



MIT SLOAN SCHOOL OF MANAGEMENT
15.483 CONSUMER FINANCE:
MARKETS, PRODUCTS, AND FINTECH

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Lecture Notes 2

**Rational Economic Choice
and Financial Decisions**

This Lecture and Next Lecture

Rational economics models

Nobel prize winners
Samuelson, Fama, etc.

Behavioral economic models

Nobel prize winners Thaler,
Shiller, etc.

Lecture outline

1. Prelude: Economic models of behavior and how to use them in household financial markets
2. Optimal consumption and saving over the life
3. How to understand, develop, and manage (or use) quantitative advising or roboadvisory program

All future lectures will be more applied and less theoretical.

Prelude

Modelling human financial behavior

- A Model** is a specification of motives & constraints that generates behavior
- **Constraints** because people have finite resources and abilities
 - **Motives** because people's behavior changes as they seek goals
 - **Mathematical** when we need to be quantitative
- A. Two ingredients:
- Human behavior (e.g. maximize wealth, maybe cognitive limitations)
 - Markets: Budget constraints (e.g. prices, money to spend)
- B. Two economic approaches to human behavior:
- Revealed preference: behavior reveals what people want
 - Behavioral: people make mistakes
- C. Two uses of models:
- Descriptive (positive)
 - Proscriptive (normative)

In this lecture we will use rational models as proscriptive guides

- Use models to describe how people should behave given reasonable objectives and constraints
 - Fintech: design financial products or advise to help people meet these goals when their behavior does not align with model predictions
 - The question – is their ex ante behavior rational or mistake?
 - is partly answered by the market: do people choose to improve behavior with your product?

Conservative approach

- People get things right, so design better tools for them to use

Riskier approach

- “We” are smarter than “them”
- Beware: being wrong

Lecture outline

1. Prelude: Economic models of behavior and how to use them in household financial markets

2. Optimal consumption and saving over the life

A. Two period example

B. A life is just a sequence of two period problems

C. What is the lifetime proscriptive advice?

3. How to understand, develop, and manage (or use) quantitative advising or roboadvising

Optimal consumption, saving, and investing

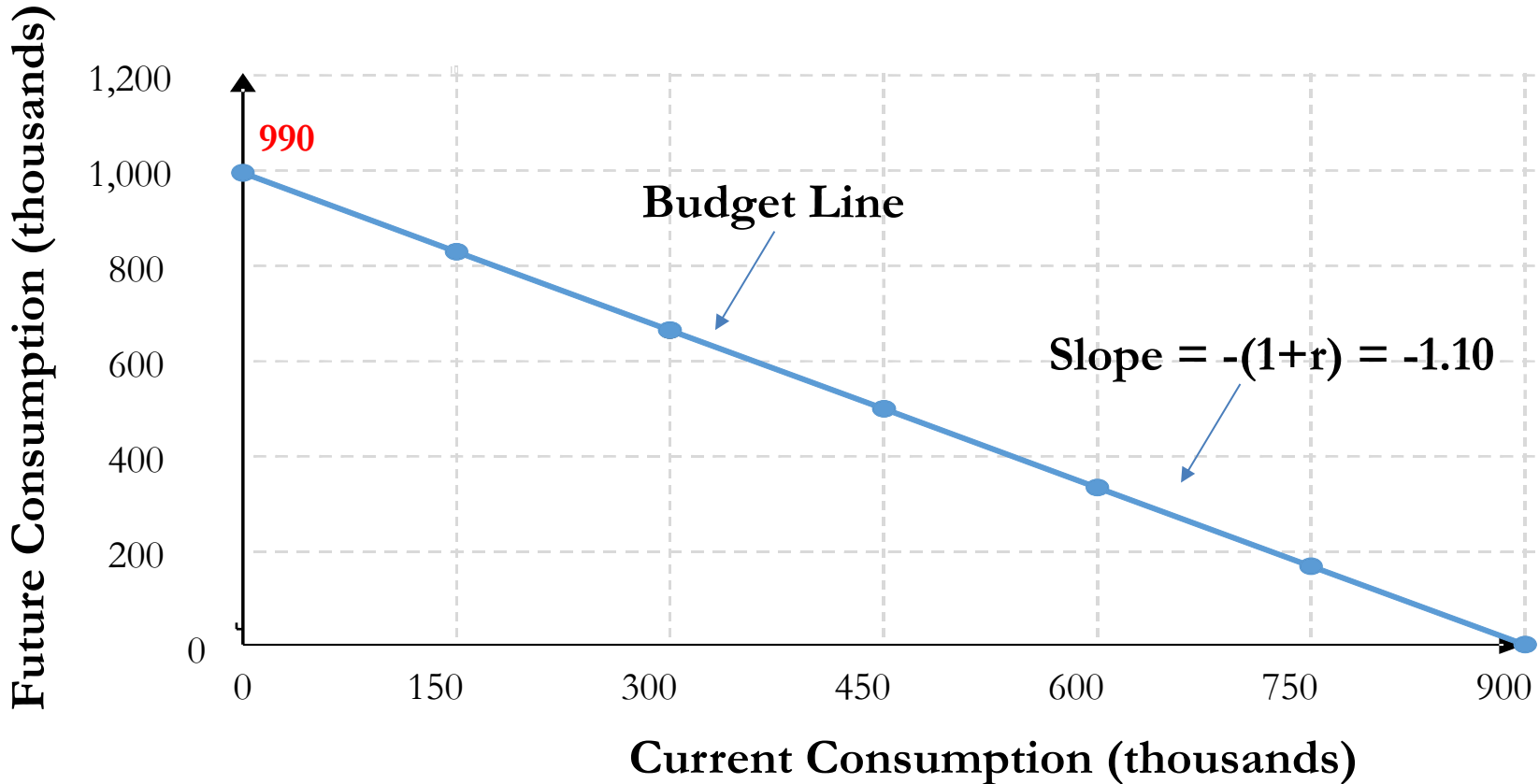
- What advice to give people? Assume they have:
 - Diminishing marginal utility
 - And solve for what they do to maximize lifetime utility
- Quantifies how people want high consumption subject to stable over time and over the possible future incomes and returns they may live through
 - a) *Should* save for retirement when income is high and dissave in retirement when income is low
 - b) *Should* insure against future risks
 - a) If income risk not insurable, should save more
 - c) *Should* invest in high-return assets only until exposure to that risk balances their high return

A. Two-period model of how much to consume and save

- Let's consider a person named Bella living in a two-period world. She receives a fixed after-tax income, measure in real term of 420,000 in the current period and expects to receive a real income of 330,000 in the future period. In addition, she enters the current period with real wealth of 180,000 in a saving account, and she can borrow or lend at a real interest rate of 10% per period
- Next, we list the symbols used to represent Bella's situation
 - y : Bella's current real income (420,000)
 - y^f : Bella's future real income (330,000)
 - a : Bella's real wealth at the beginning of the current period (180,000)
 - r : real interest rate (10%)
 - c : Bella's current real consumption
 - c^f : Bella's future real consumption
- **So what is the Budget Constraint for Bella?**

Bella's constraints

- Budget Constraint: $c^f = (y + a - c)(1 + r) + y^f$
- The budget line slopes downward, reflecting the trade-off between current and future consumption



Bella's objectives

Maximize present discounted value of utility flows from consuming:

- Choose c and c^f to maximize
$$u(c) + \beta u(c^f)$$
- β is discount factor = $1/(1+\text{discount rate})$ = degree of patience (smaller number, more impatient)
- $u(c)$ is how Bella compares different levels of c
 - Increasing
 - Concave
 - Graph

How concavity generate risk aversion

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When advising, we need to be quant

What is utility function should we use?

Industry choice: **Constant Relative Risk Aversion** utility function

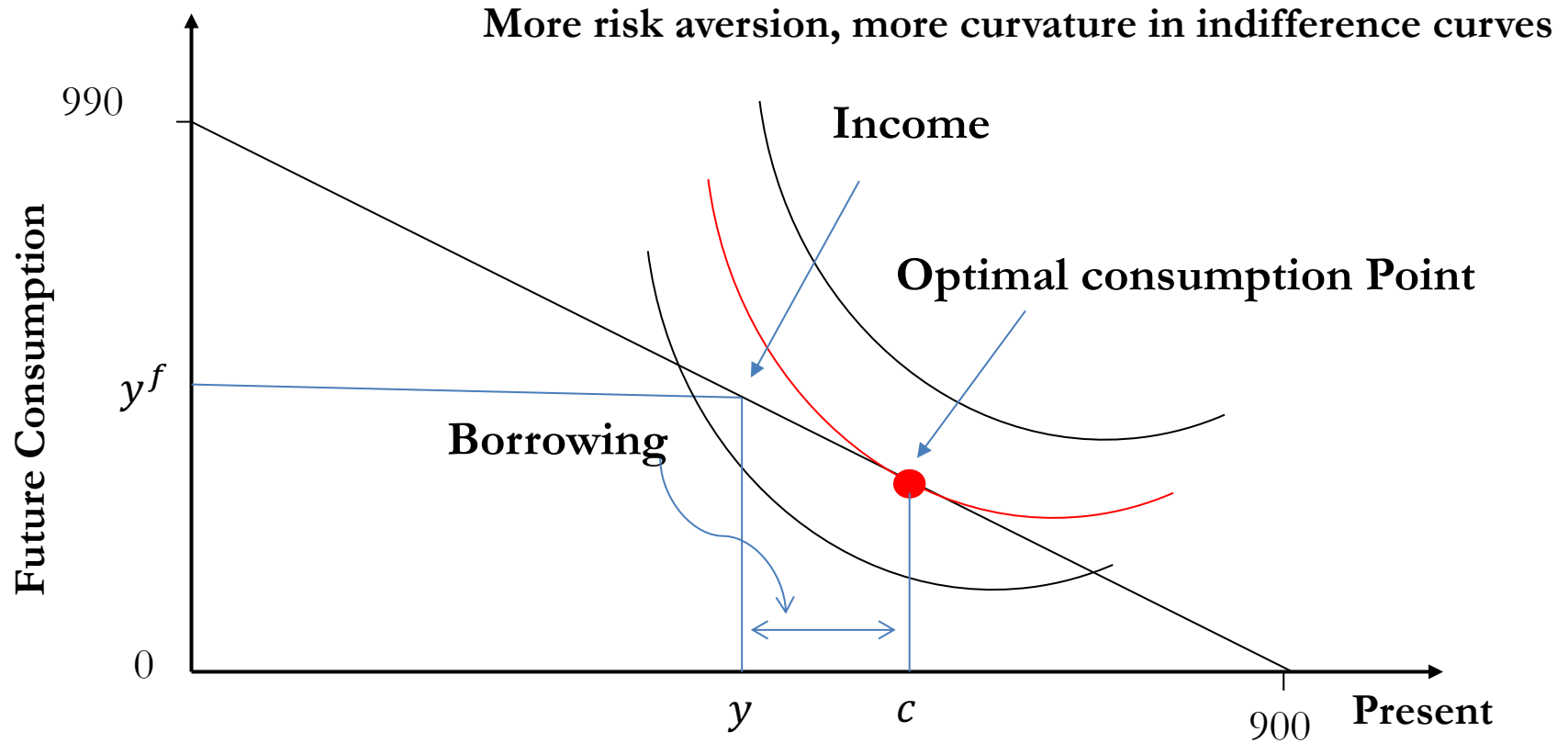
$$u(C) = \frac{C^{1-\alpha}}{1-\alpha}$$

- $\alpha > 0$ measures risk aversion, bigger α more averse
- $\alpha = 1$, then utility is $\ln(C)$
- More curvature means more aversion to variation in spending
 - Over time: how responsive to changes in interest rates?
 - Over risk: how risk averse is the person?
- How much would you pay to avoid the gamble of consumption up 10% with prob=50% and down 10% with prob=50% and what does that imply for your risk aversion?

How much in percent of lifetime consumption would you pay to avoid a fifty percent chance of 10 percent more consumption and a fifty percent chance of 10 percent less?

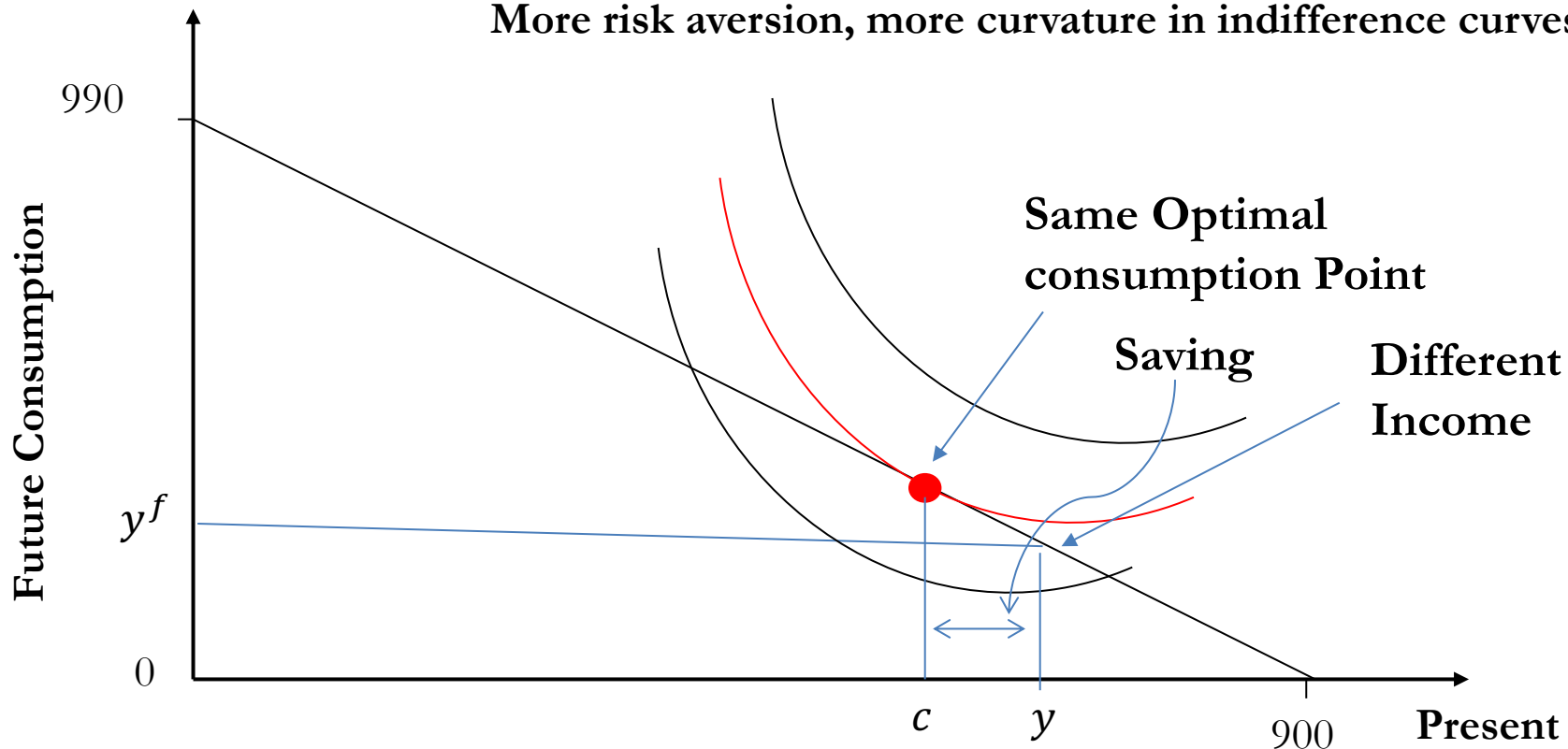
	Utility of Consumption		Expected utility	C that gives utility of G	Equivalent percent certain loss	
	Risk aversion	0.9	1.1		0.5 D+ 0.5 E	Percent below 1
(log)	0.5	1.897	2.098	1.997	0.997	0.25%
	1.0	-0.105	0.095	-0.005	0.995	0.50%
	1.5	-2.108	-1.907	-2.008	0.992	0.75%
	2.0	-1.111	-0.909	-1.010	0.990	1.00%
	2.5	-0.781	-0.578	-0.679	0.988	1.25%
	3.0	-0.617	-0.413	-0.515	0.985	1.49%
	5.0	-0.381	-0.171	-0.276	0.976	2.43%
	10.0	-0.287	-0.047	-0.167	0.956	4.42%
	25.0	-0.522	-0.004	-0.263	0.926	7.39%
	50.0	-3.564	0.000	-1.782	0.913	8.72%

How much to consume and save: utility and indifference curves



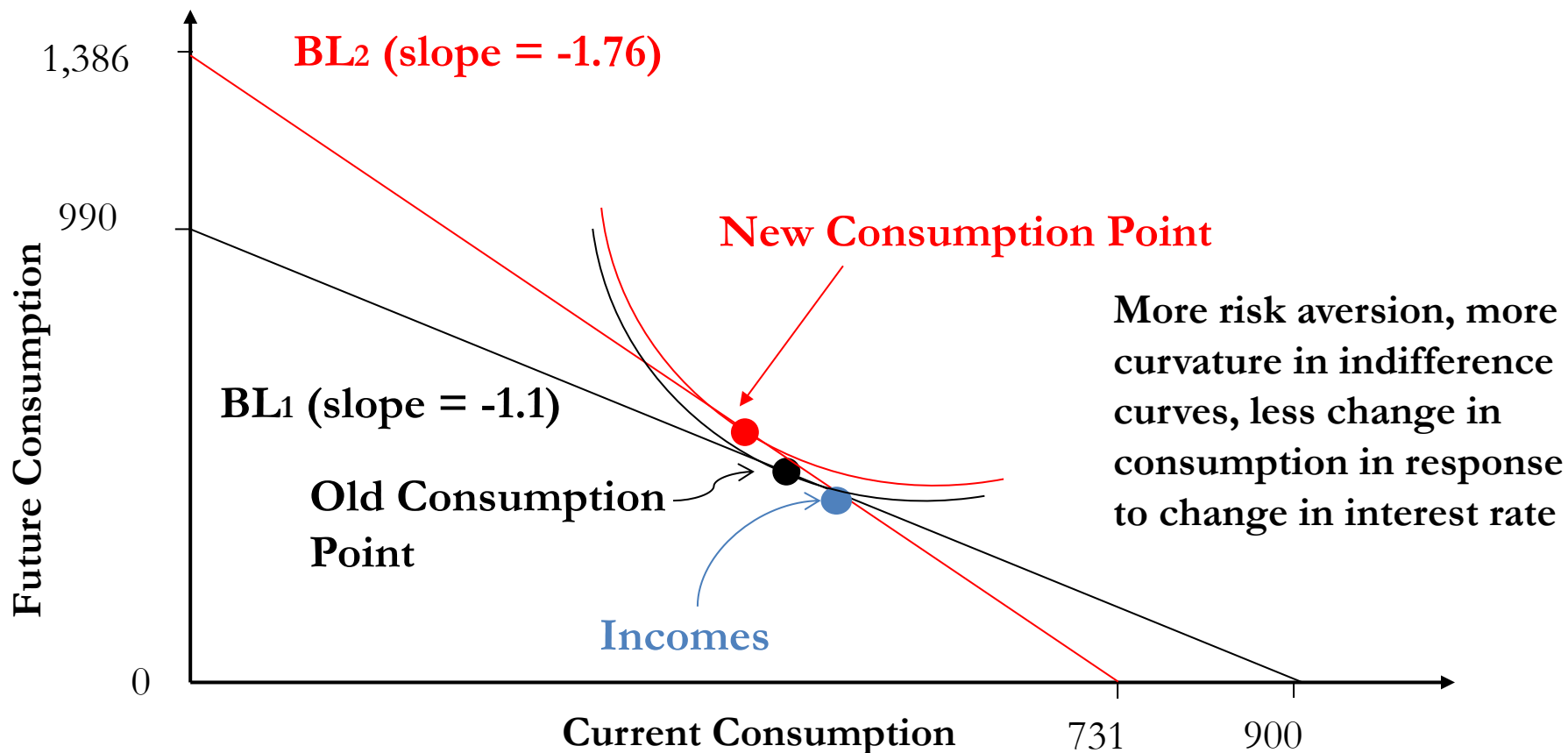
Or for someone with different income . . .

More risk aversion, more curvature in indifference curves



Example of model advantage: Can solve for optimal responses

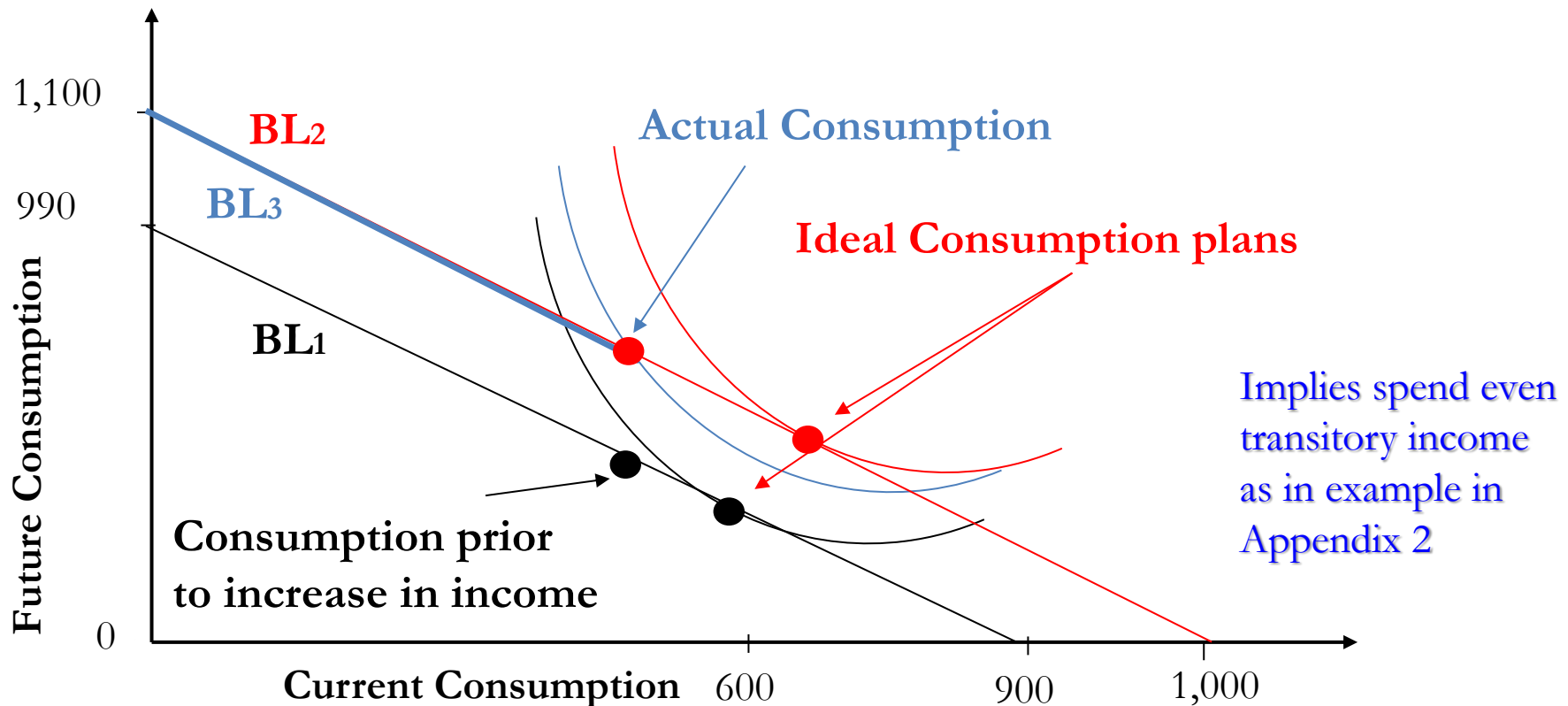
- What happens with different interest rates? Assume r jumps from 10% to 76%
- First, pivot constraints. Second, find highest indifference curve.



Interest rates increase, people consume less and save more

Example of using model to find new lending opportunities

What happens with increase in income in the future with **liquidity constraint**?
Bella cannot increase consumption today, only in the future



1. Timing of income matters.
2. Inability to borrow makes Bella worse off – role for innovation ?
3. Depends on Why. Market friction, missing product, foolish lenders?

B. But what use are quantitative prescriptions in a two-period model?

- Not much, but . . .
- Life is just a sequence of today vs. tomorrow problems
- And the today-tomorrow formulation is how actual advisory programs solve to tell you “Are you saving enough for retirement?”

Dynamic Formulation: Solving Hard Problems as Two-Period Problems

Agents maximize utility $\max_{c_t} \sum_{t=1}^T \beta^t U(c_t)$

Subject to $w_{t+1} \geq 0$ and dynamic budget constraint

$$w_{t+1} = (1 + r)(w_t + y_t - c_t)$$

Consumption tomorrow can be written in terms of wealth:

$$c_t = w_t + y_t - \frac{w_{t+1}}{1+r}$$

Then the lifetime problem becomes 2-period problems – very complicated problems become “easy” to solve (for your programmer)

Dynamic Formulation

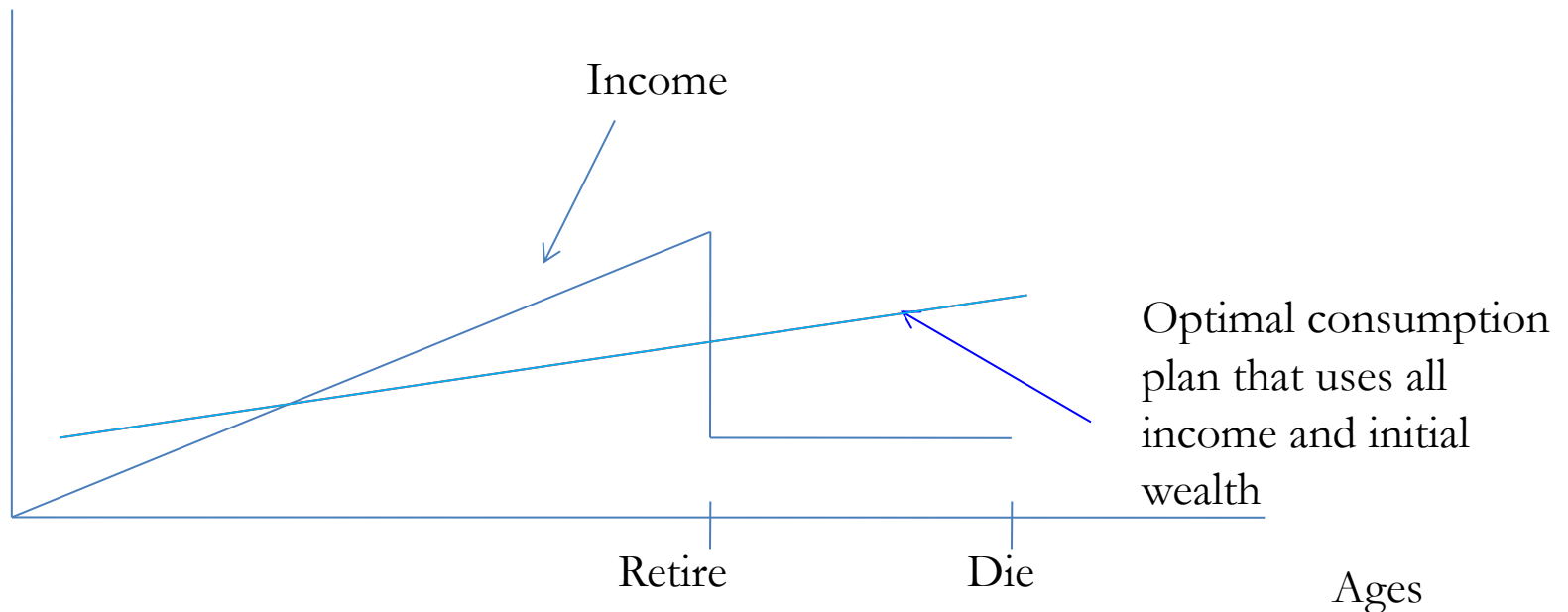
This can be written as a recursive, two period problem, just like out two-period examples

$$\begin{aligned} V_\tau(w_\tau) &= \max_{w_t} \sum_{t=\tau}^T \beta^{t-\tau} U \left(w_t + y_t - \frac{w_{t+1}}{1+r} \right) \\ &= \max_{w_{\tau+1}} U \left(w_\tau + y_\tau - \frac{w_{\tau+1}}{1+r} \right) + \beta \left[\max_{w_t} \sum_{t=\tau+1}^T \beta^{t-(\tau+1)} U \left(w_t + y_t - \frac{w_{t+1}}{1+r} \right) \right] \\ &= \max_{w_{\tau+1}} \left\{ \underbrace{U \left(w_\tau + y_\tau - \frac{w_{\tau+1}}{1+r} \right)}_{\text{Today}} + \underbrace{\beta V_{\tau+1}(w_{\tau+1})}_{\text{Tomorrow}} \right\} \end{aligned}$$

Now it is a sequence of two period problems: solve *recursively*

- In the last period of possible life, T , person consumes all their wealth, $C_T = W_T$, so $V_T(W_T) = U(W_T)$
- In the period before the last period, household faces a two period consumption problem as has wealth W_{T-1}
- Solve with a numerical solver for maximum for any W_{T-1} which gives optimal consumption in period $T-1$ as a function of the wealth: $C_{T-1}(W_{T-1})$
- In the period, $T-2$, household faces again a two period consumption problem choosing $C_{T-2}(W_{T-2})$ and knowing future consumption choices and so knowing $V_{T-1}(W_{T-1})$

Proscription: Stabilize spending over lifetime variation in income



Many real-world complexity can and should be in the real programs, e.g. Risk

Income uncertainty

Agents maximize **expected** utility

$$\max_{c_t} \mathbb{E} \left[\sum_{t=1}^T \beta^t U(c_t) \right]$$

- Uncertainty is just like time
 - Consumption smoothing over periods becomes on consumption smoothing over different possible ways the world can turn out
 - So people want insurance (at fair prices)

Proscription: Save more in response to risk

$$\max E[\ln c_t + \ln c_{t+1}]$$

$$y_t + y_{t+1} \geq c_t + c_{t+1}$$

$$y_t = 1, y_{t+1} = \left\{ \begin{array}{l} 0 \text{ with probability } \frac{1}{2} \\ 6 \text{ with probability } \frac{1}{2} \end{array} \right\}$$

If $y_{t+1} = 3$ with probability 1, then $c_t = c_{t+1} = 2$

$$c_t^* = 0.655, c_{t+1}^* = \left\{ \begin{array}{l} 0.345 \text{ with probability } \frac{1}{2} \\ 6.345 \text{ with probability } \frac{1}{2} \end{array} \right\}$$

- ⇒ Large correlation between expected income and consumption growth
- ⇒ Large propensity to spend additional income at t (but not use credit)
- ⇒ **People want income insurance – why cant they get it? Financial innovation opportunity or moral hazard and adverse selection?**

Risky assets and portfolio choice

Portfolio choice

- People want high returns
- People prefer a safe standard of living to a risky one
 - Does not imply they prefer safe investments to risky ones
 - People accept risk for return
- Theory of investment advising, insurance provision, and value of options like default
- Appendix 1 has an example with two periods, two assets, and two possible outcomes

C. What are some lessons realistic models with stock-bond portfolio choice

Figure removed due to copyright restrictions. See Figure 7.2 in Campbell, John Y. and Luis M. Viciara. [Strategic Asset Allocation](#). Oxford University Press, 2002. ISBN: 9780198296942.

- Due to risk, consumption per capita rises with income early in life, and consumption is hump-shaped due to changing risk over life
- Retirement wealth provides income during retirement: about 8x income at retirement => optimal has lots of retirement saving

Solution to realistic lifecycle model with stocks vs. bonds portfolio choice

Figure removed due to copyright restrictions. See Campbell, John Y. and Luis M. Viciara, [Strategic Asset Allocation](#). Oxford University Press, 2002. ISBN: 9780198296942.

Table 7.4. Life-Cycle Profiles

Age	Baseline	$\gamma = 10$	$\delta = 0.8$	Self-employed
Consumption				
20–35	20.22	20.13	20.53	25.09
36–50	25.48	25.12	26.50	38.39
51–65	24.61	24.23	23.94	35.23
66–80	22.43	22.65	15.95	32.67
81–100	16.98	19.04	14.27	27.26
Wealth				
20–35	5.94	8.20	3.39	12.84
36–50	29.34	39.28	7.25	65.75
51–65	75.77	100.16	10.23	173.70
66–80	77.28	105.50	5.71	159.76
81–100	13.60	30.85	0.11	46.75
Liquid portfolio share in stocks				
20–35	1.00	0.97	0.99	0.57
36–50	0.99	0.95	1.00	0.91
51–65	0.88	0.61	1.00	0.57
66–80	0.90	0.57	1.00	0.54
81–100	0.92	0.68	1.00	0.61

Most portfolio choice models strongly suggest putting most wealth in stocks for most households.

Why?

Table 7.4 Life-cycle Profiles © Oxford University Press. All rights reserved. This content is excluded from our Creative Commons license. For more information, see <https://ocw.mit.edu/help/faq-fair-use/>.

In practice: Lifecycle consumption on average rises with income but does not fall in retirement

Figure removed due to copyright restrictions. See Figure 1a Mark Aguiar and Erik Hurst. "[Deconstructing Lifecycle Expenditure](#)." Journal of Political Economy, Vol. 121, No. 3 (June 2013), pp. 437-492.

Lecture outline

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How does Fintech determine what advice to give?



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Designed by economists for households and financial planners

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How does Fintech **actually** determine what saving advice to give?

Life is complex, so the real work is solving complex problems easily, or understanding how to manage a programmer to do it.



WHY BETTERMENT

SERVICES

RESOURCES

PRICING

FAQS

FOR BUSINESS

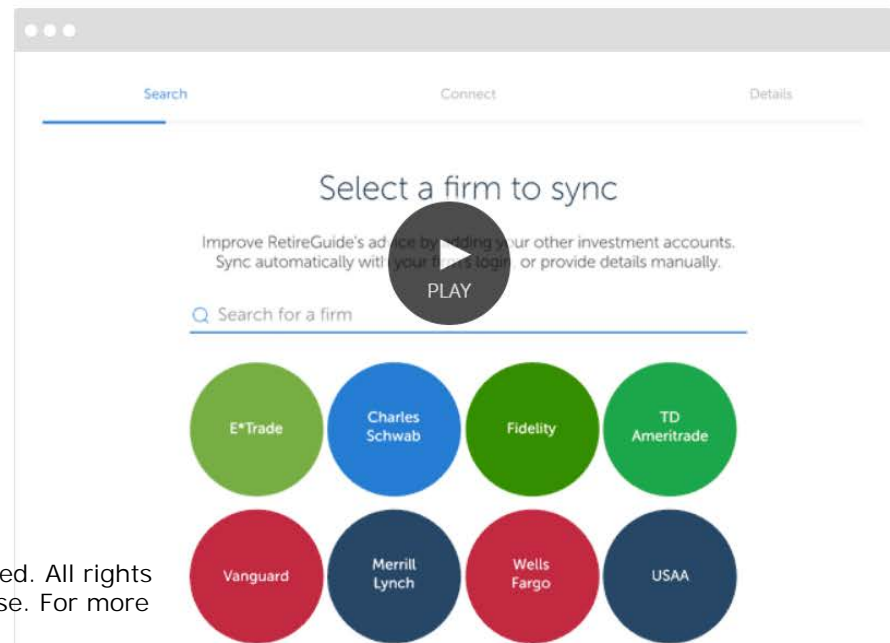
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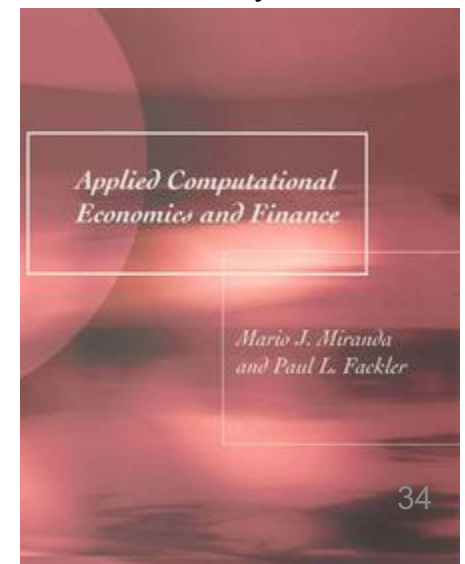
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Solution Approach (Computer)

- The previous approach is fairly simple in principle – there is a basic logic behind the numerical approach.
- Some terminology involved in programming:
 - We must discretize the space when using variables that follow continuous distributions (e.g.: normal distribution).
 - Since our grid space does not include every possible number, we need to interpolate (linear, polynomials, etc.).
 - In an infinite time problem (agents live forever) we must rely on convergence to determine the solution.
- Machine learning is dramatically increasing the complexity of problems that we can solve
- Other references:



Summary: Developing a Financial Advising Program/Robo-adviser

1. Hire a programmer
2. Design a realistic lifetime environment
 - Design environment: family dynamics, risks, taxes, investment options, probability of death from life tables . . .
 - Ask clients to give
 - Family information like children and ages, costs to send kid to college, desire to leave bequests, etc.
 - Preference information like do they want to travel in retirement, send kid to college, leave bequests, etc.
 - Income risks they face or do not face
3. Go over our notes so that you can talk at a high level about the program and
4. manage the programmer
5. Test recommendations
 - Look really closely at really bad outcomes
 - Walk clients through choices and possible outcomes

Comments

- Lots of flexibility and power in framework
- Lots of realistic complexity in any implementation
 - Big life events (cost of college, weddings etc.)
 - Durable goods and housing
 - Work and labor income flexibility like earlier/later retirement
 - Pensions, annuities, life insurance
 - Health costs and health insurance
 - Bequests
 - Etc.
- Warning: all models are wrong, so this is only one (important) input to advice and decisions
- But because people have lots of difficulty with this type of quantitative planning, this is a potentially **valuable** input

Conclusion

- Optimizing models give proscriptive advice as to how to save for retirement and how to choose an optimal portfolio
 - Betterment, ESPlanner etc. based on these types of models
- We walked through the models to:
 1. See how they work in stripped down versions
 2. See assumptions associated with the implications of the models
 3. Set out a baseline against which to define outcomes or behavior as sub-optimal
 4. Allow you to design a realistic problem that can give realistic advice
 5. Manage a programmer to solve for the realistic advice
- Some lessons for behavior, and goals for financial innovation
 - Saving for retirement not a goal, but a means to stable standard of living
 - Consumption “should be” unrelated to timing of income
 - Consumption “should be” reduced in response to higher (real) interest rates
 - Invest more in an asset the higher its expected return but less in an asset the more risk to standard of living
 - Income uncertainty leads to precautionary saving, so people prefer insurance for labor income risks or spending shocks

Appendixes

Appendix 1

How much to save in different assets

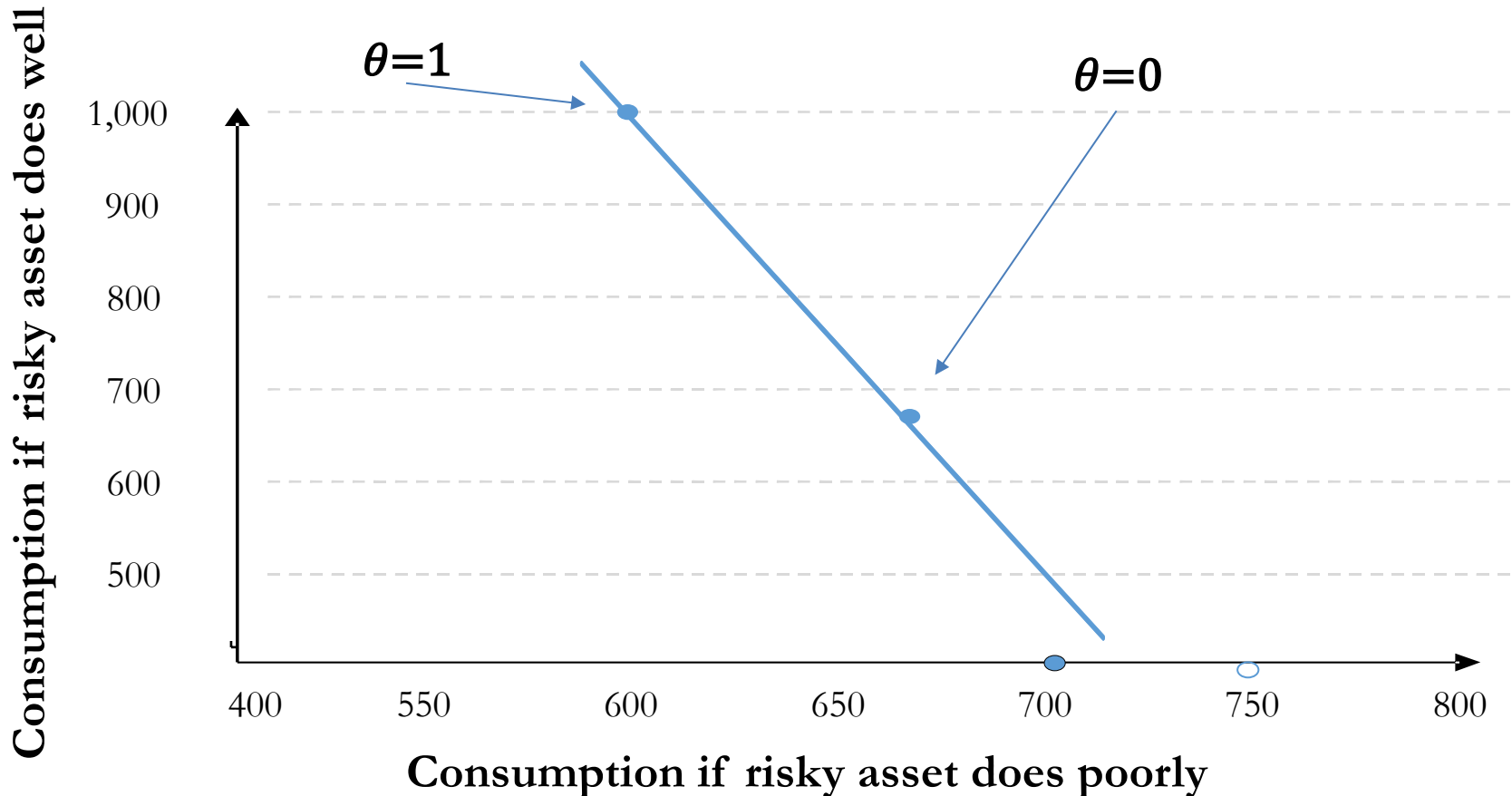
- Let's consider a “person” named Edward deciding how to invest his pension for retirement. He can invest in a risk-free asset that has a real rate of return of zero percent. Or a risky asset that returns 50% with probability one-half and **minus 10%** otherwise.
- Next, we list the symbols used to represent Edward's situation
 - w : Edward's initial wealth (666,666)
 - r^1 : Edward's real return if the risky asset does well =50%
 - r^0 : Edward's real return if the risky asset does poorly =-10%
 - c : Edward's real consumption
 - θ : the share of Edward's portfolio he invests in the risky asset
- **So what is the Budget Constraint for Edward?**

$$c^1 = (1 - \theta)w + \theta w(1 + r^1) = w(1 + \theta r^1)$$

$$c^0 = (1 - \theta)w + \theta w(1 + r^0) = w(1 + \theta r^0)$$

Edward's constraints

- The budget line slopes downward, reflecting the trade-off between consumption in the good outcome and in the bad

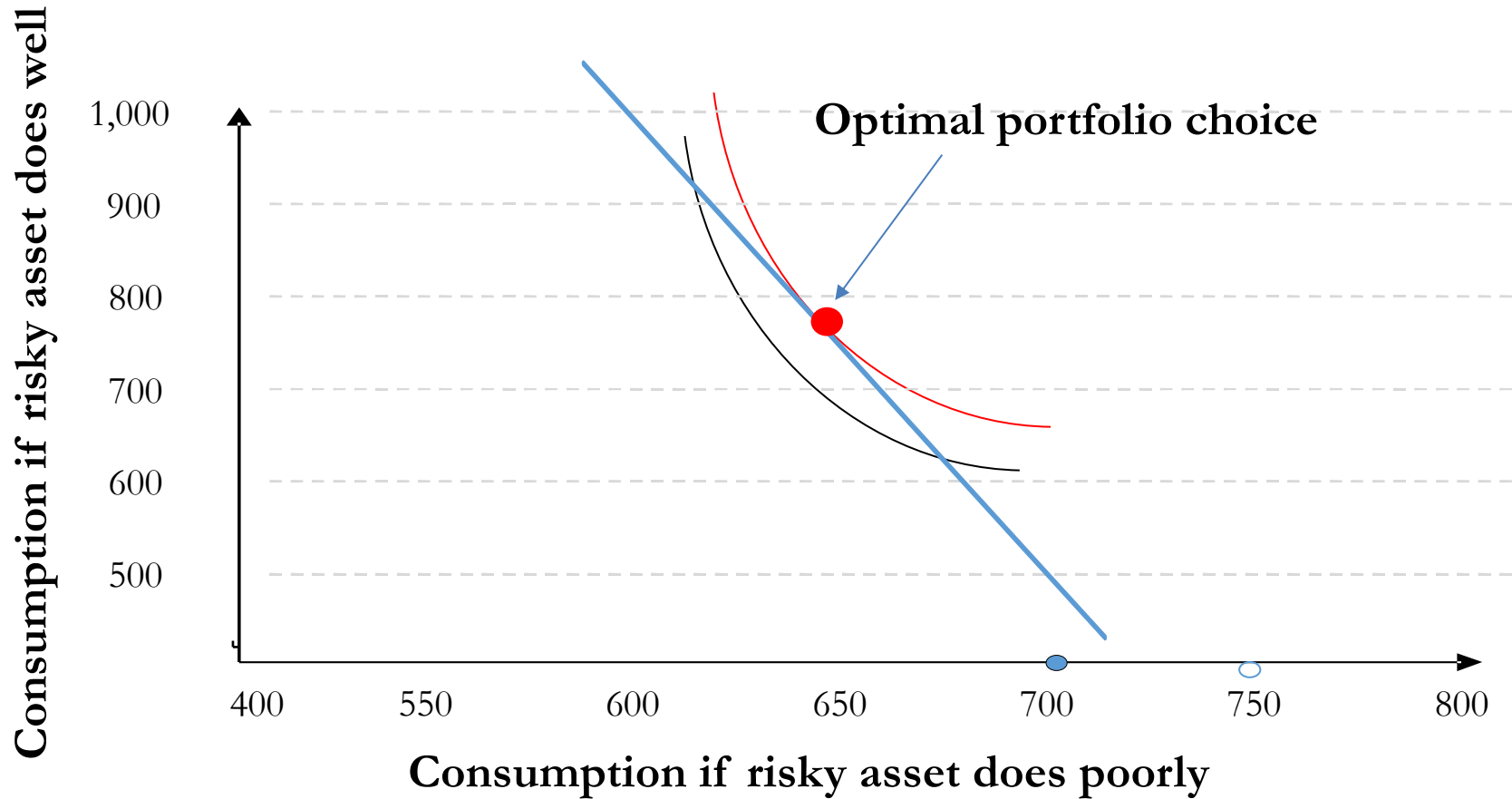


Edward's objectives

Maximize *expected* present discounted value of utility flows from consuming:

- Choose c^0 and c^1 to maximize
$$0.5 u(c^0) + 0.5 u(c^1)$$
- 0.5 and 0.5 are the probabilities of these outcomes
- $u(c)$ is how Edward compares different levels of c
 - Like Bella's utility
 - Increasing
 - Concave

How much to invest in the risky asset: utility and indifference curves



1. Utility and constraints give best investment plan
2. Given present value, timing of income irrelevant to consumption

Implications / Complications

- Changing probabilities
 - Care more about consumption in more likely outcome
 - Shift indifference curves
 - Choose more consumption in more likely states of the world
- Changing returns
 - Change slope of budget line
 - Choose more risk with higher expected return
- Short sale constraints / leverage constraints
 - Like liquidity constraints, limit choice of portfolio
- Aside: C-CAPM follows from market clearing in stock market
 - We took returns as given and solved for portfolios
 - C-CAPM takes aggregate portfolio as given and solves for market returns

Appendix 2: Parker research

Do people do what we proscribe?

EXAMPLE: Economic Stimulus Payments of 2008
Payments of \$600 individuals, \$1,200 married filing jointly+ \$300 per child eligible for CTC

- In proscriptive theory **without liquidity constraints**, households should save the vast majority of these payments to raise spending over their remaining lives
- In practice, my research shows that they spend substantial amounts, related to low liquidity

Tax stimulus rebates

Compare spending of people who randomly get their payment earlier to spending of people who randomly get their payments later

Data: Consumer Expenditure Survey, or Nielsen Consumer Panel

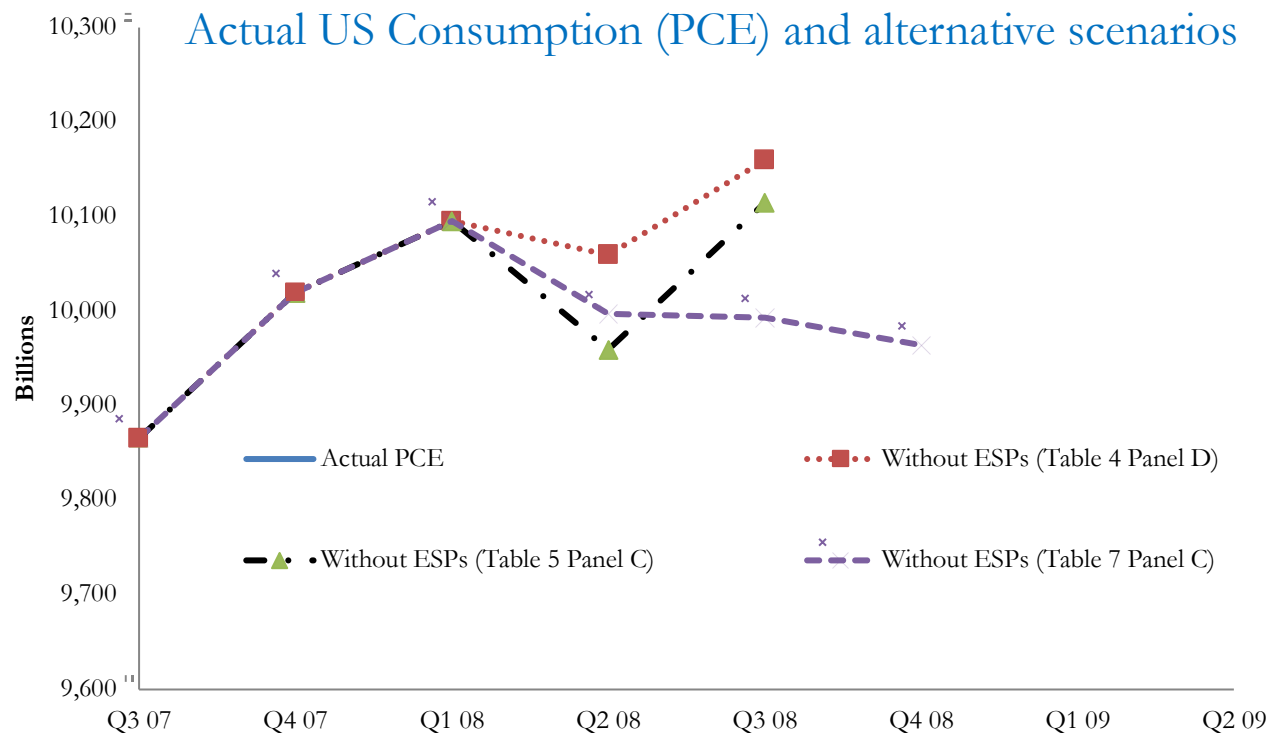
Stimulus Payment Schedule for Tax Returns Received and Processed by April 15

Direct Deposit Payments	
If the last two digits of your Social Security number are:	Your economic stimulus payment deposit should be sent to your bank account by:
00 – 20	May 2
21 – 75	May 9
76 – 99	May 16
Paper Check	
If the last two digits of your Social Security number are:	Your check should be in the mail by:
00 – 09	May 16
10 – 18	May 23
19 – 25	May 30
26 – 38	June 6
39 – 51	June 13
52 – 63	June 20
64 – 75	June 27
76 – 87	July 4
88 – 99	July 11




People spend lots from stimulus rebates

- For 2008 ESP's: on average, households spent 12-31% of ESP on non-durable and 50-90% on total expenditures during the three-months ESP arrived
 - Bigger spending effect for low-income or older
 - Bigger spending effect for homeowners



Businesses compete for the spending

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