“Centralized Ordering Policies in a Multi-Warehouse System with Lead Times and Random Demand”

A paper by Gary Eppen and Linus Schrage

Presentation by Tor Schoenmeyr

This is a summary presentation based on: Eppen, Gary, and Linus Schrage. “Centralized Ordering Policies in a Multi-Warehouse System with Lead times and Random Demand.” *TIMS Studies in the Management Sciences*, Vol. 16: Multi-Level Production/Inventory Control Systems, Theory and Practice. Edited by Leroy B. Schwarz. 1981.

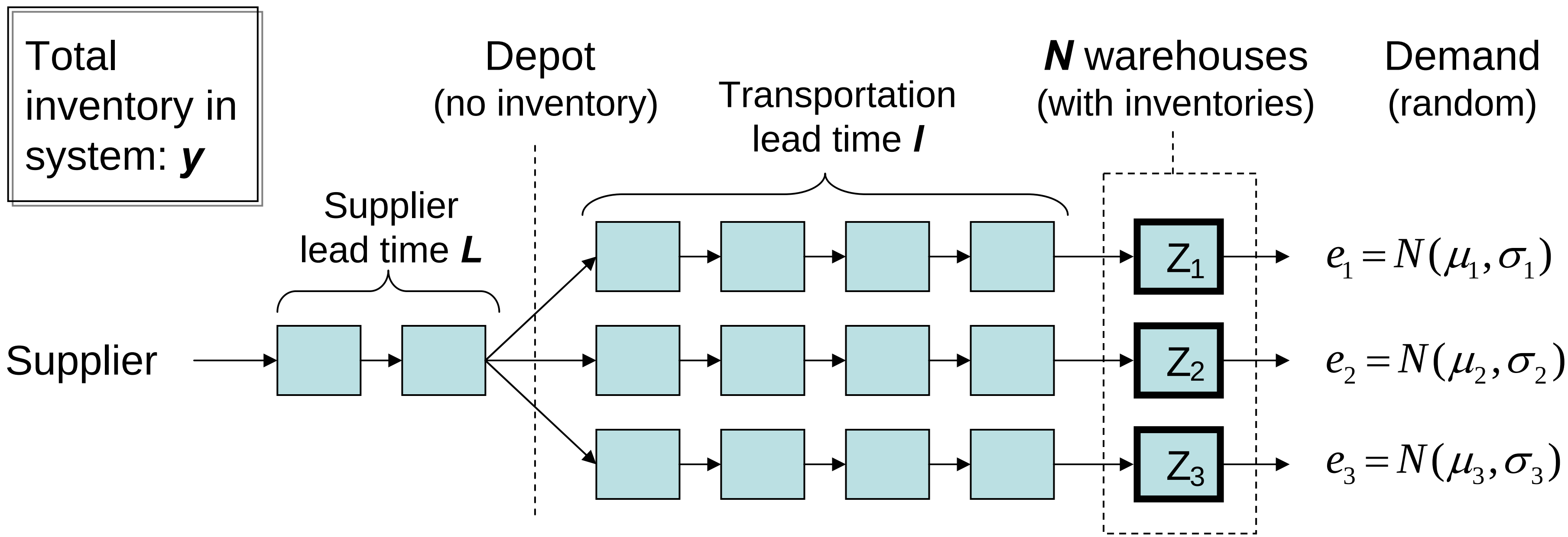
System and Problem Description

The Allocation Assumption

Policy 1: Order up to ***y*** every period

Policy 2: Order up to ***y*** every ***m*** periods

System Description and Assumptions



<u>Costs to be minimized:</u>

- Holding cost ***h*** per unit in inventory
- Penalty cost ***p*** per unit of unmet demand (placed in backlog)
- Fixed cost ***K*** for every order placed

<u>Decisions to be made every period:</u>

- How much, if anything, should be ordered from the supplier
- How should we distribute the incoming orders at the Depot

Why have a Depot?

(with no inventory)

Problem

- Separate warehouses have little purchasing power
- Demand fluctuates for the individual warehouse
- It is expensive/ impractical to build a depot
- (Demand can vary also in the aggregate)

Depot Benefit

- Exploit quantity discounts from the supplier
- Fluctuations in different warehouses even out, and you gain “statistical economies of scale”
- Depot need not to be a physical entity (the point is that goods are allocated after orders completed)
- (Maybe a depot with inventory can do even better)

Applicability of model

	Good application: Steel for conglomerate	Questionable application: Coca-Cola for 7-Eleven
Production lead times:	Long	Short
Inventory surplus:	Holding costs (expensive)	Cheap, not to say desirable (up to shelf capacity)
Inventory shortfall:	Order placed on “backlog” at some penalty	Customer walks (or buys a substitute)

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Allocation Assumption

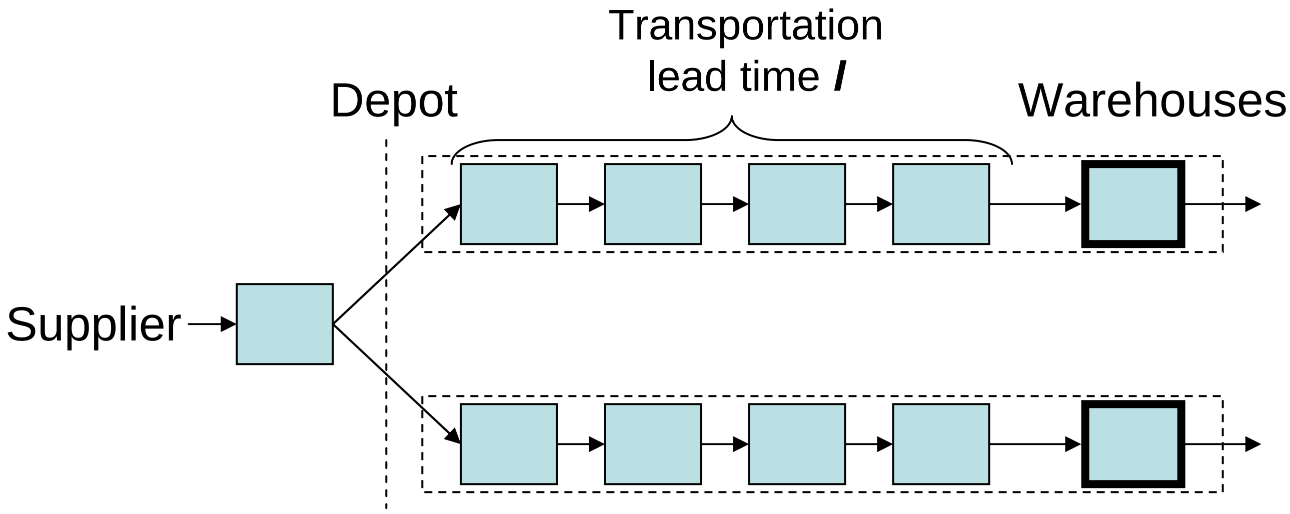
(for every period ordering, normal demand)

"Every period ***t***, we can make an allocation (at the depot) such that the probability of running out at each warehouse is the same at period ***t*+*l***"

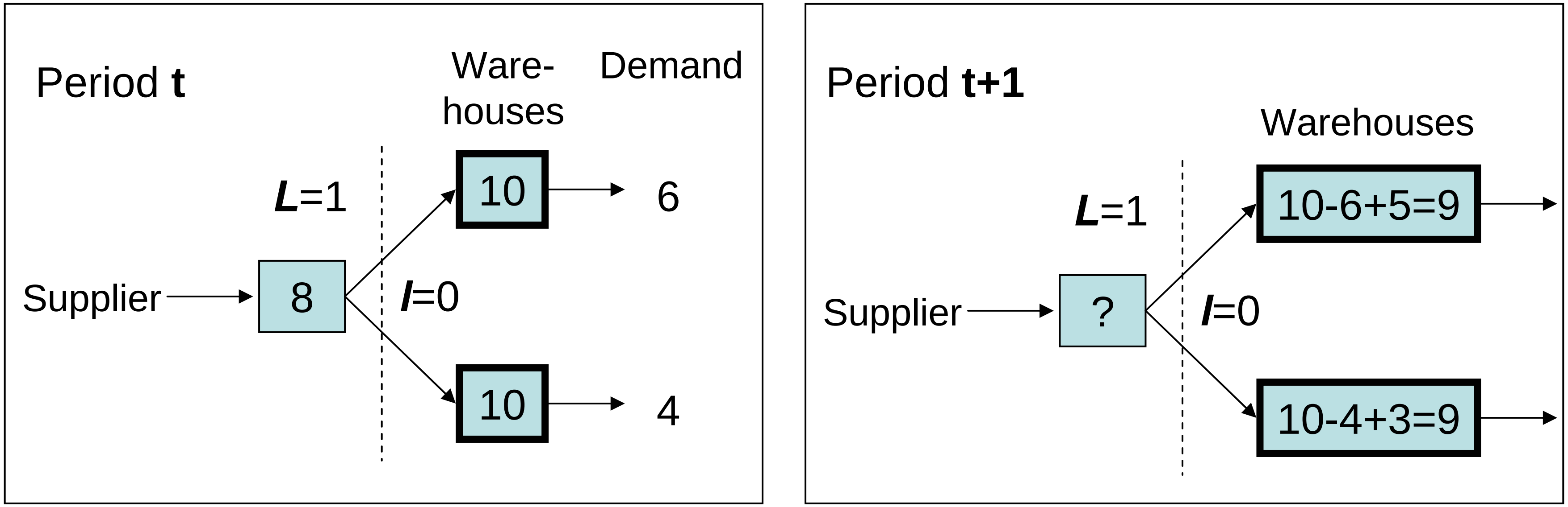
⟺

"Every period, we can find a constant v, such that the total inventory at and in transit to the ***i*** th warehouse is:

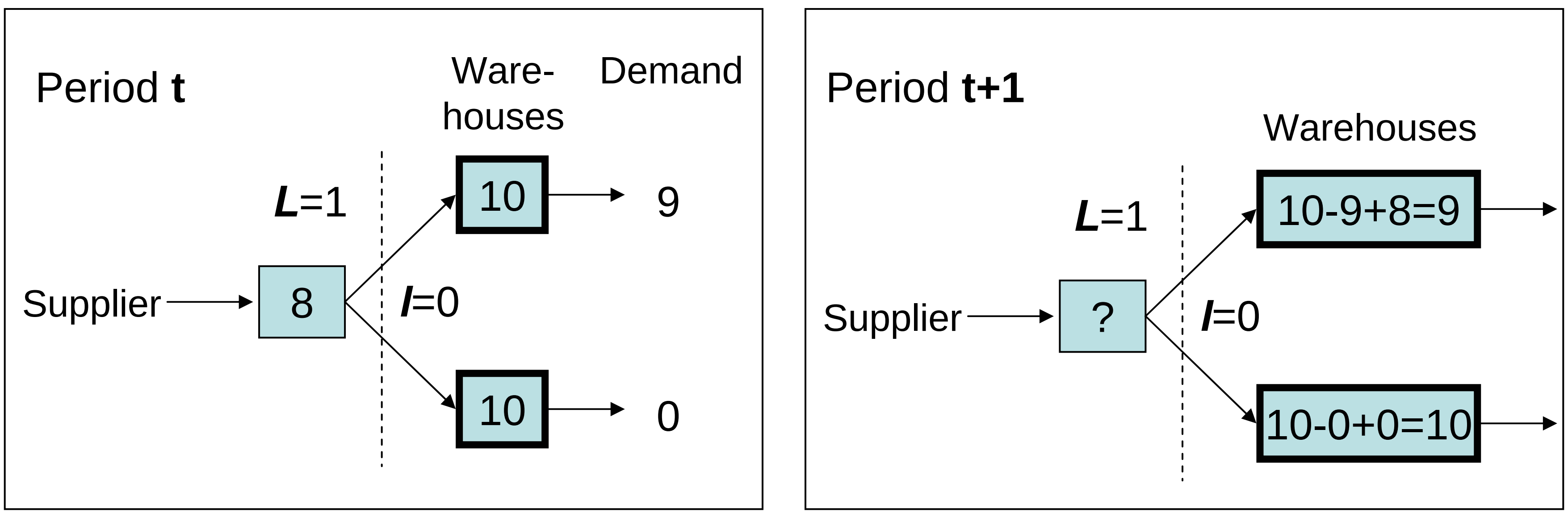
$$(l+1)\mu_i + v\sigma_i\sqrt{l+1}$$



Example when Allocation Assumption holds (identical warehouses)



Example when Allocation Assumption is violated (identical warehouses)



The Allocation Assumption holds for high μ/σ and low N

Theoretical Result

Eppen and Schrage derive a good theoretical approximation formula for the probability of A.A. being true.

Probability of A.A. being true according to experiment presented in paper (my experiment in parenthesis) Percent

μ/σ / N	½	1	3/2	2	5/2
2	32.6 (35.9)	66.3 (66.3)	85.8 (86.5)	95.2 (95.5)	98.8 (98.8)
3	20.1 (19.8)	54.7 (53.2)	79.8 (79.0)	93.0 (93.5)	98.1 (98.2)
4	11.4 (10.0)	43.1 (41.0)	73.3 (72.0)	90.1 (90.5)	97.3 (97.4)
5	7.6 (4.9)	36.5 (30.6)	68.6 (65.8)	88.3 (88.0)	96.5 (96.9)
6	4.6 (2.5)	29.9 (22.3)	63.2 (60.7)	86.4 (85.2)	96.1 (95.8)
7	2.8 (1.2)	24.5 (15.8)	59.1 (53.5)	84.1 (83.2)	95.5 (95.5)
8	1.6 (0.5)	20.4 (11.0)	54.3 (46.6)	82.0 (80.5)	94.6 (95.3)

The paper does not explain how "negative demand" should be interpreted. This happens frequently in the lower left corner where my experiments gave different results than those of the paper

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Policy 1: Order Every Period (fixed ordering costs *K* = 0*)*

Problem:	**Intuitive answer:**	**But…**
How should we distribute the goods that come in to the depot every period?	We should distribute goods so that total goods at and en route to every factory is “the same”	But is this always possible? If we make the A.A., then yes!
How much should we order from the factory every period?	We should order so that the same total inventory level ***y*** is achieved every period. (=order last period’s demand)	What should be the value of ***y***?

Eppen and Schrage find an analytical expression for the inventory at each warehouse

We know how much is ordered every period (as a function of ***y***)

We know how the incoming goods are split up at the Depot

We know the (random function for) demand at each warehouse

Eppen and Schrage derive this expression for the inventory ***S*** at each warehouse (simplified form for the case of identical warehouses):

$$S_j = (l+1)\mu + \frac{y}{N} - (l+1)\mu - \frac{\sum_{t=1}^{L}\sum_{i=1}^{N} e_{it}}{N} - \sum_{t=L+1}^{L+l} e_{jt}$$

Fixed component

Random component

The problem is now equivalent to the newsboy problem, and can be solved analytically

Newsboy problem

"The newsboy buys ***i*** newspapers, at a cost ***c*** each. He sells what is demanded ***d*** (random variable), or all he has got ***i***, whichever is less, at a price ***r.*** Any surplus is lost."

	Deterministic inventory	Random inventory	Cost of surplus (per unit)	Cost of shortage (per unit)
Newsboy	i	-d	c	r-c
Our system (at warehouse)	$(l+1)\mu + \frac{y}{N}$	$-(l+1)\mu - \frac{\sum_{t=1}^{L}\sum_{i=1}^{N} e_{it}}{N} - \sum_{t=L+1}^{L+l} e_{jt}$	h	p

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Policy 2: Order Up to level ***y*** every ***m*** periods

+ We can select ***m*** and ***y*** to minimize total costs, including fixed ordering costs ***K***
+ Periodic ordering policy easy to implement in practice
+ Authors claim that theoretical results on this policy has wider applicability
– Certain approximations have to be made to find the best ***m*** and ***y***
– Even if the best ***m*** and ***y*** were to be found, the periodical ordering policy isn’t necessarily optimal

Several new assumptions lead to an analytical solution for the periodic ordering policy

2. The allocation rule

$$(l+m)\mu_i + v\sigma_i$$

is used, but not proven to be optimal

1. "Stock can only run out in the last period before order arrivals"

Analytical expression for optimal ***y*** and cost, given ***m***

3. As before, we make the a.a. – it is always possible to make this allocation

In typical cases, we can then find the best solution by optimizing ***y*** for ***m***=1,2,3…