Order Full Rate, Leadtime Variability, and Advance Demand Information in an Assemble-To-Order System

by Lu, Song, and Yao (2002)

Presented by Ping Xu

This summary presentation is based on: Lu, Yingdong, and Jing-Sheng Song. "Order-Based Cost Optimization in Assemble-to-Order Systems." To appear in *Operations Research*, 2003.

Preview

- Assembly-to-order system
  - Each product is assembled from a set of components,
  - Demand for products following batch Poisson processes,
  - Inventory of each component follows a base-stock policy
  - Replenishment leadtime *i.i.d.* random variables for each component.

- Model as a $M^X/G/\infty$ queue, driven by a common multiclass batch Poisson input stream
  - Derive the joint queue-length distribution,
  - Order fulfillment performance measure.

Model

- *M* different components, and $F = \{1,2,\ldots,m\}$ are the component indices.
- Customer orders arrive as a stationary Poisson process, $\{A(t), t>=0\}$, with rate λ .
- Order type *K*: it contains positive units of component in *K* and 0 units in *F\K*.
  - An order is of type *K* with probability q^k, $\sum_K q^K = 1$
  - Type K order stream forms a compound Poisson process with rate $\lambda^K = q^K \lambda$
  - A type-K order has Q_j^K units for each component *j*, $Q_K = (Q_j^K, j \in K)$ has a known discrete distribution.
- For each component *i*, the demand process forms a compound Poisson process.

Model

- Demand are filled on a FCFS basis.
- Demand are backlogged (if one or more components are missing), and are filled on a FCFS basis.
- Inventory of each component is controlled by an independent base-stock policy,
  - Where s_i is the base-stock level for component i
  - For each component *i*, replenishment leadtimes, L_i, are *i.i.d.*, with a cdf of G_i
  - Net inventory at time *t*, $I_i(t) = s_i - X_i(t),\ i = 1,\dots m$, where $X_i(t)$ is the number of outstanding orders of component *i* at time *t*.
- Immediate availability of all components needed for an arriving demand as the "off-the-shelf" fill rate.
  - Off-the-shelf fill rate of component *i*, $f_i = \mathrm{P}[X_i + Q_i \le s_i]$
  - Off-the-shelf fill rate of demand type K, $f^K = \mathrm{P}[X_i + Q_i^K \le s_i, \forall i \in K]$
  - Average (over all demand types) off-the-shelf fill rate, $f = \sum_K q^K f^K$

Performance Analysis

- Derive the joint distribution and steady state limit of vector $X(t) = (X_1(t), \ldots, X_m(t))$

(See "Suppliers/Arrivals Replenishment Orders" diagram in Lu, Song, and Yao paper)

  - Each component i, the number of outstanding orders is exactly the number of jobs in service in an $M_i^{Q_i} / G_i / \infty$ queue with Poisson arrival λ_i and batch size Q_i
  - The *m* queues are not independent.
  - Given the number of demand arrivals up to t, the $X_i(t)s$ are independent of one another.

Performance Analysis

- Proposition 1: $X(t) = (X_1(t), \ldots, X_m(t))$ has a limiting distribution. Derive the generating function of *X*.
- In the special case of unit arrival, $Q_i \equiv 1$, the generating function of *X* corresponds to a multivariate Poisson distribution. For each *i*, X_i is a Poisson variable with parameter $\lambda_i \ell_i = (\Sigma_{K \in \Re_i} \lambda^K) \ell_i$
- The correlation of the queue is solely induced by the common arrivals. If the proportion of the demand types that require both *i* and *j* are very small, the correlation between X_i and X_j is negligible.
- Level of correlation is independent of the demand rate.
- Reducing the variability of leadtime or batch sizes will result in a higher correlation among the queue lengths of outstanding jobs.

Response-time-based order fill rate

1) $f^K(w)$ is the probability of having all the components ready within *w* units of time.

2) $D_i(t, t+u] := D_i(t+u) - D_i(t)$

3) Total number of departures from queue *i* in $(\tau, \tau + w) = X_i(\tau) + D_i(\tau, \tau + w] - X_i(\tau + w)$

4) $I_i(\tau) + \{X_i(\tau) + D_i(\tau, \tau + w] - X_i(\tau + w)\} \geq 0$

5) $X_i(\tau + w) - D_i(\tau, \tau + w] \leq s_i, \ i \in K$

6) $X_i(\tau + w) = X_i^w(\tau) + \sum_{n=1}^{Q_i^K} 1\{L_i^n > w\} + X_i(\tau, \tau + w]$

7) Demand at τ can be supplied by $\tau + w$ iff

$$X_i^w(\tau) + X_i(\tau, \tau + w] - D_i(\tau, \tau + w] \leq s_i - \sum_{n=1}^{Q_i^K} 1\{L_i^n > w\}, \ i \in K$$

$$Y_i := X_i^w - Y_i^w$$

8) Order fill rate of type-K demand within time window *w*, $f^K(w) = \mathrm{P}\left[Y_i + \sum_{n=1}^{Q_i^K} 1\{L_i^n > w\} \leq s_i, \ \forall i \in K \right]$

9) Mean:

$$E[Y_i] = \left(\sum_{\Im \in \Re_i} \lambda^{\Im} E(Q_i^{\Im}) \right) (\ell_i - w)$$

Connection to advance demand information

- Suppose each order arrival epoch is known *w* time units in advance, where *w>0* is a deterministic constant.
- Suppose a type-K order arrives at τ , and this information is known at $\tau + w$, we can fill this order upon its arrival with probability,

$$f_A^K(0) = \mathsf{P}\left\{ X_i^w(\tau - w) + X_i(\tau - w, \tau] - D_i(\tau - w, \tau] \leqslant s_i - \sum_{n=1}^{Q_i^K} \mathbf{1}[L_i^n > w],\ i \in K \right\} = f^K(w).$$

- Advance demand information improves the off-the-shelf fill rate: $f^K(w) \geqslant f^K(0)$
- Compare $f_A^K(0)$ with that of the modified system, $\hat{f}^K(0)$, where leadtime is reduced from L_i to $\hat{L}_i = [L_i - w]^+$

$$\hat{f}^K(0) = \mathrm{P}\left\{ X_i^w + \sum_{n=0}^{Q_i^K} 1[\hat{L}_i^n > 0] \le s_i,\ \forall i \in K \right\} \le f^K(w)$$

- Knowing demand in advance (by w time units) is more effective, in terms of order fill rate, than reducing the supply leadtime of components.