbond rate = 4.31%

Interest rates, 1953 – 2001



Issue 2

Market risk premium?

$r_{project} = r_f + \beta_{project} E[R_M - r_f]$

Historical estimates

1872 – 1999: 5.73% (std error = 1.63%)
1926 – 1999: 8.26% (std error = 2.24%)
1963 – 1999: 6.44% (std error = 2.51%)

r = DY + g

1872 – 1999: 3.64% (std error = 1.15%)
1872 – 1949: 3.79% (std error = 1.78%)
1950 – 1999: 3.40% (std error = 0.99%)

Constant growth model

$$P = \frac{D}{r - g}$$

Going forward? My guess, 4 – 6%

Market risk premium

Survey of CFOs

Source: Graham and Harvey, 2002

Southwest

Cost of capital

Firm's beta: $\beta_{SW} = 1.25$

Industry's beta: $\beta_{Airlines} = 1.36$

$\beta \approx 1.30$

Riskfree rate = Tbond rate = 10.39%

Market risk premium = 5.0%

Discount rate*

$$\mathbf{r = r_f + \beta_{project}\, E[R_M - r_f] = 10.39 + 1.30 \times 5.00 = 16.89\%}$$

* If no debt

Issue 3

Debt financing, part 1

If the firm has debt, the cost of capital (discount rate) is a weighted average of the costs of debt and equity financing.

Cost of equity: $r_E = r_f + \beta_E E[R_M - r_f]$

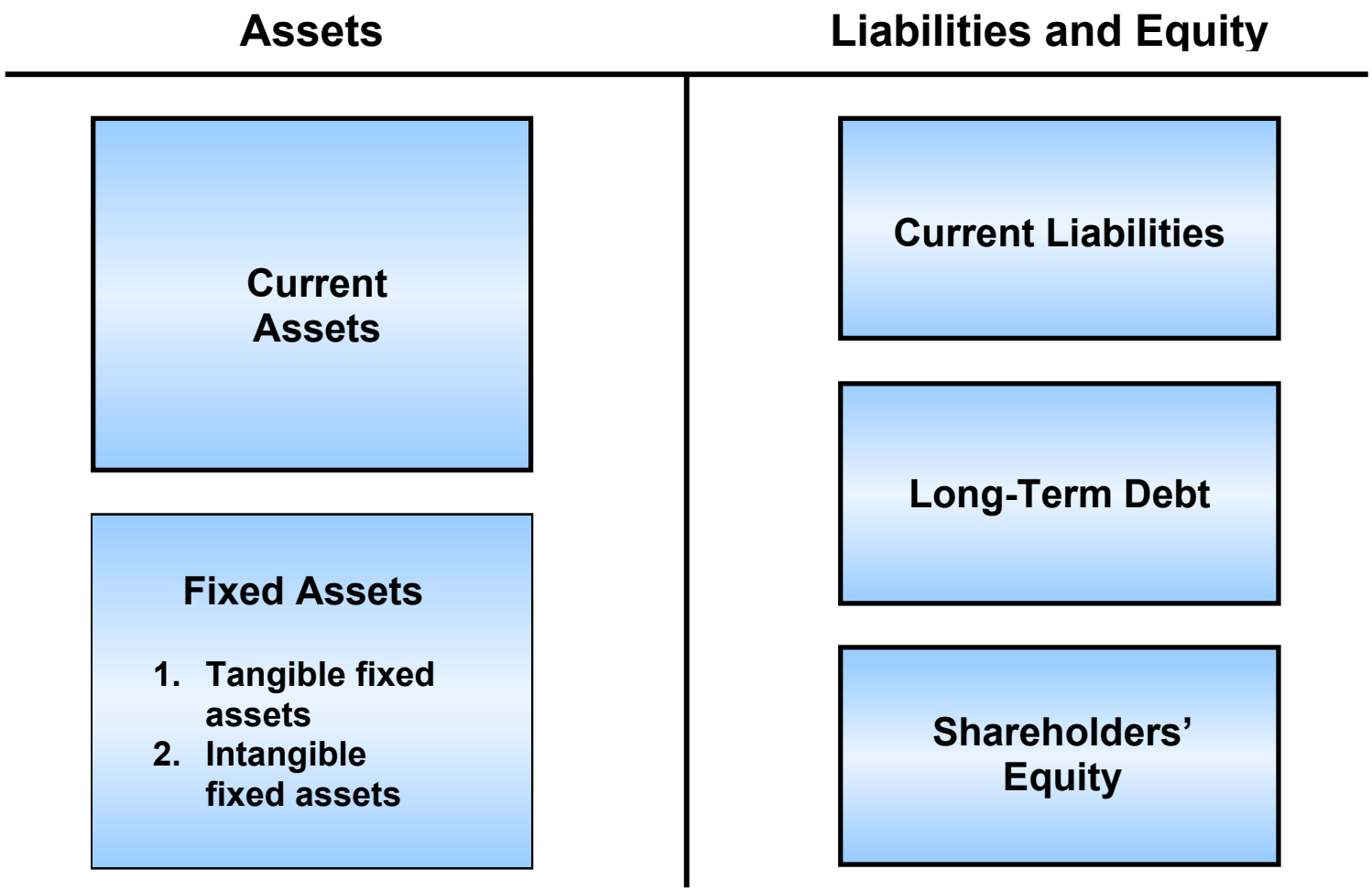
Cost of debt: (1) $r_D = r_f + \beta_D E[R_M - r_f]$

(2) r_D = yield on the firm's bonds

After-tax weighted average cost of capital

$$\mathbf{WACC} = \frac{D}{A}(1-\tau)r_D + \frac{E}{A}r_E$$

Balance sheet



Southwest

In 1979, Southwest was financed with 20% debt (debt / firm value). The borrowing rate was 11.4% and the tax rate was 35%. What is Southwest's WACC?

- **Cost of equity**

  $\beta_E = 1.30 \quad \Rightarrow \quad r_E = 10.39 + 1.30 \times 5.00 = 16.89\%$

- **Weighted-average cost of capital**

  $WACC = 0.20 \times (1 - 0.35) \times 11.4\% \; + \; 0.80 \times 16.89\% = 14.9\%$

- **Discount rate = 14.99%**

Issue 3

Debt financing, part 2

If firms have different debt ratios, we cannot directly compare the stock betas of firms in the same industry.

Firms with higher leverage should have riskier equity
Higher D/V $\rightarrow$ higher β_E
Complicates the use of industry betas.

(1) Estimate equity betas for each firm

(2) Calculate r_E and WACC for each firm

(3) Use the industry's WACC to estimate the cost of capital for the project

Southwest

Airline industry

Equity betas

Airline	β_E	Airline	β_E
American	1.42	Northwest	1.35
Continental	1.18	United	1.55
Delta	1.30	USAir	1.37

Leverage ratios

Airline	D/V	Airline	D/V
American	42%	Northwest	22%
Continental	30%	United	37%
Delta	53%	USAir	25%

Southwest

The tax rate is 35%, r_D = 11.4%, r_f = 10.39%, and $E[R_M - r_f]$ = 5.0%.

$$r_E = r_f + \beta_E \, E[R_M - r_f]$$

$$WACC = \frac{D}{A}(1-\tau)\, r_D + \frac{E}{A} r_E$$

Airline	β_E	r_E	D/A	WACC
American	1.42	17.5%	42%	13.3%
Continental	1.18	16.3	30	13.6
Delta	1.30	16.9	53	11.9
Northwest	1.35	17.1	22	15.0
United	1.55	18.1	37	14.2
USAir	1.37	17.2	25	14.8
Average	1.36	17.2%	35%	**13.8%**

Issue 4

Multifactor models

Beta might not fully summarize all relevant risks. Additional risk factors could be important.

Measuring risk

Regress R_i on macroeconomic risk factors, $F_1 \ldots F_K$

$$R_i = \alpha_i + \beta_{i1} F_1 + \beta_{i2} F_2 + \ldots + \beta_{iK} F_K + \varepsilon_i$$

β_{ik} is firm i's sensitivity to the factor.

Expected returns

Expected returns are linearly related to risk

$$E[R_i] = \gamma_0 + \gamma_1 \beta_{i1} + \gamma_2 \beta_{i2} + \ldots + \gamma_N \beta_{iN}$$

γ_k is the risk premium for factor k.

Multifactor models

Fama-French 3-factor model*

- CAPM misses risk factors associated with size and B/M
- What are the risks?

R_M = Market portfolio return
SMB = Small stock return – Big stock return
HML = High-B/M stock return – Low-B/M stock return

$$R_i = \alpha_i + \beta_i R_M + s_i R_{SMB} + h_i R_{HML} + \varepsilon_i$$

$$E[R_M - r_f] \approx 5.0\%, \quad E[R_{SMB}] \approx 3.0\%, \quad E[R_{HML}] \approx 4.0\%$$

*http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

Betas, 1960 – 2001

B/M portfolios			Size portfolios		
Decile	β	R^2	Decile	β	R^2
Low B/M	1.10	0.88	Smallest	1.33	0.56
2	1.08	0.90	2	1.06	0.73
3	1.05	0.92	3	1.13	0.79
4	0.99	0.89	4	1.14	0.84
5	0.91	0.87	5	1.14	0.86
6	0.86	0.84	6	1.10	0.88
7	0.93	0.76	7	1.04	0.91
8	1.04	0.74	8	1.10	0.93
9	1.16	0.64	9	1.00	0.96
High B/M	1.29	0.54	Largest	0.90	0.97

Southwest Airlines

Cost of capital

$$R_{SW} = \alpha + \beta_{SW} R_M + s_{SW} R_{SMB} + h_{SW} R_{HML} + \varepsilon_i$$

$\hat{\beta}_{SW} = 1.123$
$\hat{s}_{SW} = 0.623$
$\hat{h}_{SW} = 0.442$

Cost of equity

$$r_E = 10.4 + 1.123 \times 5.0 + 0.623 \times 3.0 + 0.442 \times 4.0 = 19.7\%$$

WACC

$$\text{WACC} = 0.20 \times (1 - 0.35) \times 11.4\% + 0.80 \times 19.7\% = \mathbf{17.2\%}$$