15.414 - Recitation IV

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There may be some truth in these CAPM (and APT) theories, but last year some stocks did much better than these theories predicted, and others did much worse. Comment.

Final Comment... Robustness Issue

- Keep in mind that portfolio theory (and the CAPM) makes the assumption that investors absolutely know the joint distribution of asset returns (i.e. they know each asset's true mean, variance, and all the correlations).

- This is not an accurate assumption. We don't know the this distribution and must use historical return data to estimate it. These estimates though are subject to statistical error. Even in the most ideal statistical setup, these estimates will imprecise, though they will become more precise as we increase the number of observations we use in making them.

- An important question to ask is whether these estimation problems cause noticeable inefficiencies in our portfolio selection. The answer is: yes, it does. Let's look at a numerical example.

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● Assume you estimate the following joint distribution using historical price data. We also include the portfolios you choose given your estimates (assume no short sales restrictions, etc).

	E[R]	V[R]	Variance-Covariance Matrix				
X	0.09	0.029344	1	0.458	0.032		
Y	0.08	0.03258	0.458	1	0.022		
Z	0.12	0.068906	0.032	0.022	1		
T	0.05	0					

Portfolios

E[Rp]	V[Rp]	w(X)	w(Y)	w(Z)			S[Rp]
0.05	0.134619	0.1976	1.601825	-0.79942	0.049999	1	0.366904
0.075	0.038872	0.321651	0.883762	-0.20541	0.075	1	0.19716
0.1	0.019853	0.445697	0.165728	0.388576	0.1	1	0.1409
0.125	0.077555	0.569743	-0.55231	0.982564	0.125	1	0.278487
0.15	0.21198	0.693789	-1.27034	1.576553	0.15	1	0.460412
0.175	0.423136	0.81784	-1.9884	2.170565	0.175001	1	0.650489
0.2	0.711007	0.941886	-2.70644	2.764553	0.200001	1	0.843212

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● Meanwhile, assume that the TRUE distribution of returns is given below (along with the portfolios you would have chosen if you had known this true distribution).

	E[R]	V[R]	Variance-Covariance Matrix						
X	0.11	0.029344	1	0.458	0.032				
Y	0.09	0.03258	0.458	1	0.022				
Z	0.11	0.068906	0.032	0.022	1				
T	0.05	0							

Portfolios

E[Rp]	V[Rp]	w(X)	w(Y)	w(Z)			S[Rp]	E[Rp*]	S[Rp*]
0.05	0.237824	-1.82749	3	-0.17251	0.05	1	0.487672	0.077963	0.366904
0.075	0.079067	-0.77128	1.75	0.021278	0.075	1	0.281188	0.092325	0.19716
0.1	0.01815	0.284929	0.5	0.215071	0.1	1	0.134722	0.106685	0.1409
0.125	0.055075	1.341136	-0.75	0.408864	0.125	1	0.234681	0.121046	0.278487
0.15	0.189841	2.397343	-2	0.602657	0.15	1	0.435708	0.135407	0.460412
0.175	0.422448	3.45355	-3.25	0.79645	0.175	1	0.64996	0.149768	0.650489
0.2	0.752897	4.509757	-4.5	0.990243	0.2	1	0.867696	0.164129	0.843212

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• As you can see, there are significant differences in the portfolios you choose. This is a major problem if it affects your bottow line (i.e. moves you far away from the efficiency frontier). The Figure below shows that estimation error indeed can cause you to select highly inefficient portfolios.

Efficiency Frontier: Robustness