A Dynamic model for requirements planning with application to supply chain optimization

by

Stephen Graves, David Kletter and William Hezel (1998)

15.764 The Theory of Operations Management

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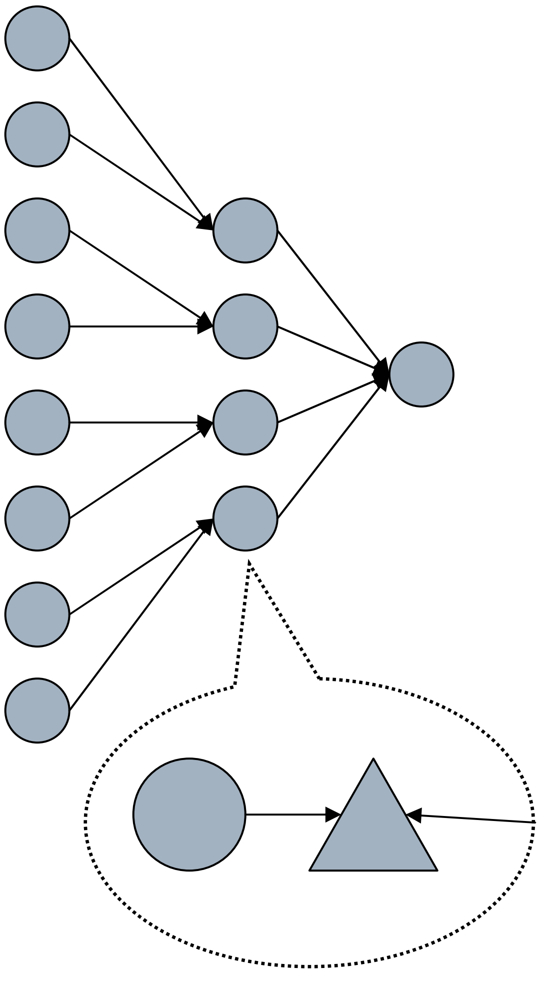
This summary presentation is based on: Graves, Stephen, D.B. Kletter and W.B. Hetzel. "A Dynamic Model for Requirements Planning with Application to Supply Chain Optimization." *Operations Research* 46, supp. no. 3 (1998): S35-49.

Objective of the paper

- Overcome limits of Materials Requirements Process models
- Capture key dynamics in the planning process
- Develop a new model for requirements planning in multi-stage production-inventory systems

Quick Review of MRP



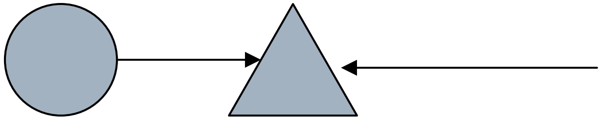
- At each step of the process, production of finished good requires components
- MRP methodology: iteration of
  - Multiperiod forecast of demand
  - Production plan
  - Requirements forecasts for components (offset leadtimes and yields)
- Assumptions:
  - accuracy of forecasts
  - Deterministic parameters
- Widely used in discrete parts manufacturing firms: MRP process is revised periodically
- Limits: deviations from the plan due to incertainty

Literature Review

- Few papers about dynamic modeling of requirements
- No attempt to model a dynamic forecast process, except:
  - Graves (1986): two-stage production inventory system
  - Health and Jackson (1994): MMFE
- Conversion forecasts-production plan comes from previous Graves' works.

Methodology and Results

- We model:
  - Forecasts as a stochastic process
  - Conversion into production plan as a linear system
- We create:
  - A single stage model

production  demand

  - From which we can built general acyclic networks of multiple stages
- We try it on a real case (LFM program, Hetzel)

Overview

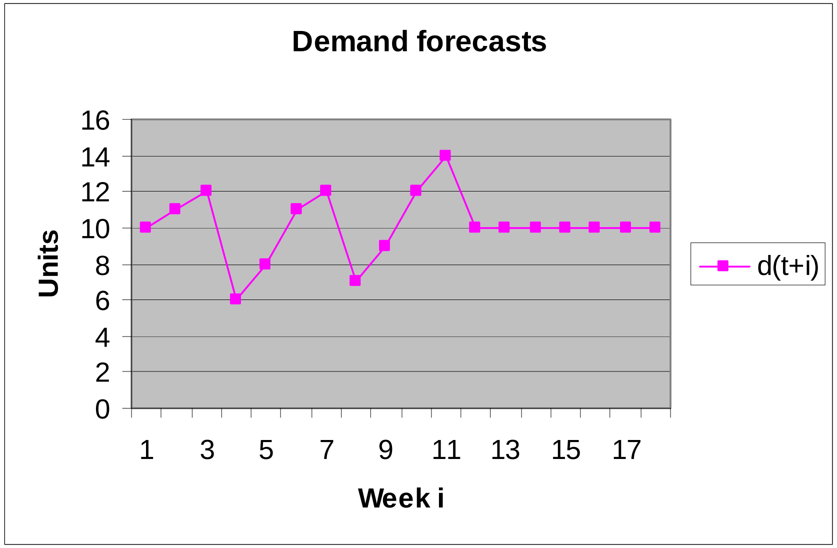
- Single-stage model
- Extension to multistage systems
- Case study of Kodak
- Conclusion

Overview

- Single-stage model
  - Model
    - Forecasts process
    - Conversion into production outputs
    - Measures of interest
  - Optimality of the model
- Extension to multistage systems
- Case study of Kodak
- Conclusion

The forecast process model

- Discrete time
- At each period t:
  - $f_t(t+i)$, $i<=H$
  - $f_t(t+i)=$ Cte , $i>H$
  - $f_t(t)$ is the observed demand



- Forecast revision:

$$\Delta f_t(t+i) = f_t(t+i) - f_{t-1}(t+i) \qquad i = 0,1,\ldots H$$

$$\underline{\Delta f_t} \text{ is iid random vector} \qquad E[\underline{\Delta f_t}] = 0, \mathrm{var}[\underline{\Delta f_t}] = \Sigma$$

- i-period forecast error: $f_t(t) - f_{t-i}(t)$

The forecast process properties

- Property 1: The i-period forecast is an unbiased estimate of demand in period t

$$f_t(t) = f_{t-i}(t) + \sum_{k=0}^{i-1} \Delta f_{t+k}(t)$$

- Property 2: Each forecast revision improves the forecast

$$Var[f_t(t) - f_{t-i}(t)] = Var[\sum_{k=0}^{i-1} \Delta f_{t-k}(t)] = \sum_{k=0}^{i-1} \sigma_k^2$$

- Property 3: The variance of the forecast error over the horizon MUST equal the demand variance

$$Var[f_t(t)] = Var[f_t(t) - f_{t-H+1}(t)] = \sum_{k=0}^{H} \sigma_k^2 = Tr(\Sigma)$$

Production plan: assumptions

- At each period t:
  - $F_t(t+i)$ is the planned production (i=0, actual completed)
  - $I_t(t+i)$ is the planned inventory
  - We introduce the plan revision:

$$\Delta F_t(t+i) = F_t(t+i) - F_{t-1}(t+i)$$

- We SET the production plan $F_t(t+i)$ so that $I_t(t+H)=ss>0$

(cumulative revision to the PP=cumulative forecast revision)

- From the Fundamental conservation equation:

$$I_t(t+i) = I_t(t) + \sum_{k=1}^{i} (F_t(t+k) - f_t(t+k))$$

- We find: $$\sum_{k=0}^{H} \Delta F_t(t+k) = \sum_{k=0}^{H} \Delta f_t(t+k)$$

We assume the schedule update as a linear system

Linear system assumption

- we model:

$$\Delta F_t(t+i) = \sum_{j=0}^{H} w_{ij}\,\Delta f_t(t+j) \qquad i = 0,1,\ldots,H$$

$$\sum_{i=0}^{H} w_{ij} = 1 \qquad 0 \le w_{ij} \le 1$$

- w_{ij} = proportion of the forecast revision for t+j that is added to the schedule of production outputs for t+i

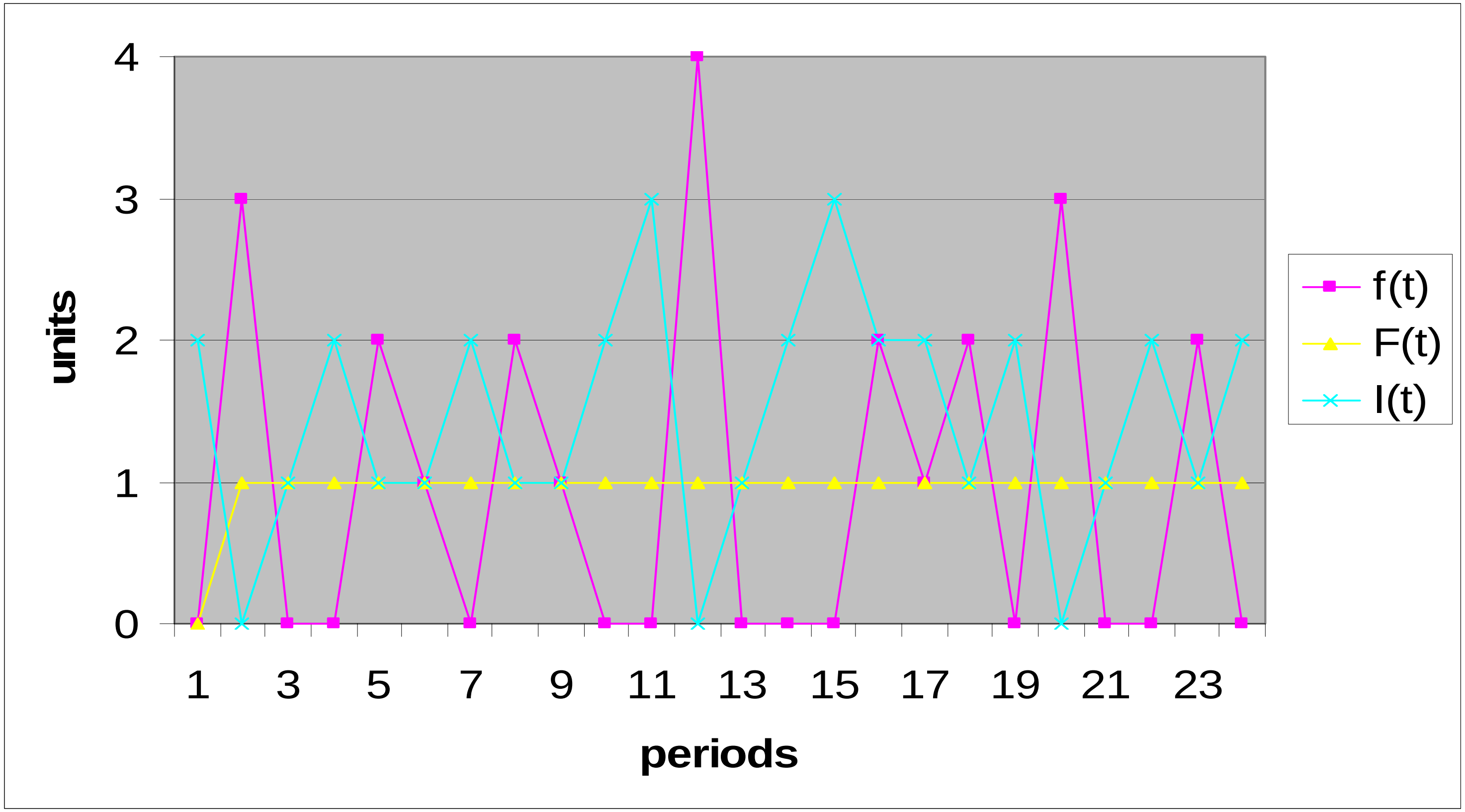
- Weights express tradeoff between Smooth production and Inventory

wij =1 / (H+1)

wii =1 ; wij = 0 (i!=j)

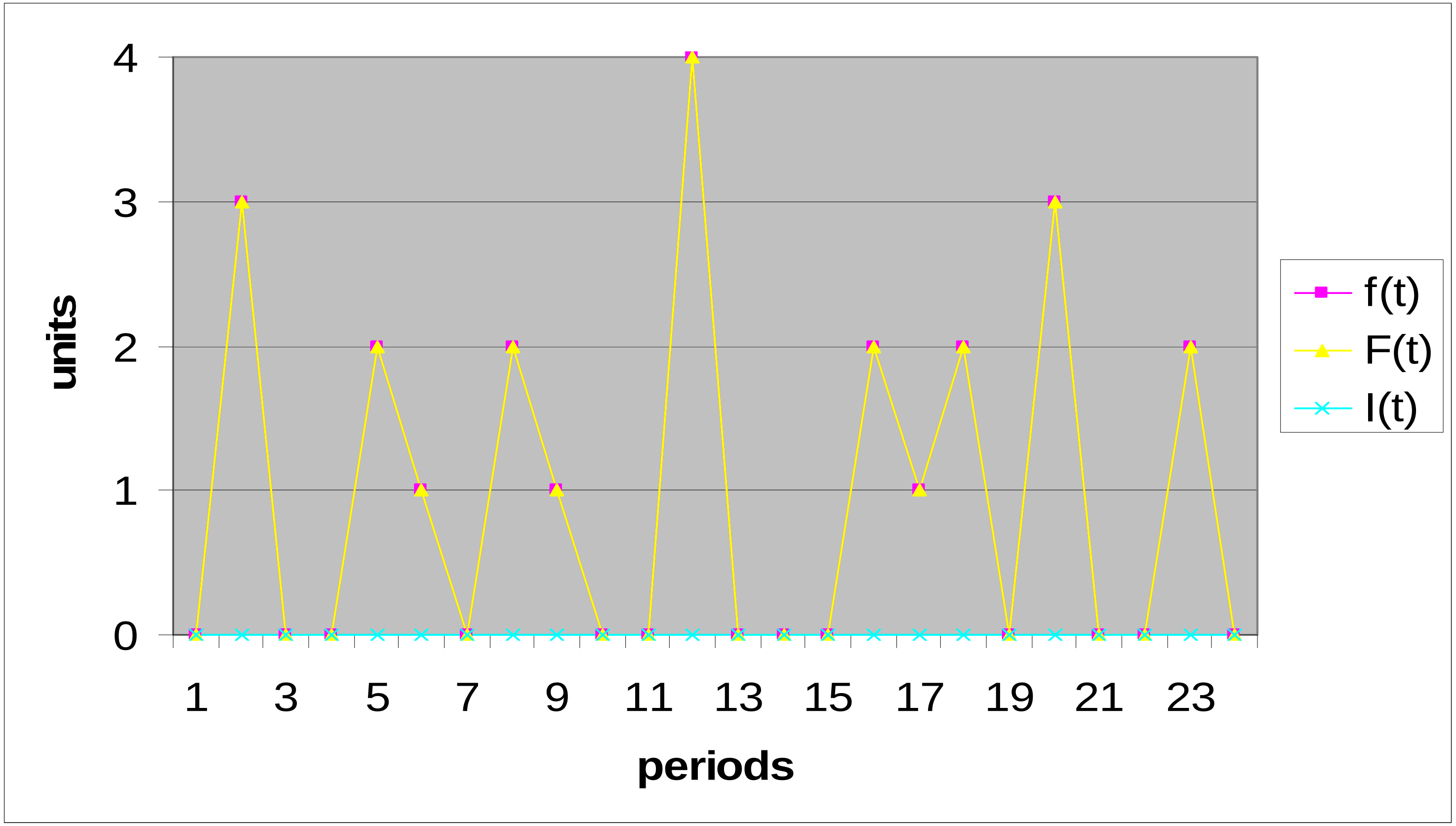
- Decisions variables OR parameters

Smoothing production



F(t) constant ; needed capacity = 1 ; Average inventory = 1,5

Minimizing inventory cost



F(t) = f(t) ; needed capacity = 4 ; Average inventory = 0

Measures of interest

- we note: $\underline{\Delta F_t} = \mathbf{W} \underline{\Delta f_t}$ ← (H+1) by (H+1) matrix
- We obtain :

$$\underline{F_t} = \underline{\mu} + \sum_{i=0}^{H} \mathbf{B}^i \mathbf{W} \underline{\Delta f_{t-i}}$$

- We can now measure the <u>smoothness</u> AND the <u>stability of the production outputs</u>, given by:

(see section 1.3, "Measures of Interest," in the Graves, Kletter, and Hetzel paper)

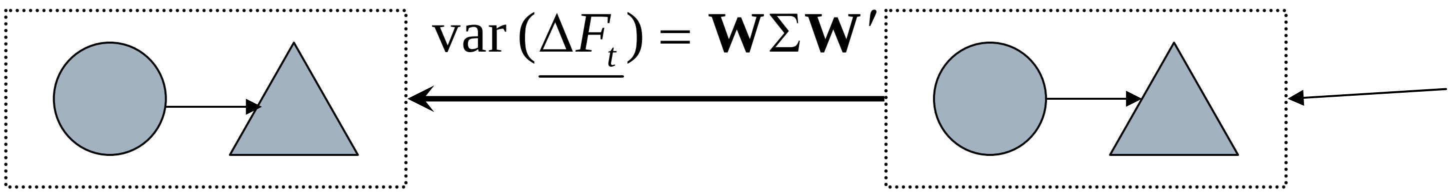
- And if we consider <u>the stock</u>:

(see section 1.3, "Measures of Interest," in the Graves, Kletter, and Hetzel paper)

Global view

- smoothness and stability of the production outputs
- smoothness and stability of the forecasts for the upstream stages

$$\mathrm{var}(\underline{\Delta F_t}) = \mathbf{W}\Sigma\mathbf{W}'$$



$$\mathrm{var}[\underline{\Delta f_t}] = \Sigma$$

- Service level = stock out probability

$$E(I_t) > k\sigma(I_t)$$ = ss

How to choose W?

Optimization problem

- Tradeoff between production smoothing and inventory:

$$\min \operatorname{var}[F_t(t)]$$ = Min(required capacity)

subject to

$$\operatorname{var}[I_t(t)] \le K^2$$ = constraint on amount of ss

$$\sum_{i=0}^{H} w_{ij} = 1 \qquad \forall j$$

Resolution (1/2)

- Lagrangian relaxation:

$$L(\lambda) = \min \operatorname{var}[F_t(t)] + \lambda \operatorname{var}[I_t(t)] - \lambda K^2$$

- We assume:

$$\Sigma = \begin{pmatrix} \sigma_0^2 & & & & \\ & \sigma_1^2 & & 0 & \\ & & \dots & & \\ & 0 & & \sigma_{H-1}^2 & \\ & & & & \sigma_H^2 \end{pmatrix}$$

Forecast revisions are uncorrelated

- We get a decomposition into H+1 subproblems

$$L(\lambda) = \sum_{j=0}^{H} L_j(\lambda) - \lambda K^2 \qquad L(\lambda) = \min \sum_{i=0}^{H} (w_{ij}\sigma_j)^2 + \lambda \sum_{i=0}^{H} (b_{ij}\sigma_j)^2$$

Resolution (2/2)

- Convex program → Karush-Kuhn-Tucker are necessary AND sufficient

$$w_{ij} + \lambda \sum_{k=i}^{H} (w_{0j} + \ldots + w_{kj} = u_{kj}) = \gamma$$

- Which can be transformed

$$w_{ij} = P_i(\lambda) w_{0j} \qquad i = 0, 2, \ldots, j$$

$$w_{ij} = P_i(\lambda) w_{0j} - R_{i-j}(\lambda) \qquad i = j+1, \ldots, H$$

- Which defines all elements (with the convexity constraint)

$$\sum_{j=0}^{H} w_{ij} = 1$$

Solution

- The Matrix W is symmetric about the diagonal and the off-diagonal
  - Wij >0, increasing and strictly convex over i=1...j
  - Wij >0, decreasing and strictly convex over i=j...H
- The optimal value of each subproblem is

$$L_j(\lambda) = w_{jj}\sigma_j^2 \qquad j = 0,1,\ldots,H$$

- we show:

$$w_{j+k,j} = w_{j-k,j} = \alpha\left[\frac{(1-\alpha)}{(1+\alpha)}\right]^k \qquad \text{where } \alpha = \sqrt{\frac{\lambda}{(\lambda+4)}}$$

- And then:

$$L(\lambda) = \min \operatorname{var}[F_t(t)] + \lambda \operatorname{var}[I_t(t)] - \lambda K^2$$
$$\approx tr(\Sigma)\sqrt{\frac{\lambda}{(\lambda+4)}} - \lambda K^2$$

Computation

$$W^{-1} = \begin{pmatrix} \frac{\lambda+1}{\lambda} & \frac{-1}{\lambda} & & & \\ \frac{-1}{\lambda} & \frac{\lambda+2}{\lambda} & \frac{-1}{\lambda} & 0 & \\ & \cdots & \cdots & \cdots & \\ & 0 & \frac{-1}{\lambda} & \frac{\lambda+2}{\lambda} & \frac{-1}{\lambda} \\ & & & \frac{-1}{\lambda} & \frac{\lambda+1}{\lambda} \end{pmatrix}$$

interpretation

- Wjj→1 when Lambda increases
- →low inventory but no production smoothness
- Wjj→(1/H+1) when Lambda → 0
- →High inventory but great production smoothness

(See Figures 2 and 3 on pages S43 of the Graves, Kletter, and Hetzel paper)

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Multi stage model assumptions

- Acyclic network on n stages, m end-items stages, m<n
- 1…m End items forecasts independent
- Downstream stages decoupled from upstream ones.
- Each stage works like the single-one model

- We note: $\underline{f_i},\ i = 1, \ldots, n$

  $\underline{F_i},\ i = 1, \ldots, n$

- We find again:

$$\underline{\Delta F_i} = \mathbf{W}_i \underline{\Delta f_i} \text{ for each stage } i$$

Stages linkage assumptions

- At each period, each stage must translate outputs into production starts
- We use a linear system $\underline{G_i} = \mathbf{A}_i \underline{F_i}$
  - Ai can model leadtimes, yields, etc.
  - Push or pull policies
- We show that each $\underline{\Delta f_i}$ is an iid random vector

→ All previous assumptions are satisfied

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Kodak issue

- Multistage system: film making supply chain. Three steps, with
  - Growth of items
  - Growth of value
  - Decrease of leadtimes
- How to determine optimal safety stock level at each stage ?
- Use DRP model
  - Wide data collection
  - W = I (no production smoothing)
  - Estimation of forecasts covariance matrix
  - A captures yields

Practical results

- -20 % recommendation

- Inventory pushed upstream
  - Risk pooling
  - Lowest value

- Shortcomings of DRP model:
  - Lead time variability
  - Stationary average demand

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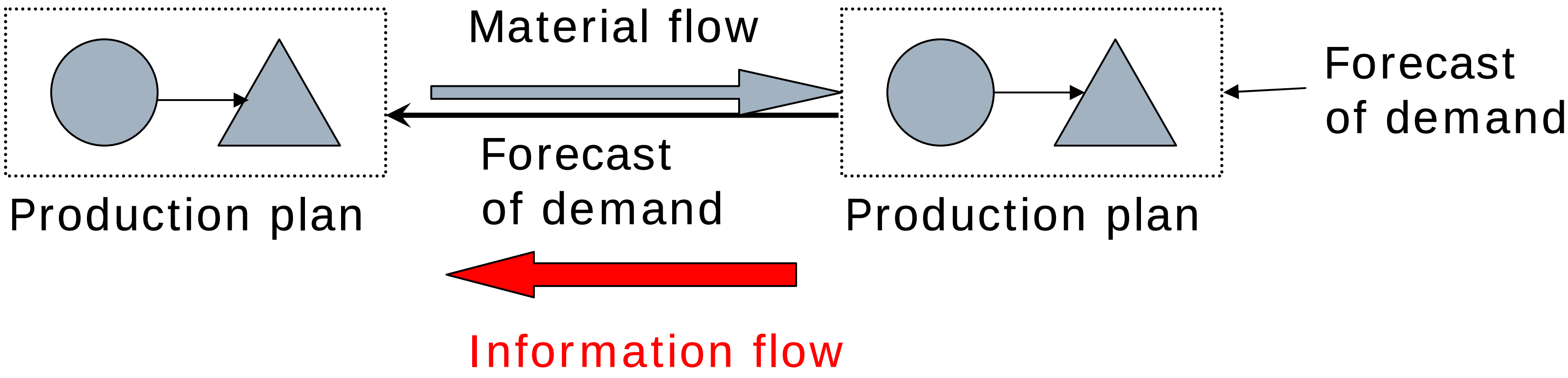
Conclusion

- Model a single inventory production system as a linear system
- Multistage Network:



- Following research topics...