

An Introduction to Sensitivity Analysis

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7. VENSIM EXAMPLES

Abstract

This paper is an introduction to a series of papers on sensitivity analysis. It contains three exploratory exercises demonstrating the effects of various parameter and initial value changes on system behavior.

The paper assumes that the reader is able to build and understand a multiple-level model, and has experience with the sensitivity feature in the STELLA software.¹ We encourage the reader to build all the models and to run the simulations described.

¹ For a review of the sensitivity feature in STELLA, please refer to your STELLA user manual or to *Road Maps 3: A Guide to Learning System Dynamics* (D-4503-3), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, p. 12-13. STELLA is a registered trademark of High Performance Systems, Inc.

Introduction

Sensitivity analysis is used to determine how “sensitive” a model is to changes in the value of the parameters of the model and to changes in the structure of the model. In this paper, we focus on parameter sensitivity. Parameter sensitivity is usually performed as a series of tests in which the modeler sets different parameter values to see how a change in the parameter causes a change in the dynamic behavior of the stocks. By showing how the model behavior responds to changes in parameter values, sensitivity analysis is a useful tool in model building as well as in model evaluation.

Sensitivity analysis helps to build confidence in the model by studying the uncertainties that are often associated with parameters in models. Many parameters in system dynamics models represent quantities that are very difficult, or even impossible to measure to a great deal of accuracy in the real world. Also, some parameter values change in the real world. Therefore, when building a system dynamics model, the modeler is usually at least somewhat uncertain about the parameter values he chooses and must use estimates. Sensitivity analysis allows him to determine what level of accuracy is necessary for a parameter to make the model sufficiently useful and valid. If the tests reveal that the model is insensitive, then it may be possible to use an estimate rather than a value with greater precision. Sensitivity analysis can also indicate which parameter values are reasonable to use in the model. If the model behaves as expected from real world observations, it gives some indication that the parameter values reflect, at least in part, the “real world.”

Sensitivity tests help the modeler to understand dynamics of a system. Experimenting with a wide range of values can offer insights into behavior of a system in extreme situations. Discovering that the system behavior greatly changes for a change in a parameter value can identify a leverage point in the model—a parameter whose specific value can significantly influence the behavior mode of the system.²

² In this paper, the term “behavior mode” refers to the general kind of behavior, such as exponential growth, asymptotic growth, S-shaped growth, or oscillation.

Exploratory Exercises

In this section we look at two models and explore how sensitive they are to changes in parameters and initial values of stocks. The first exploration shows that parameter changes produce some change in the appearance of behavior of the system, but they do not change the behavior mode. The second exploration demonstrates that changes in different parameters create different types of changes in the behavior of the system.

In the first exploration, we conduct sensitivity analysis on all the constant parameters in the model. However, in a large model, such an extensive treatment of sensitivity analysis is often impossible.³ The modeler must pick the parameters he expects to have most influence on the behavior, or the ones that he is most uncertain about, and only use those in the sensitivity analysis. We will see in the second exploration that an examination of the structure of the model can indicate, without running the sensitivity tests, what kind of effect changes in some parameters would have. An explanation of how to choose the parameters for testing will be offered in later papers in the sensitivity analysis series.

Exploration 1: Lemonade Stand

In the first exploration, let's look at a lemonade stand located on a college campus. As usual, we are particularly interested in the behavior of the stock, the number of cups of lemonade that are ready to be sold to customers. The stand is open eight hours every day. Howard, the owner, is the only person working in the stand.

We encourage you to try to build the model and then compare it with the model suggested in Figure 1. The model documentation and equations can be found in section 7.1 of the Appendix.⁴

³ Some system dynamics software, such as Vensim, can make automatic sensitivity tests of all parameters. Vensim is a registered trademark of Ventana Systems, Inc.

⁴ The Lemonade stand model is based on: John Sterman, 1988. *Formulation of a simple business model* (D-4148), 15.874, System Dynamics for Business Policy, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology.

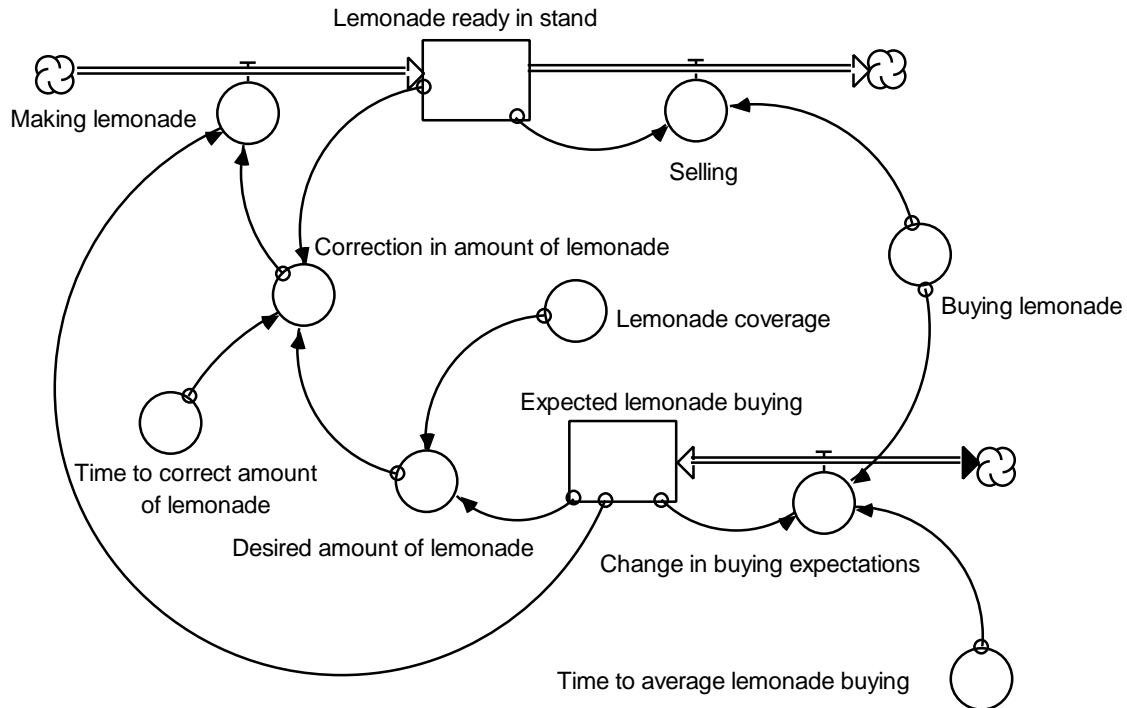


Figure 1: Model of the Lemonade stand

The stock, called “Lemonade ready in stand,” is increased by an inflow called “Making lemonade,” and decreased by the outflow, “Selling.” “Selling” (cups per hour) is equal to “Buying lemonade,” an exogenous parameter (not present in a feedback loop of the system). Howard has found out that he sells approximately 20 cups of lemonade every hour. However, he cannot sell more than what he has. Therefore, if “Buying lemonade,” or the demand for lemonade, at any given time is greater than the amount of “Lemonade ready in stand,” he can only sell the smaller one of these amounts, that is, the amount he has ready.

Based on his experience, Howard expects to sell a certain number of cups every hour. If the demand for lemonade changes, it usually takes him approximately an hour to recognize the shift in sales as opposed to just random fluctuations. This 1 hour time constant is the “Time to average lemonade buying.”

Howard also wants to have lemonade prepared for some time in advance. A parameter called “Lemonade coverage” is the number of hours worth of lemonade he wants to have ready. For example, a “Lemonade coverage” of 2 hours means that if he stops making lemonade at four o’clock, he will be able to sell all the remaining lemonade by six o’clock, provided that the demand for lemonade does not change during this time. Therefore, “Lemonade coverage” determines his “Desired amount of lemonade.”

Howard then compares the desired amount to the actual amount of “Lemonade ready in the stand.” He needs some time to compensate for this difference, the “Time to correct amount of lemonade.” The difference between the desired and the actual amount, divided by the “Time to correct amount of lemonade,” is his hourly “Correction in amount of lemonade.” He then adds the correction to the amount of lemonade he expects to sell, and makes a corresponding number of cups of lemonade. This number is the hourly

inflow to the stock. Of course, “Making lemonade” cannot be negative. So, if the correction is negative (if the desired amount is smaller than the actual amount), and if the sum of the correction and his expectations is still negative, he will not make any lemonade until the sum of the correction and his expectations becomes positive again.

Let’s now look at the behavior of the system when started in equilibrium. It remains in equilibrium during the entire eight hours of the simulation. The value of “Lemonade ready in stand” is 40 cups, as one can quickly check by multiplying the 2 hour “Lemonade coverage” by the constant value of “Buying lemonade” of 20 cups per hour. Figure 2 shows the base run.

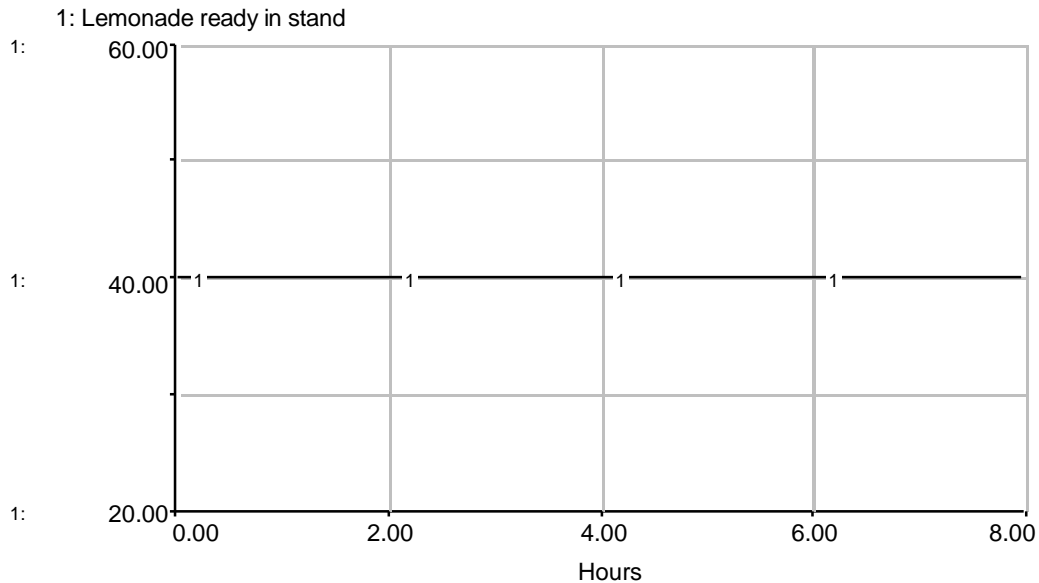


Figure 2: Base run of the Lemonade Stand model

Now let’s assume that the demand for lemonade changes. At time 1 hour, we introduce a step increase of 5 cups in “Buying lemonade.” Figure 3 shows the resulting behavior of “Lemonade ready” and “Buying lemonade.”

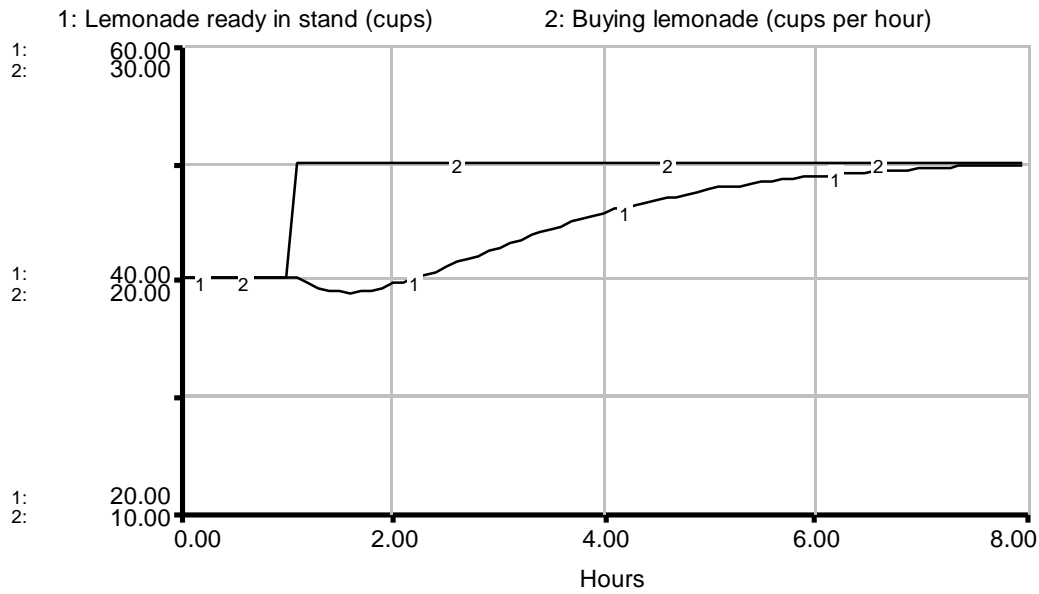


Figure 3: The system responding to a step increase in Buying lemonade

The system starts off in equilibrium, but only remains in equilibrium during the first hour. Immediately after the change in demand for lemonade, the amount of “Lemonade ready in stand” decreases slightly because “Selling” increases together with “Buying lemonade.” In the meantime, it takes Howard some time to perceive the change and then to adjust his expectations, so he will not be able to react immediately by making more lemonade. It is the difference between the “Desired amount of lemonade” and the actual amount of “Lemonade ready in stand” that prompts him to make more lemonade so that the stock can start increasing and approach its new equilibrium value. To obtain the equilibrium value of 50 cups, one only needs to multiply the new value of “Buying lemonade,” 25 cups per hour, by the “Lemonade coverage,” 2 hours.

Exercise

Using the change in “Buying,” we can now experiment with the model to see how the behavior resulting from disturbance changes under different parameter settings.

When building a model, we are often uncertain about the exact value of a parameter. However, it is important to be able to pick reasonable values for the parameters. This means that the values chosen should stay in a plausible range that could occur in the real world system. For example, in the Lemonade stand model it would not be reasonable to expect the “Time to correct amount of lemonade” to be just a few minutes. Howard needs a longer time to perceive the change in the stock, and then to correct the amount of lemonade so that it is ready to be sold. On the other hand, a value of 5 or 10 hours to correct the amount of lemonade would prevent him from responding to a change in his selling rate. The stock of lemonade might deplete quickly, resulting in a loss of customers, or overproduction. However, values of one or two hours seem quite reasonable: he will have enough time to find all the necessary ingredients, mix them together, and prepare the lemonade to be sold. The results of the sensitivity tests will

indicate whether the conclusions drawn from the model can be supported even without being certain about the exact values of the parameters.⁵

Three parameters and the initial value of the stock “Lemonade ready in the stand” can be used to explore the sensitivity of this model. The three parameters are “Time to average lemonade buying,” “Time to correct amount of lemonade,” and “Lemonade coverage.”

It would probably be difficult to find out exactly how long it takes Howard to realize that “Buying lemonade” has changed. We can try to use several values for the “Time to average lemonade buying” to see if and how much the behavior of the system changes when the time is shorter or longer. Three values that seem reasonable are 30 minutes, 1 hour, and 1 hour and a half. The comparative run is shown in Figure 4.

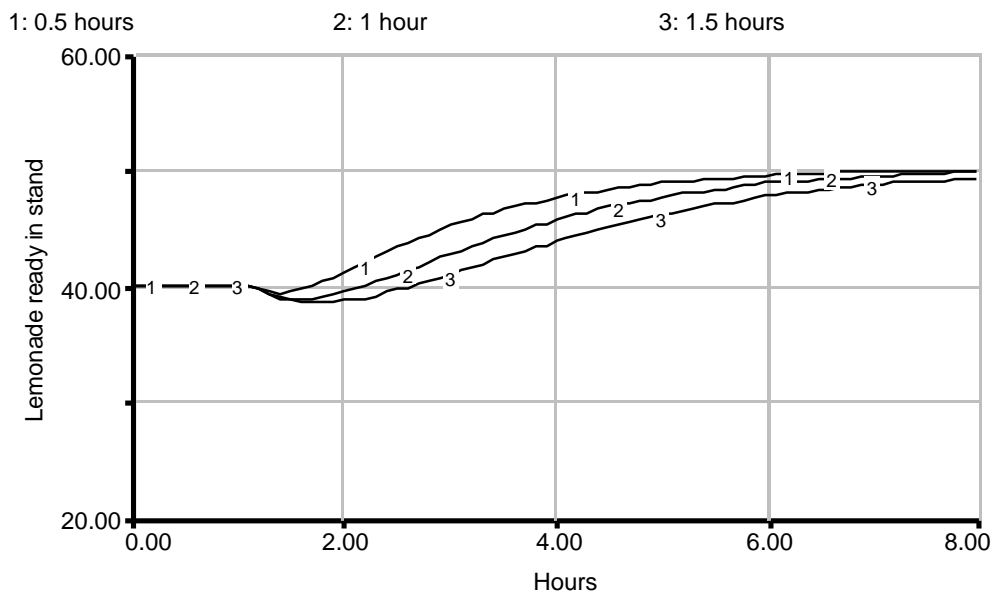


Figure 4: The effect of changes in “Time to average lemonade buying”

Although the three curves do not look exactly the same, these parameter changes do not affect the general mode of behavior of the system. All three curves show a small decrease in the stock right after the step increase and then a slow approach to the new equilibrium value of 50 cups. The similarity of the results shows that Howard does not have to be certain how long it would take him to perceive a change in his “Selling” rate to be able to estimate the overall behavior of his stock of lemonade.

The curves indicate that the faster Howard adjusts his expectations, the faster the stock of lemonade will approach equilibrium. The initial decrease in the stock is greater for larger values of this parameter. If it takes Howard longer to perceive a decline in the stock, the stock declines for a longer time and to a lower value.

⁵ The parameter values used in this paper are of course not the only reasonable values, and they were only picked as examples. We encourage you to use other values if you feel that they would represent the situation more realistically.

“Time to correct amount of lemonade” is another parameter about whose value Howard is uncertain. Three values that seem plausible are one hour, one hour and a half, and two hours. Figure 5 shows the comparative run.⁶

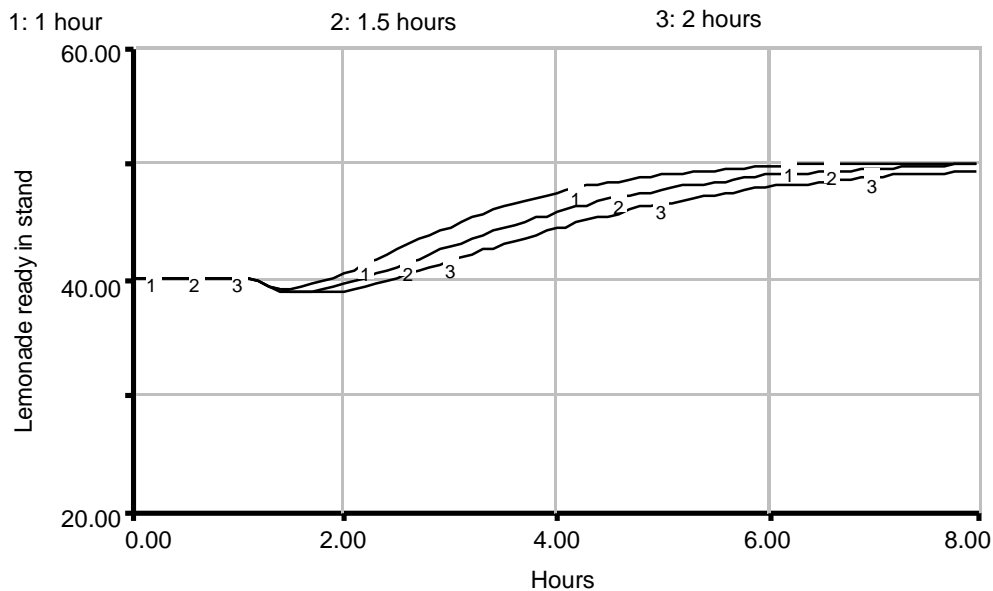


Figure 5: The effect of changes in “Time to correct amount of lemonade”

Again, notice that the general type of behavior remains the same for these parameter changes, but each curve is slightly different. As before, this means that Howard only needs to know an approximate range of values for this parameter to be confident in the behavior simulated by the model. The effect of changing the value of “Time to correct amount of lemonade” is similar to the “Time to average lemonade buying.” A lower value of “Time to correct amount of lemonade” makes the stock reach the equilibrium value faster because the “Correction in amount of lemonade” is greater, so Howard prepares the lemonade faster. Also, an increase in the value of this parameter makes the initial decrease in the stock larger. If it takes Howard a longer time to correct the amount of “Lemonade ready,” he will not react quickly enough by starting to make more lemonade, so he will not prevent his stock of lemonade from decreasing.

Next, let’s look at changes in “Lemonade coverage.” It is Howard himself who decides how many hours worth of lemonade he wants to keep ready at all times. He always wants to have a sufficient amount of lemonade ready, but he also wants to be sure that the lemonade he sells is always fresh. He can therefore run tests with several values of “Lemonade coverage” to see how the behavior of his stock of “Lemonade ready” would change if he decided to have smaller or larger “Lemonade coverage.” Some values that he can try for this parameter are one and a half hours, two hours, and two and a half hours. Figure 6 shows the comparative run.

⁶ Note that the sensitivity feature in STELLA automatically resets the value of the previous parameter to its base run value when you select another parameter for the sensitivity test.

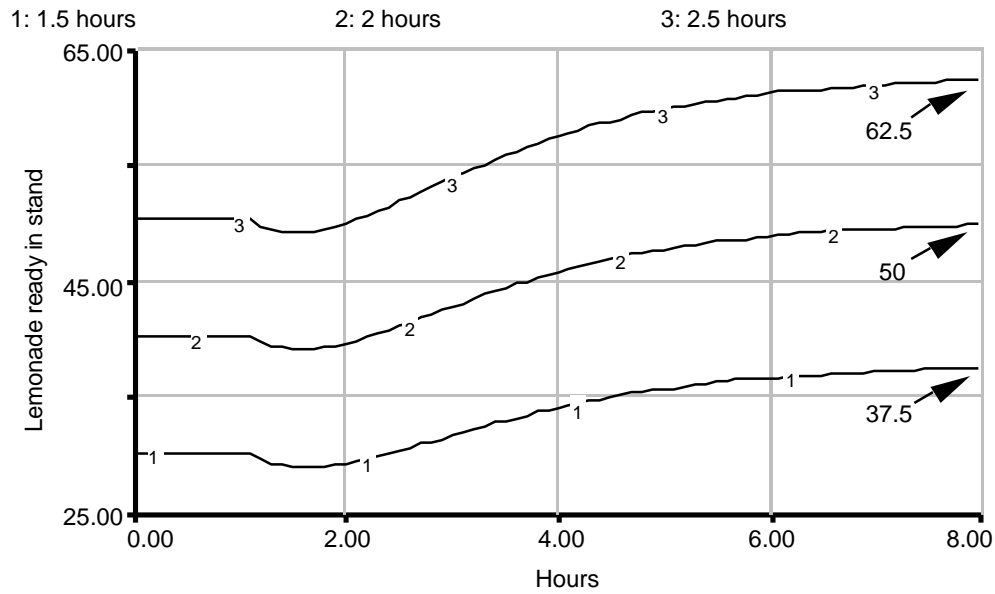


Figure 6: The effect of changes in Lemonade coverage

The effect of a change in “Lemonade coverage” is different from changes in the two previous parameters.⁷ It is important to notice, however, that even though the behavior of the three curves is not exactly the same the general behavior is again unchanged. The curves start out in equilibrium with different initial values. Immediately after the step increase, there is always a slight decrease in the stock, followed by a slow asymptotic approach to equilibrium. Contrary to the changes in the first two parameters that we examined, changing “Lemonade coverage” does not influence the time it takes the stock to approach equilibrium. Instead, it changes the equilibrium value of the amount of “Lemonade ready in stand.” The equilibrium values can be obtained by multiplying “Buying lemonade” by the respective values of “Lemonade coverage.” Therefore, if Howard changes his “Lemonade coverage,” only the equilibrium value of the stock of “Lemonade ready” will be affected. Because “Lemonade coverage” compensates for possible variations and fluctuations in demand for lemonade, a higher value of “Lemonade coverage” protects Howard from running out of lemonade. Therefore, to make sure that he always has enough “Lemonade ready” for the customers, Howard should not choose a very small value for “Lemonade coverage.”

Finally, we can make changes in the initial value of “Lemonade ready in stand.” This value is set when Howard prepares lemonade before opening the stand every morning. He could have initial values of 30, 40, or 50 cups ready, as shown in Figure 7.

⁷ Please note that to start this simulation in equilibrium, you should set the initial value of “Lemonade ready in stand” to “Buying lemonade * Lemonade coverage.”

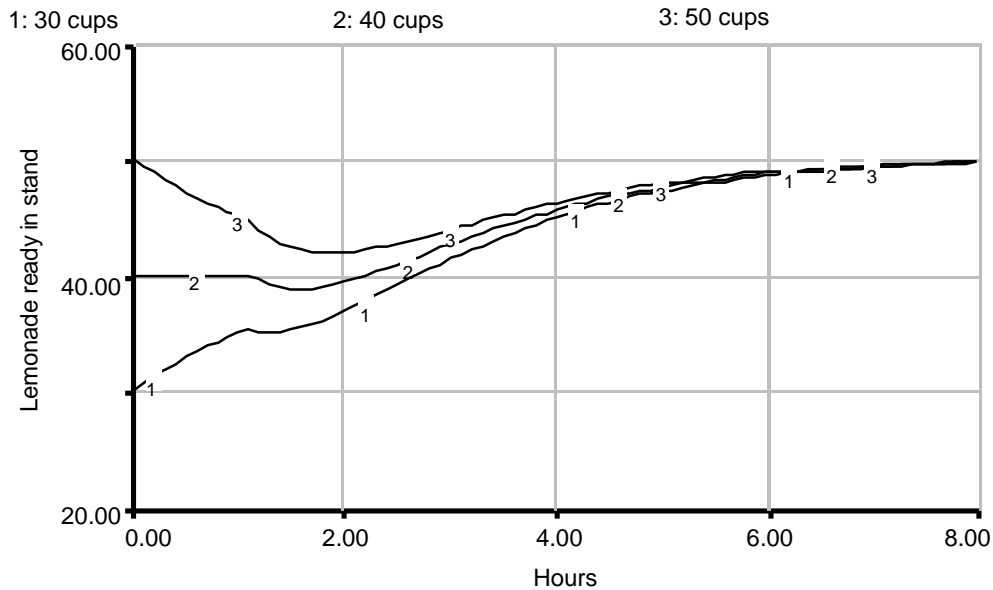


Figure 7: The effect of changes in the initial value of “Lemonade ready in stand”

Notice again that even though the behavior of the curves looks different from the ones in the previous simulations of this model the general behavior has not changed. Even when Howard starts out with a larger or smaller amount of lemonade, the behavior of the stock of Lemonade ready does not change greatly. When the initial amount of Lemonade ready is lower than what the equilibrium value should be, that is less than 40 cups, the stock will be increasing during the first hour, as in curve 1. When the initial amount is larger than 40 cups, the stock will decrease for the first hour, as in curve 3. However, after the step increase, all the curves immediately decline slightly, and then start slowly increasing to approach the same equilibrium value of 50 cups nearly at the same time.

Debrief

As expected, changing the value of parameters in the model does make some difference in the behavior observed. Also, the sensitivity tests indicate that some parameter changes result in “greater,” or more significant, changes than others. For example, compare Figures 5 and 6. In Figure 5 the changes in “Time to correct amount of lemonade” produce little difference in the behavior, while in Figure 6 the curves show the same behavior, but at different values of the stock. This measure of more significant changes is studied through sensitivity analysis. In all cases, however, it is the structure of the system that primarily determines the behavior mode. In general, but with exceptions, parameter values, when altered individually, only have a small influence on behavior.

Exploration 2: Epidemics

In the second exploration we look at an epidemics model. The model was already used in a previous chapter in Road Maps, so it is possible that you have already built it.⁸

⁸ For more information about the model, see: Terri Duhon and Marc Glick, 1994. *Generic Structures: S-Shaped Growth I* (D-4432), System Dynamics in Education Project, System Dynamics Group, Sloan

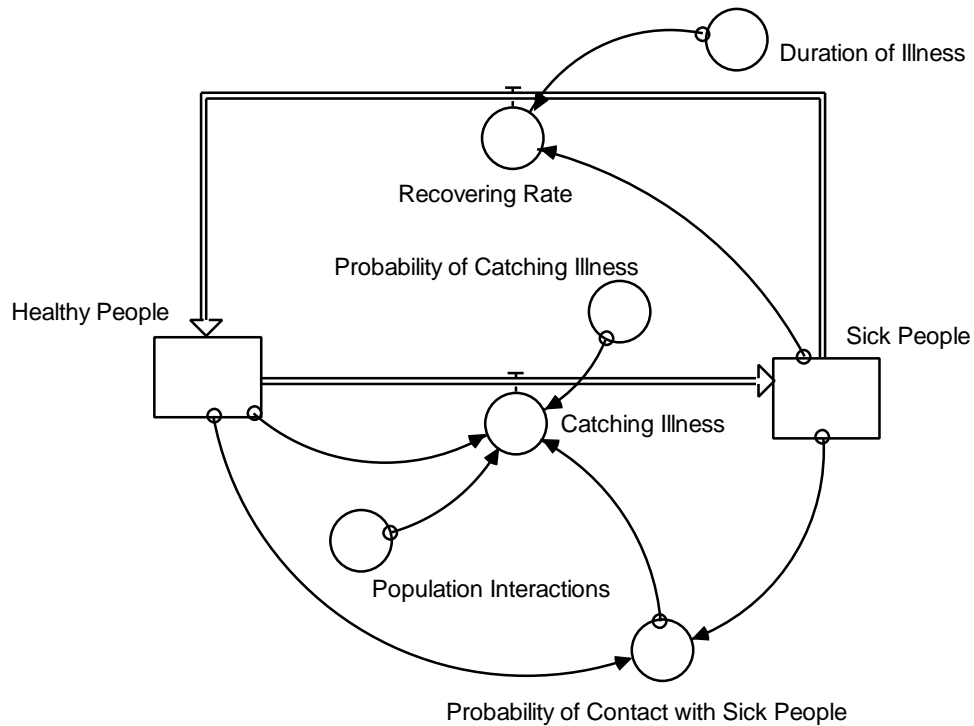


Figure 8: The Epidemics Model

The model shown in Figure 8 has two stocks, “Healthy People” and “Sick People.” The total population is a constant 100 people in all the simulations, so the initial value of “Healthy People” is:

$$\text{Initial number of Healthy People} = 100 - \text{Sick People}$$

In the base run, the initial value of “Sick People” is 1. The flow of “Catching Illness” converts the “Healthy People” to “Sick People.” It is affected by three variables: “Probability of Contact with Sick People,” “Probability of Catching Illness,” and “Population Interactions.” The “Probability of Contact with Sick People” is the ratio of “Sick People” to the total population of 100 people. Therefore, the more “Sick People” there are in the population, the greater the probability of coming into contact with a sick person. When a healthy person comes into contact with a sick person, there is a certain probability that the healthy person will catch the illness. This is the “Probability of Catching Illness.” The value of “Population Interactions” determines the total number of interactions or contacts that any one person has with other people in the population during a month. The “Recovering Rate” converts “Sick People” back to “Healthy People,” with a time constant called “Duration of Illness.” See section 7.2 of the Appendix for the model documentation and equations.

Figure 9 shows the base run of the model, using the parameter values indicated in the Appendix.

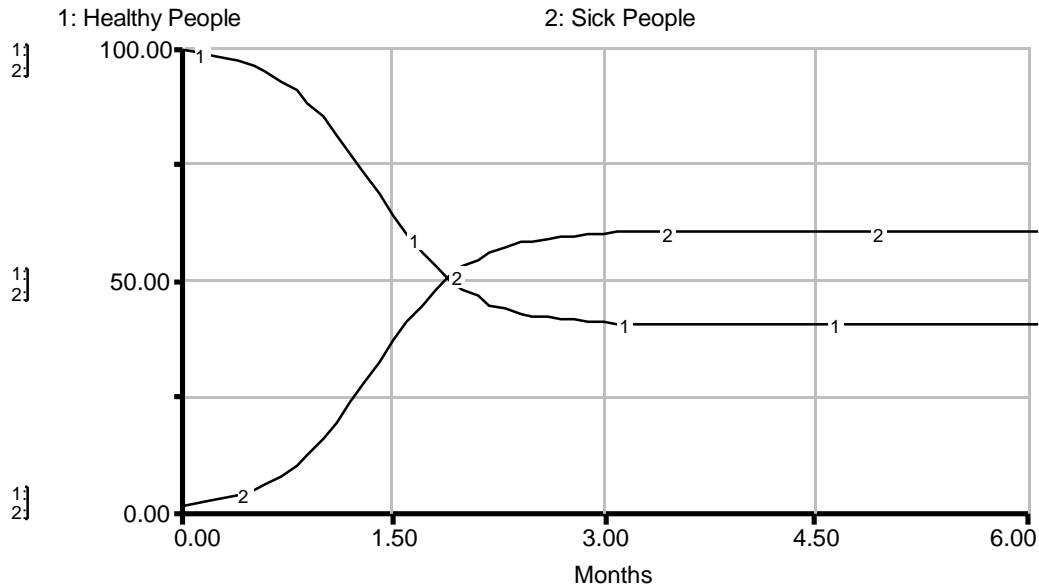


Figure 9: Base run of the Epidemics Model

Both stocks exhibit S-shaped growth. The number of “Sick People” at first increases exponentially, but then shifts to asymptotic growth to approach the equilibrium value of 60 “Sick People.” The number of “Healthy People,” on the other hand, decreases, at first exponentially and then asymptotically to the equilibrium value of 40 “Healthy People.”

Exercise

The initial value of “Sick People,” as well as three parameters—“Duration of Illness,” “Population Interactions,” and “Probability of Catching Illness”—can be used to evaluate the sensitivity of this model.

In this exercise, we use some parameter values or initial values that correspond to extreme cases. Such tests are very helpful in better understanding the dynamics of the system. For example, the extreme values of the initial number of “Sick People” would be 0 or 100 people. An extreme value of “Duration of Illness” would be a very large value, say one million months, illustrating the case of a terminal disease with no recovery. When using such extreme values, however, it is important to make sure that they stay in the range of values that are possible in the real-world system (e.g., do not use a negative initial value for a stock that cannot be negative). Sensitivity tests using extreme values are performed mostly to learn about the dynamics of the system.

The initial value of “Sick People” represents the number of people who are sick when the simulation begins. The stock can, for example, start with no sick people, one sick person, 20, 50, or even 100 sick people. The behavior of both stocks, Healthy People and Sick People, is shown in Figure 10.

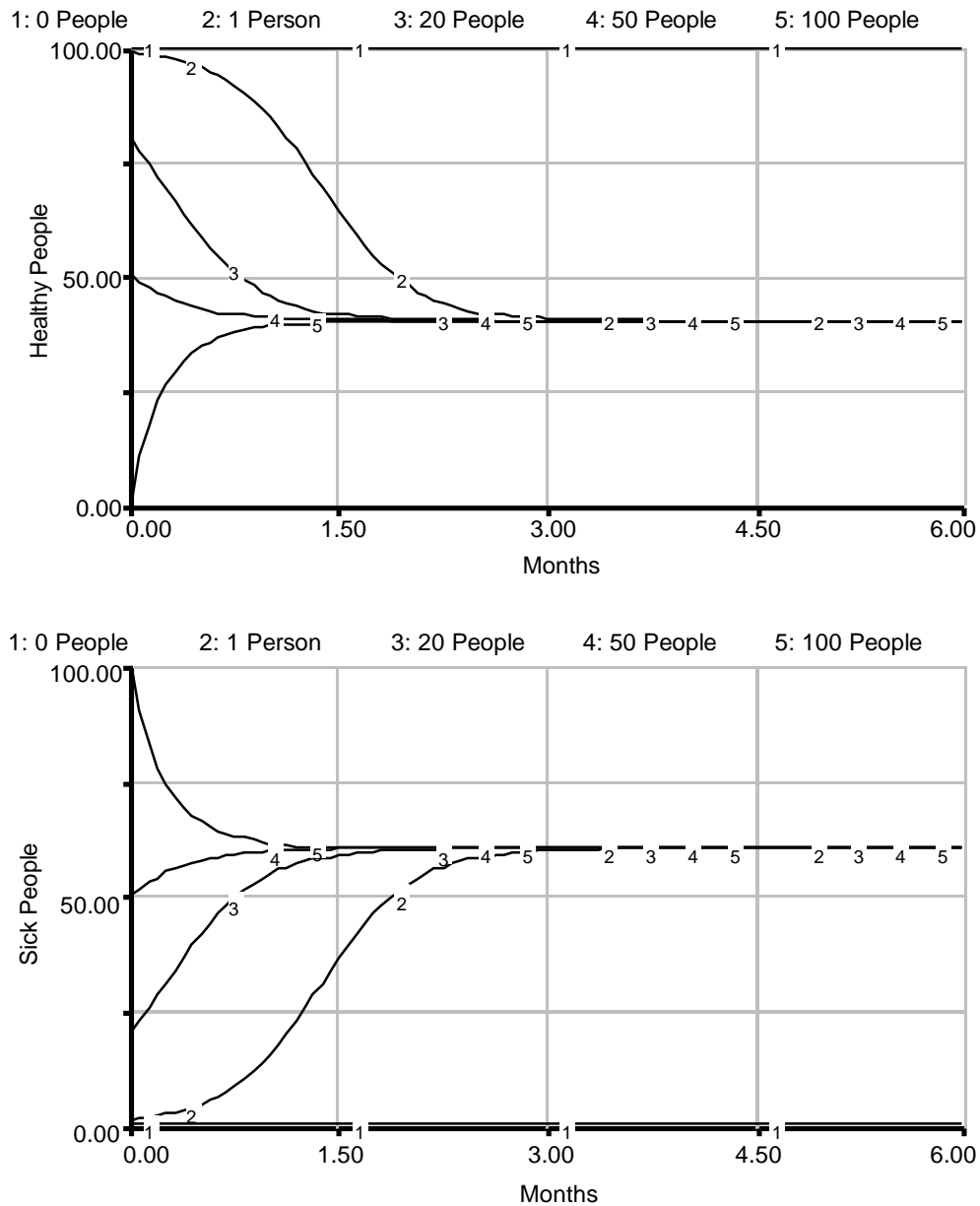


Figure 10: The effect of changes in the initial value of Sick People

Let's look at the individual curves. Curve 1 represents the case of zero "Sick People." During the entire simulation, all the hundred people in the population stay healthy and no one becomes sick. If there are no "Sick People," the "Probability of Contact with Sick People" and therefore the flow "Catching Illness" are both zero. Thus, it is obvious that there must be at least one sick person to obtain the characteristic S-shaped growth behavior. Curve 2 corresponds to the case of one sick person initially. The presence of only one sick person makes a great difference in the behavior of the system. The one person starts spreading the disease to more people. Just like in the base run, the two stocks follow the S-shaped growth pattern, and stabilize at the base run equilibrium values. Curves 3 and 4 behave similarly to curve 2, but approach their respective equilibrium values faster because their initial values are closer to equilibrium.

In curve 5, the behavior of the stocks is reversed—the number of “Healthy People” increases to its equilibrium value of 40, while the number of “Sick People” decreases to 60. As long as there is initially at least one sick person present in the population the equilibrium value does not depend on the initial number of “Sick People.”

The parameter “Duration of Illness” determines how long it takes a sick person to become healthy again. Try using the values of 3 days, two weeks, a month, six months, and an extremely large value, such as a million months, as shown in Figure 11.⁹

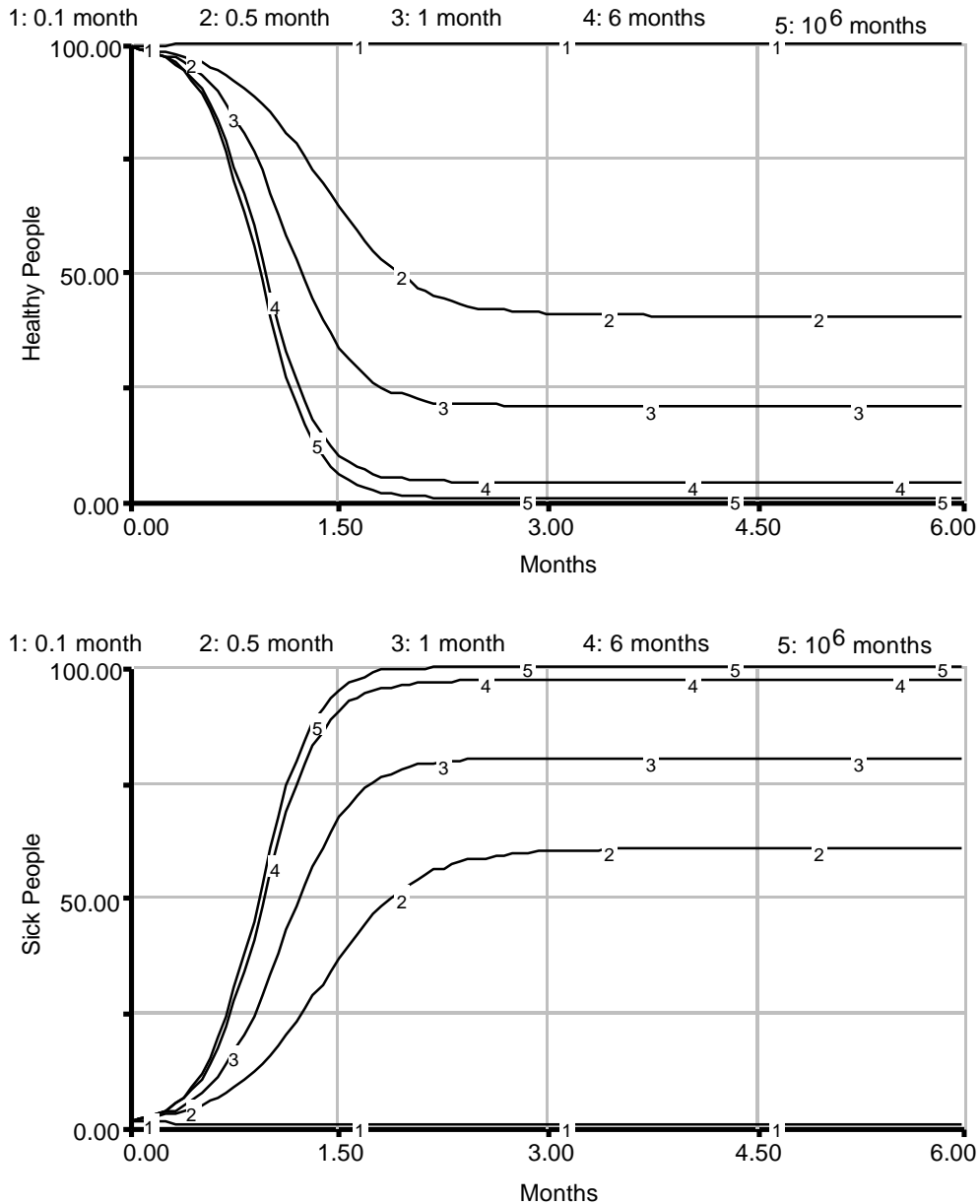
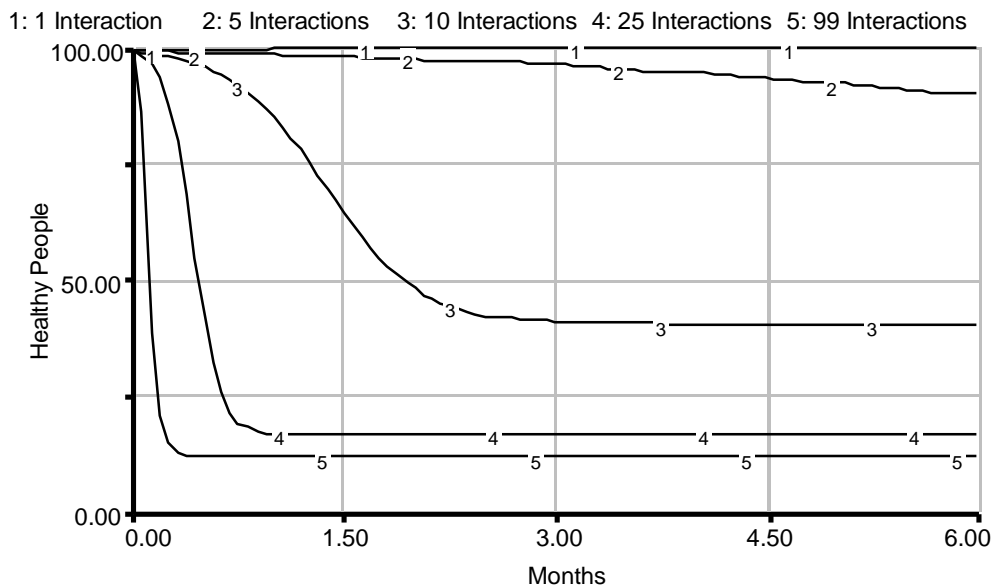


Figure 11: The effect of changes in “Duration of Illness”

⁹ Note that you should set the time interval of the simulation to 0.1 or less to test for the value of “Duration of Illness” of 0.1.

Let's again analyze the individual curves. Curve 1 represents a case of a very short disease that only lasts a few days. The only sick person becomes healthy before spreading the disease to anyone else, and all the "Healthy People" stay healthy. Curve 2 corresponds to the base run. Notice from comparing curves 2, 3, 4, and 5, that increasing the "Duration of Illness" results in higher equilibrium values of "Sick People" and lower equilibrium values of "Healthy People." The reason for this behavior is that when the disease is longer, each sick person can spread it to more people. In particular, curve 5 shows that when the disease is extremely long, or terminal (no recovery), all 100 people become sick. The longer the "Duration of Illness," the sooner the stocks approach equilibrium. This result should be expected—the longer the disease, the more people get sick and have a longer time to spread the disease further and faster, so equilibrium occurs sooner.

The parameter "Population Interactions" determines how many contacts any person has with other people during a unit of time. The value of a parameter such as "Population Interactions" is quite uncertain—it can be different for each person and can even vary over time. Therefore, the value chosen for the model can only be an estimate of the average of the actual value. A sensitivity test that uses different values of "Population Interactions" indicates how the behavior of the system is affected when its value changes. Depending on the nature of the population, the number of "Population Interactions" can vary significantly. A person can, for example, meet only one other person each month, 5 people, 10 people, 25 people, or even all the rest of the population—99 people a month. Figure 12 shows the behavior of both stocks.



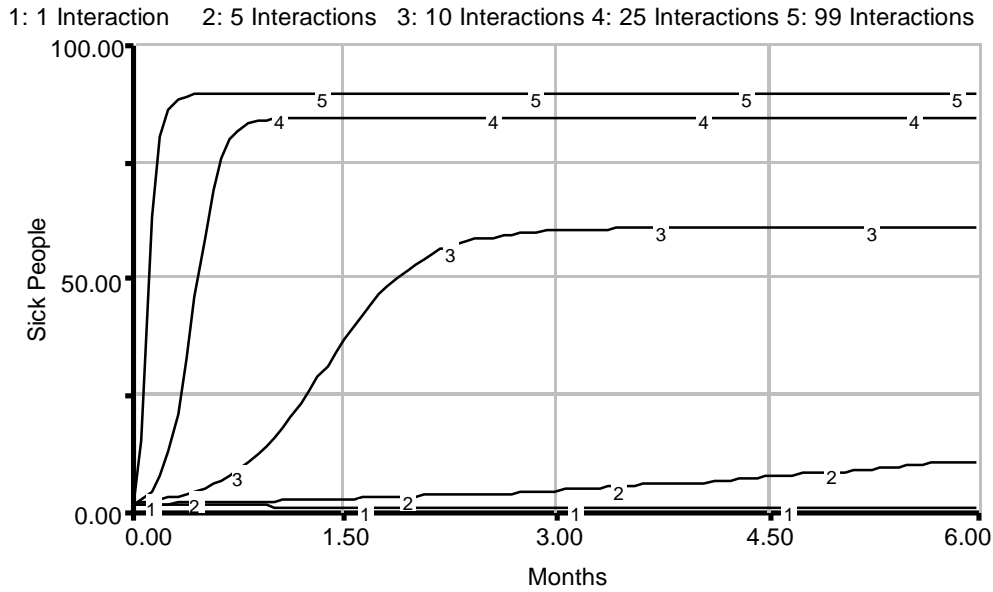


Figure 12: The effect of changes in “Population Interactions”

If there are few interactions in the population, the only sick person becomes healthy and no one else becomes sick, as in curve 1. By comparing curves 2, 3, 4, and 5, notice that increasing the number of interactions increases the equilibrium value of “Sick People” and decreases the equilibrium value of “Healthy People.” Therefore, if you want to study the spread of an epidemic in a specific population, and if you are interested in the equilibrium number of people who will be affected, you will have to have an idea of the value of “Population Interactions” in that population. Also, an increase in “Population Interactions” makes the stocks reach their equilibrium values faster—it increases the value of “Catching Illness,” and thus “Healthy People” become sick sooner.

Before looking at the effects of changing “Probability of Catching Illness,” turn back to Figure 8 showing the Epidemics model. Notice that both “Population Interactions” and “Probability of Catching Illness” enter the rate of “Catching Illness” in a similar way. The rate equation is:

$$\text{Catching Illness} = \text{Healthy People} * \text{Probability of Contact with Sick People} * \underline{\text{Population Interactions}} * \underline{\text{Probability of Catching Illness}}$$

Both “Population Interactions” and “Probability of Catching Illness” are linear factors of the rate equation—they both simply multiply “Healthy People” and “Probability of Contact with Sick People.” One would therefore expect that their effect on the system behavior should be similar, and that they should produce the same kind of changes in the behavior when their value changes. We encourage you to run sensitivity tests with different values of “Probability of Catching Illness.” Because this value is also very uncertain in the real world, try values of 0.05, 0.25, 0.5, 0.75, and 1, for example. The graphs you obtain should resemble the ones in Figure 12 above.

Debrief

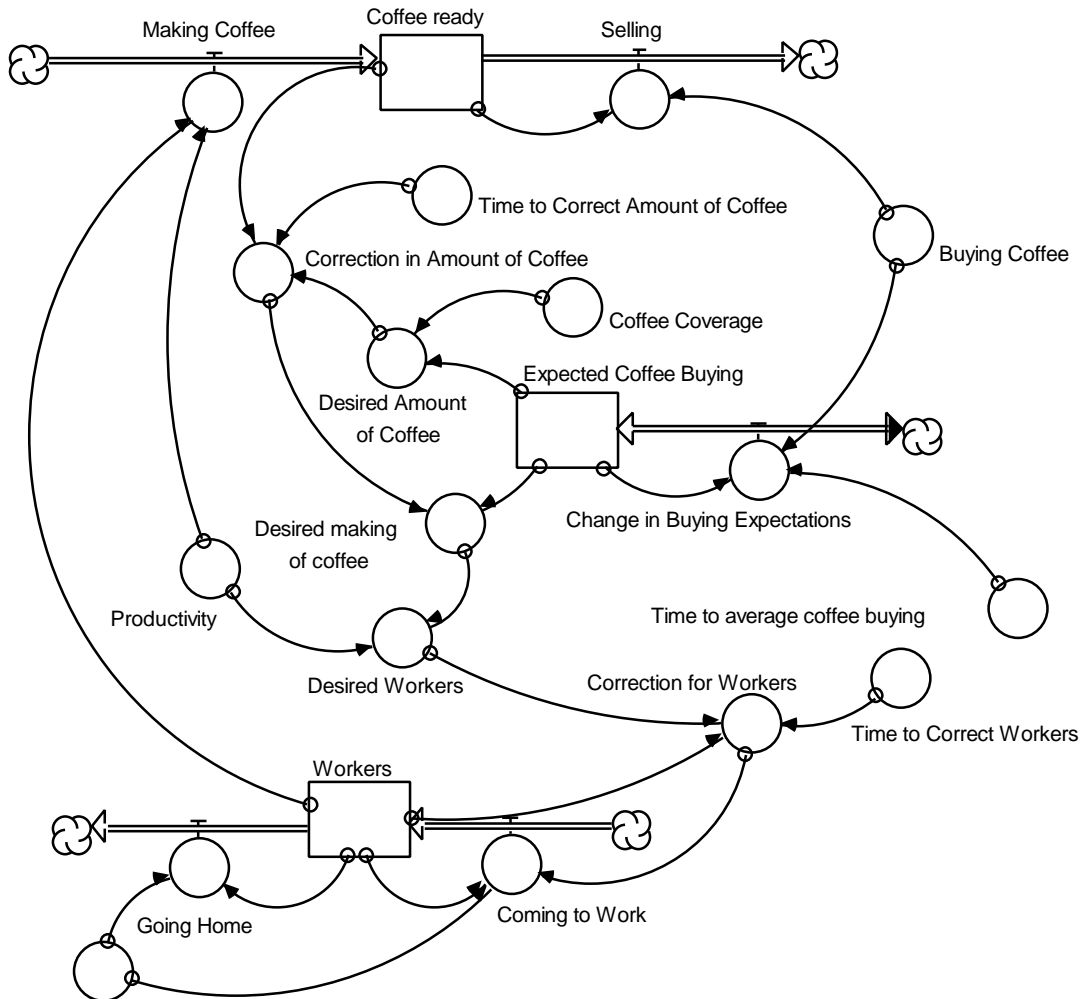
Sensitivity analysis again showed that changing the value of parameters makes some difference in the behavior of the model, while the general behavior mode is relatively insensitive to parameter changes. Some parameter changes affect the behavior to a larger extent than others. Changes in some parameters affect the equilibrium values, such as “Lemonade coverage” from the Lemonade stand model. Other parameters affect the time necessary to approach equilibrium—for example “Time to correct amount of lemonade” from the Lemonade stand model, or the initial value of “Sick People” from the Epidemics model. Others again have influence over both—“Population Interactions” for example. When two parameters have similar roles in the model, they usually affect the behavior in a similar way, such as “Population Interactions” and “Probability of Catching Illness.” Using extreme values in sensitivity tests is an excellent tool for gaining a deeper understanding of the behavior generated by the structure of the model as well as for testing the assumptions about the model.

Independent Exploration: Coffeehouse

We now return to Howard, the owner of the lemonade stand on a college campus. Howard realized that it could be more profitable for him to sell coffee because students tend to drink more coffee than lemonade, and they drink it at any time of the day and night. Therefore, he closed his lemonade stand and opened a 24-hour Coffeehouse. Howard bases the Coffeehouse model on the model he used in his lemonade stand to model the number of cups of “Coffee ready.” We will run the simulation over a period of two days, or 48 hours.

Figure 13 shows the full model. The model documentation and equations are in section 7.3 of the Appendix.¹⁰

¹⁰ The Coffeehouse model is based on: John Sterman, 1988. *Understanding Business Fluctuations—The Causes of Oscillations* (D-4149), 15.874, System Dynamics for Business Policy, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology.



Average Length of Working

Figure 13: The Coffeehouse Model

Because Howard cannot work 24 hours a day all by himself, he needs to hire student workers to work in the Coffeehouse. To represent the new situation, he adds a sector to his model for the number of workers. He employs several workers, but not all of them are at work at the same time. The stock of “Workers” in the model measures only the number of workers who work in the Coffeehouse at a specific time.

Each worker is able to prepare a certain number of cups every hour, determined by the parameter called “Productivity.” In the base run, “Productivity” is 20 cups per worker per hour. When Howard divides the “Desired making of coffee” by “Productivity,” he obtains the “Desired Workers,” or the number of workers necessary to prepare the desired amount of coffee.

Howard then compares the “Desired Workers” to the actual number of “Workers.” The difference between these two values, divided by a time constant called “Time to Correct Workers,” gives his hourly “Correction for Workers.” The “Time to Correct Workers,” 3 hours in the base run, is the time constant that he needs to compensate for the difference between the desired and actual number of workers. It is the time in which he wants to call them on the phone and have them come to work.

Howard then adds the Correction to the number of workers who leave the Coffeehouse every hour. He has to make sure that a corresponding number of workers also come to work at the Coffeehouse each hour—this is the inflow called “Coming to Work” that increases the number of “Workers.” The outflow, “Going Home,” decreases the number of present “Workers.” “Going Home” is the number of “Workers” divided by 4 hours, the “Average Length of Working.” Finally, Howard multiplies the number of “Workers” by their “Productivity” to obtain the flow of “Making Coffee.”

Without any outside disturbance, the system starts out and remains in equilibrium at 40 cups (Coffee coverage * Buying coffee) during the 48 hours of simulation. Figure 14 shows the base run of the model, using the parameter values from section 6.3 of the Appendix.

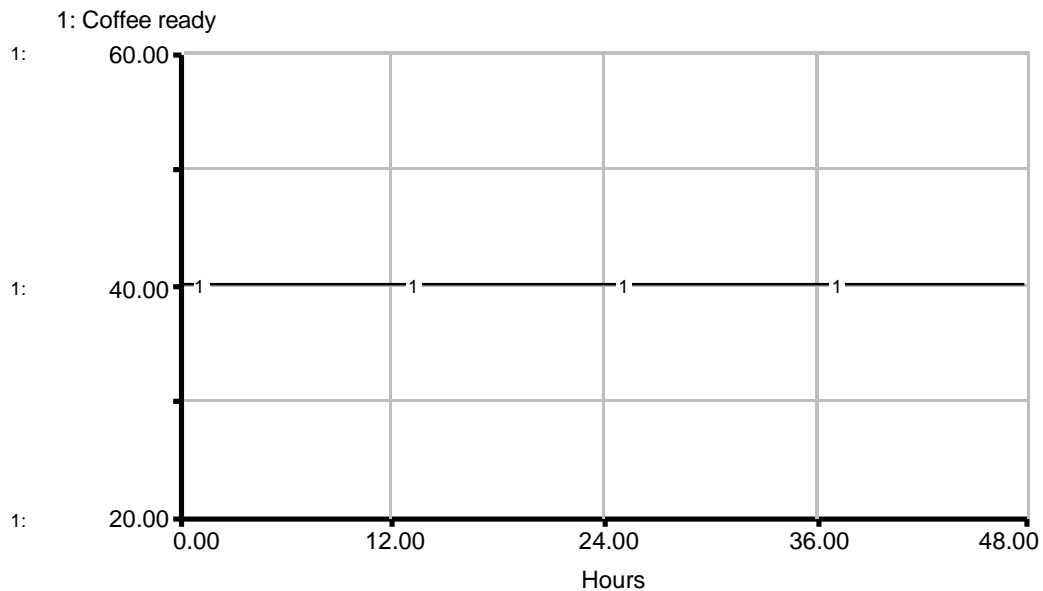


Figure 14: Base run of the Coffeehouse model

Let’s assume again that the demand for coffee suddenly changes. The system starts off in equilibrium, with 40 cups of coffee ready. At hour 3, we introduce a step increase of 5 cups in the buying of coffee. Figure 15 shows the resulting behavior of “Coffee ready” and “Buying Coffee.”

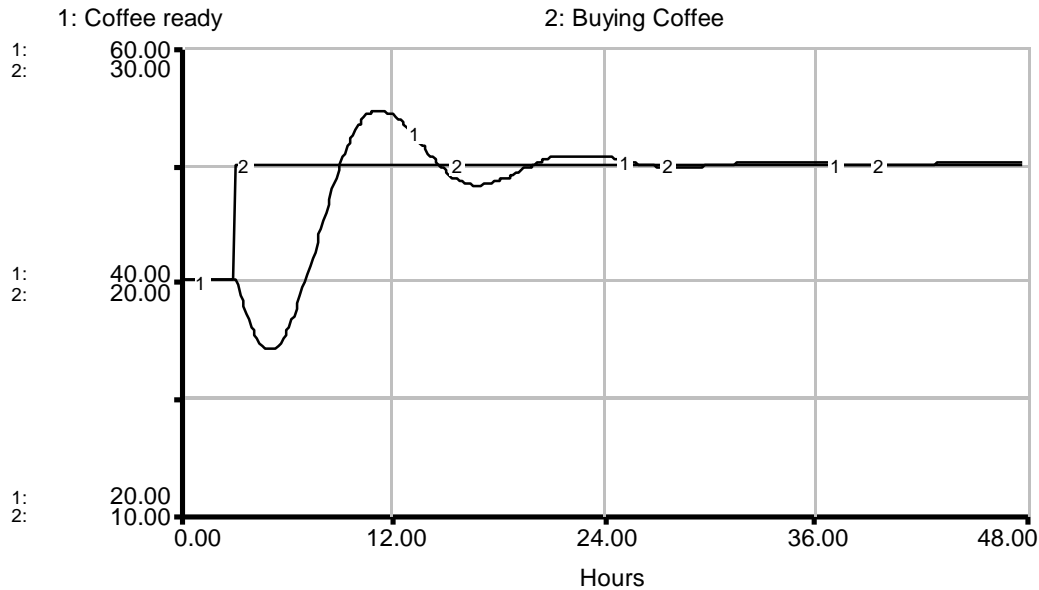


Figure 15: The system responding to a step increase in Buying Coffee

The additional structure added to the Lemonade stand model significantly changes the behavior of the system—the stock of coffee now exhibits “damped oscillations.”

After the increase in buying, “Coffee ready” decreases because “Selling” steps up together with demand. However, “Making Coffee” has not changed yet. It first takes Howard a certain time to perceive the change in buying as opposed to random noise and to find out how much coffee the workers should be making. He then determines how many more workers should be working in the Coffeehouse, and calls them to make them come to work. However, because “Desired making of coffee” is high, Howard keeps calling more workers than the equilibrium number. As they come to work, the stock of coffee starts increasing, reaches its new equilibrium value of 50 cups,¹¹ overshoots it, and continues to increase until the number of workers decreases again to its equilibrium value. The oscillations continue, but they become smaller and smaller, until both stocks eventually approach their equilibrium values.¹²

Choosing Parameters for Sensitivity Analysis

Howard is interested in the behavior of “Coffee ready.” What parameters and initial values should he use in a sensitivity analysis of the Coffeehouse model?

Performing Sensitivity Tests

Now take the parameters and initial values that you determined in section 4.1 one by one. For each of them, choose three values that seem reasonable to you and that you

¹¹ As in the first exploration, the equilibrium value can be obtained by multiplying “Coffee Coverage” (2 hours) by “Buying Coffee” (25 cups per hour).

¹² For a more detailed explanation of the causes of oscillations, see: Kevin Agatstein, 1996. *Oscillating Systems II: Sustained Oscillation* (D-4602), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology.

believe would offer insights about the behavior of the system. BEFORE running the sensitivity tests, think about the parameter as part of the structure of the model. Compare it to the parameters from the Lemonade stand and Epidemics models, and try to guess the effect of changing it on the behavior of “Coffee ready.” Why would Howard want to know the effect? Then simulate the model and compare the graphs to your predictions. If a behavior seems surprising at first, try to explain it through the structure of the model.

Conclusion

Specific parameter values can change the appearance of the graphs representing the behavior of the system. But significant changes in behavior do not occur for all parameters. System dynamics models are in general insensitive to many parameter changes. It is the structure of the system, and not the parameter values, that has most influence on the behavior of the system.

Sensitivity analysis is an important tool in the model building process. By showing that the system does not react greatly to a change in a parameter value, it reduces the modeler’s uncertainty in the behavior. In addition, it gives an opportunity for a better understanding of the dynamic behavior of the system.

We encourage you to experiment with the three models from this paper (as well as any other models that you have built) on your own. For example, try to change several parameters at the same time, observe the behavior produced, and compare it to the conclusions in this paper. Can you suggest any parameter values that would produce the “optimal,” or most desirable behavior? The use of sensitivity analysis in such policy analysis will be explored in a later paper in this series.

Suggested Solutions to Section 4

Solution to Section 4.1

The initial value of “Coffee ready,” as well as six parameters—“Time to average coffee buying,” “Time to correct amount of coffee,” “Coffee coverage,” “Time to correct workers,” “Productivity,” and “Average length of working”—can be used to explore the sensitivity of this model.¹³

Solution to Section 4.2

The initial value of “Coffee ready” represents the number of cups ready in the Coffeehouse before the beginning of the simulation. It is usually set to “Coffee coverage * Buying coffee,” but Howard may want to find out what would happen if he prepared more or less coffee at the beginning of the simulation. Three possible initial values are 30, 40, and 50 cups. Figure 17 shows the comparative run.

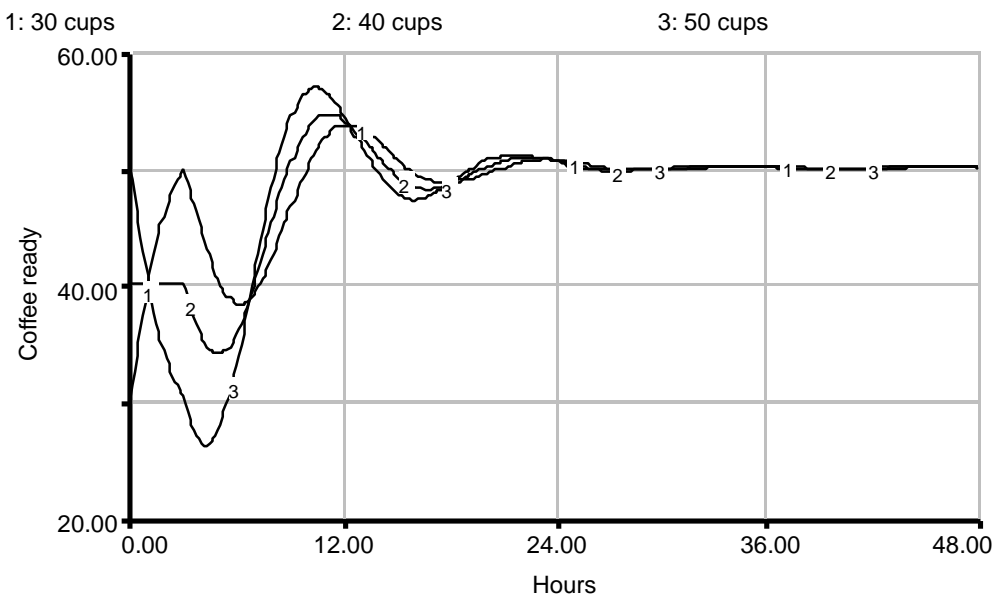


Figure 16: The effect of changes in the initial value of “Coffee ready”

As with the initial value of “Lemonade ready” in the Lemonade stand model, changing the initial value of “Coffee ready” creates differences in the behavior of the three curves, but does not affect the behavior mode of the system. Therefore, in the long run it is not very important how much coffee Howard initially makes. A change in the initial value of “Coffee ready” changes the magnitude of oscillations, but the curves approach equilibrium at nearly the same time. Notice that except for the initial value of 40 cups, the simulation does not start in equilibrium. Also, a higher initial value leads to a lower drop in “Coffee ready.” A larger initial amount of coffee means initially fewer workers

¹³ In the Coffeehouse model, there are three stocks whose behavior can be of interest: Coffee ready, Expected Coffee Buying, and Workers. In this paper, we usually show only the behavior of Coffee ready. In some cases, however, the behavior of the other stocks will also be necessary to understand the behavior of Coffee ready. We encourage you to perform similar runs with the other stocks in all the sensitivity tests.

necessary at the Coffeeshouse. As the stock decreases and undershoots the equilibrium value, it takes longer for the additional workers to be called and put to work, so the stock declines to a lower value.

The “Time to average coffee buying” is the equivalent of the “Time to average Lemonade buying” in the first exploration, so their effects on the behavior should be similar. The value of “Time to average coffee buying” is again uncertain, and Howard is not sure how the behavior of “Coffee ready” changes when it takes the workers less or more time to recognize a change in “Buying Coffee.” This time, let’s use values of 1, 2 and 3 hours, as shown in Figure 17.

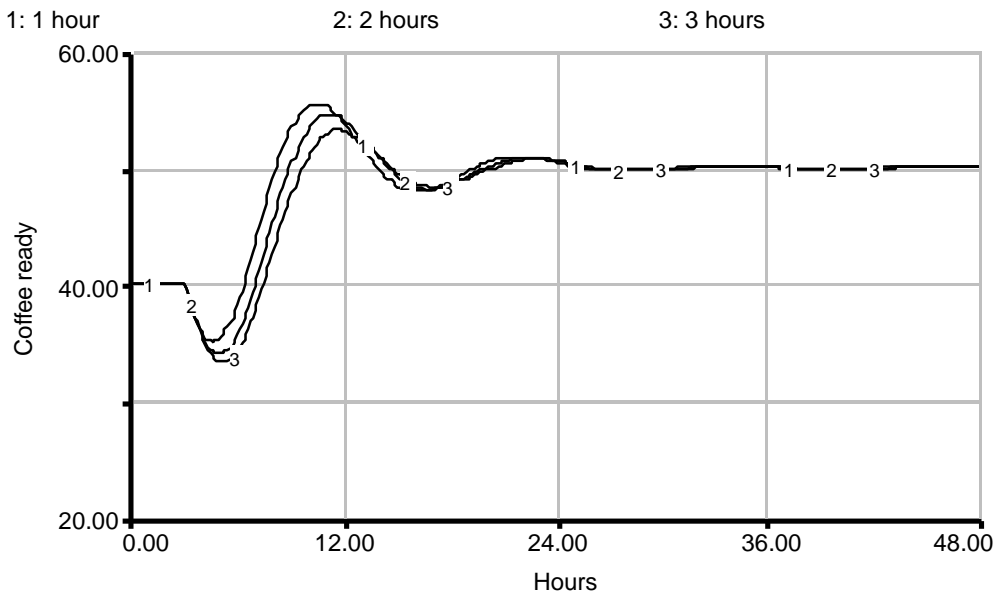


Figure 17: The effects of changes in “Time to average coffee buying”

Notice again that even though the three curves are not identical, the general type of behavior—damped oscillations—is the same in all three curves. More accuracy is not needed for this parameter because the behavior is not very sensitive to a change in it. An increase in the “Time to average coffee buying” increases the initial decline of the stock, but makes the overshoot smaller. When the “Time to average coffee buying” is long, it takes the stock longer to reach equilibrium.

As in the first exploration, the “Time to correct amount of coffee” indicates how long it takes to change the amount of coffee as a result of a sudden change in buying. The value of this parameter is also quite uncertain, so Howard wants to know how precisely he needs to know it to support the conclusions of his model. Three possible values, 0.5 hour, 1 hour, and 1.5 hours are shown in Figure 18.

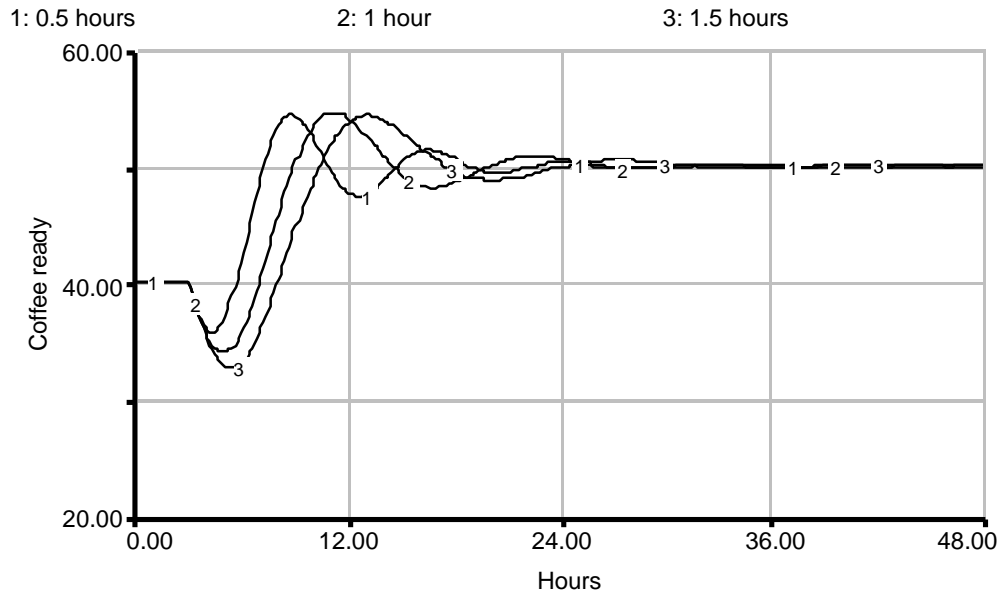


Figure 18: The effect of changes in the “Time to correct amount of coffee”

Again, although the three curves are not exactly the same, they all show the same type of behavior. Howard can now be more confident that it is not a specific value of “Time to correct amount of coffee” that creates the system behavior. Increasing the “Time to correct amount of coffee” increases the initial decline of the stock because the workers are not able to respond quickly enough and start preparing more coffee. A longer “Time to correct amount of coffee” also increases the time it takes the stock to reach equilibrium.

The parameter “Coffee Coverage” determines how many hours worth of coffee Howard would like the workers to keep ready in the Coffeehouse. It has the same role as the “Lemonade coverage” in the first exploration. As with lemonade, Howard wants to make sure that he always has a sufficient amount of “Coffee ready,” while it is still fresh. He would like to know what would be the optimal value to set for “Coffee Coverage.” Three possible values that he could test for are 1, 2, and 3 hours, as in Figure 19.

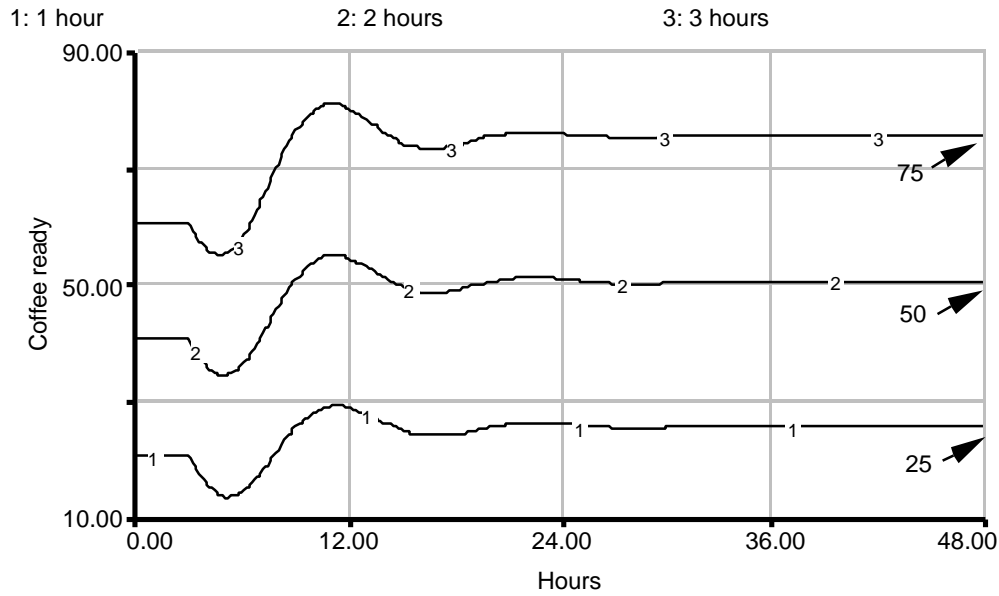


Figure 19: The effect of changes in “Coffee coverage”

As with “Lemonade coverage,” changes in “Coffee coverage” only affect the initial and equilibrium values of the stock, but do not affect the time it takes to reach equilibrium.¹⁴

The “Time to correct workers” indicates how long it takes Howard to make more workers come to work. Because the workers in the Coffeehouse are students, they are usually not available immediately, and it can take Howard up to a few hours to make the necessary number of workers come to work. But he is not completely certain about the exact time required to make them come over, so he wants to try out different values to see how the behavior of the system changes. He knows that it usually takes him somewhere between 2 and 4 hours to call them up and make sure they come. However, he also wants to know whether the behavior would change in any way if the workers could come to work when he needed them—that is, if “Time to correct workers” was close to zero. He therefore tries one very low value, such as 0.25 hours, along with two more reasonable values of 2 and 4 hours, as in Figure 20.¹⁵

¹⁴ To start the simulation in equilibrium, again set the initial value of “Coffee ready” to “Coffee coverage * Buying coffee.”

¹⁵ If you try a low value, make sure that it is at least twice the value of the solution interval (DT) you are using, to avoid a “DT error.”

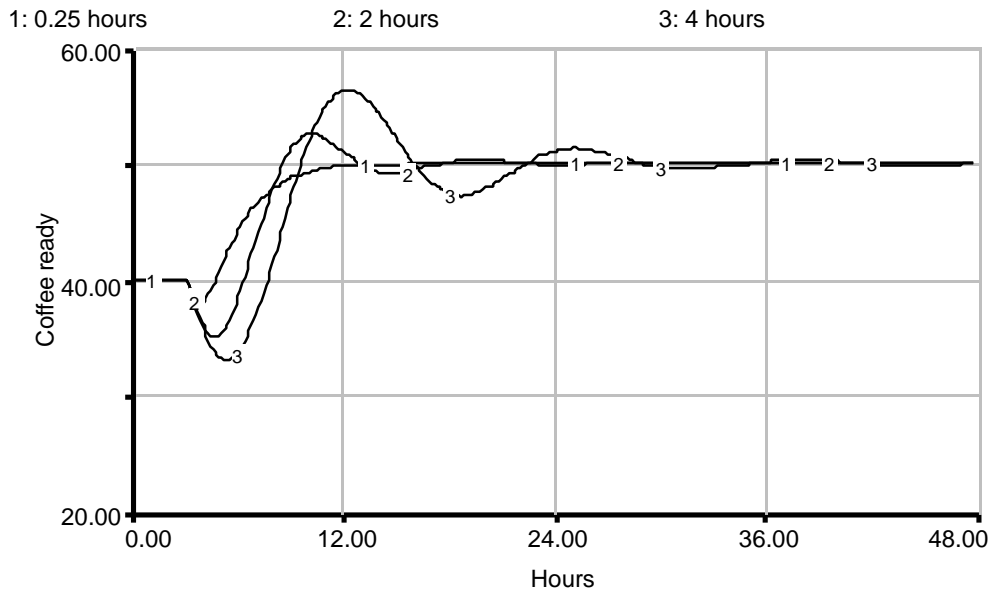


Figure 20: The effect of changes in “Time to correct workers”

A comparison of the curves again indicates that, except for very low values of “Time to correct workers,” the general type of behavior remains the same. Notice how curve 1 resembles the behavior observed in all runs of the Lemonade stand model from section 3.1. In the Lemonade stand, there was no “Time to correct workers” because Howard was the only worker and was always immediately available. If the Coffeehouse workers were able to come to work immediately, the structure and thus the behavior of the Coffeehouse model would be the same as that of the Lemonade stand model. It is therefore the additional structure in the “Workers” sector that creates a time lag between the decision to make coffee, the time when the workers are available, and the time when the coffee is made; thus, it causes the Coffeehouse model to exhibit oscillation. However, a situation in which the workers would be available immediately is unlikely to happen, and the higher values of “Time to correct workers” are probably more reasonable. Howard thus does not need to know exactly how long it takes him to find workers who are available to come to work. The higher the value of “Time to correct workers,” the greater the initial decrease of the stock, and the higher the magnitude of oscillations. Also, increasing the value of Time to correct workers increases the time it takes the stock to approach the same equilibrium value of 50 cups of coffee.

“Productivity” measures the amount of coffee that one worker can prepare in one hour. Because the workers also have other responsibilities in the coffeehouse, they cannot spend the entire hour preparing coffee. Howard does not know exactly how productive his workers are, so he tries three possible values of 10, 20, and 30 cups per worker per hour, as in Figure 21.

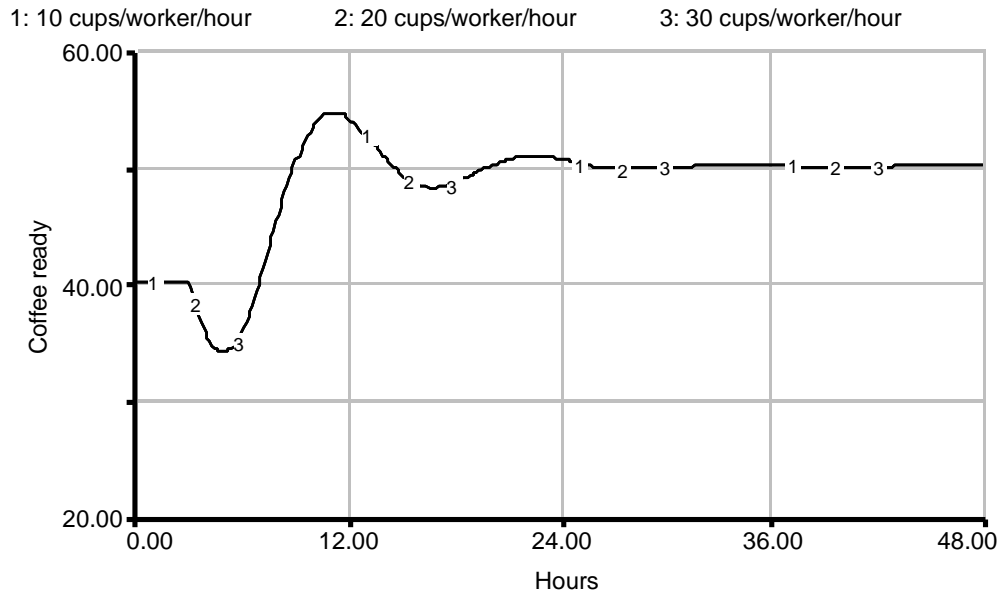


Figure 21: The effect of changing “Productivity”

Changing the value of “Productivity” does not have any effect on the amount of “Coffee ready.” To explain this result, let’s look at the behavior of the stock of “Workers” for the same three parameter values of “Productivity,” shown in Figure 22.

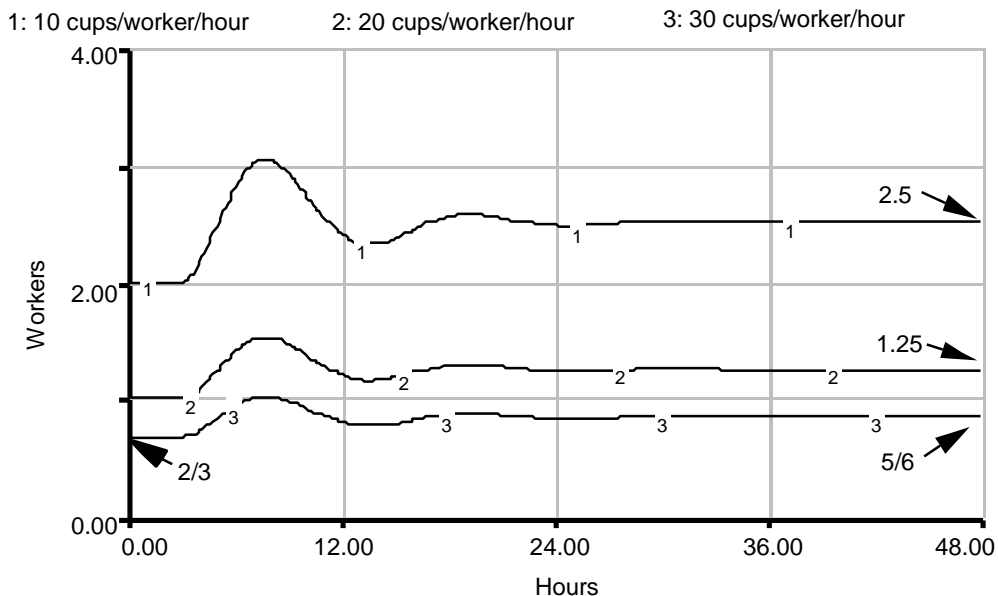


Figure 22: The effect of changing “Productivity” on the number of “Workers”

A change in “Productivity” creates a simultaneous change in the initial number of “Workers.” Let us first compare the behavior of the three curves before the increase in buying. In curve 1, where “Productivity” is low, more workers have to come to the Coffeehouse to maintain the desired amount of coffee. Their number can be obtained by dividing the value of “Making Coffee” by the value of “Productivity.” For equilibrium, the number of cups made every hour must equal the number of cups sold every hour, i.e., initially 20 cups per hour. Dividing 20 cups per hour by 10 cups per worker per hour

gives 2 “Workers,” which is the initial value in curve 1. Similarly, in curve 2, the initial number of “Workers” is 1. Finally, in curve 3, the initial number of “Workers” is $2/3$.¹⁶ After the step increase in “Buying Coffee,” the three curves show similar behavior. The oscillations occur at the same time, but their magnitude decreases as “Productivity” increases because with a higher “Productivity,” the workers can react better to sudden changes in buying. The curves approach their equilibrium values at the same time—curve 1 approaches the value of 2.5 workers, curve 2 approaches 1.25 workers, and for curve 3 the equilibrium value is $5/6$ of a worker.¹⁷ Therefore, it is the number of “Workers” that compensates for the changes in “Productivity” so that the amount of “Coffee ready” remains unchanged.¹⁸

The “Average length of working” indicates the average number of hours a worker spends in work. Howard can easily find out this value, but he would like to know whether there would be any differences if his workers stayed in the Coffeehouse for a longer time. Possible values for this parameter that would be convenient for him and for the workers are, for example, 2, 4, and 6 hours, represented in Figure 23.

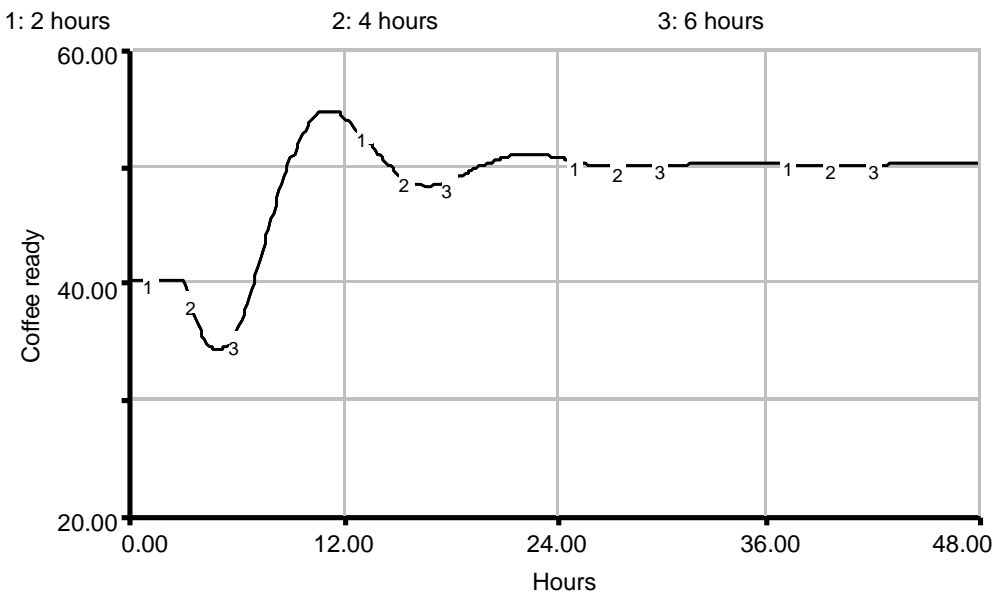


Figure 23: The effect of changes in “Average length of working”

A change in the value of “Average length of working” does not influence the amount of “Coffee ready.” Let us again look at the behavior of “Workers” for an explanation. It is shown in Figure 24.

¹⁶ Of course, in the real world, there could not be $2/3$ of a worker working in the Coffeehouse. But a model is just a representation of the real world. In this case, when our primary purpose is to test the model for sensitivity, we do not have to be concerned about this problem. The manager would have to round up the number of workers to the next whole number.

¹⁷ Again, the manager should just round up the equilibrium values to the next whole number.

¹⁸ However, if we were to model the profits of the Coffeehouse, “Productivity” would certainly make a large effect.

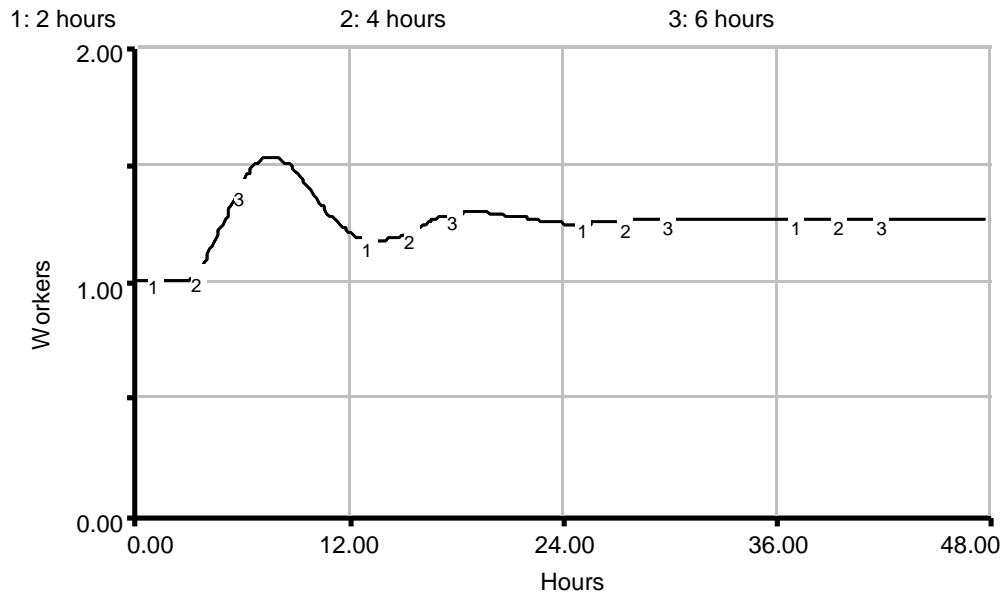


Figure 24: The effect of changes in “Average length of employment” on the number of “Workers”

Unlike “Productivity,” changes in the “Average length of working” do not influence the number of “Workers” presently working in the Coffeehouse.¹⁹ The reason is that “Average length of working” affects both flows into the stock of “Workers.” Increasing the value of Average length of employment will simultaneously increase the value of “Coming to Work” and “Going Home,” so the net flow into “Workers” will not change. In addition, when the number of “Workers” does not change, the amount of “Coffee ready” will not change either (assuming that all the other variables in the model remain unchanged).

Debrief

The exercise confirmed the conclusions established in the two explorations: changing parameter values makes a difference to the behavior of the model, but the overall behavior mode stays the same. A change in behavior corresponds to a change in structure. Also, some parameters cause a more significant or different change in the behavior than others. For example, compare Figures 19, 20, and 21. Changes in three different parameters (“Coffee coverage,” “Time to correct workers,” and “Productivity”) can produce different changes in the behavior of the stock of “Coffee ready,” or no change at all, as in the case of “Productivity.”

¹⁹ However, for large values of Average length of employment, starting at approximately 14 hours, the behavior starts to change—the oscillations become larger as Average length of employment increases. You should still keep in mind that the values in the sensitivity tests should always stay in a range that is possible in the real world. Fourteen hours does not seem to be a reasonable value for the number of hours that a worker spends in work.

Appendix: Model Documentation

Before simulating the following three models, please make sure that stocks can become negative and that flows that can change direction are set to be bi-directional. This can help to identify errors in the formulation of the model. For directions, please consult your STELLA user manual.

Lemonade Stand Model

Expected_lemonade_buying(t) = Expected_lemonade_buying(t - dt) + (Change_in_buying_expectations) * dt

INIT Expected_lemonade_buying = Buying_lemonade

DOCUMENT: The hourly demand for lemonade that Howard expects.

Units: cups/hour

INFLOWS:

Change_in_buying_expectations = (Buying_lemonade - Expected_lemonade_buying) / Time_to_average_lemonade_buying

DOCUMENT: The rate at which Howard's expectations about demand for lemonade change.

Units: (cups/hour)/hour

Lemonade_ready_in_stand(t) = Lemonade_ready_in_stand(t - dt) + (Making_lemonade - Selling) * dt

INIT Lemonade_ready_in_stand = Buying_lemonade * Lemonade_coverage

DOCUMENT: The number of cups of lemonade that Howard has ready in the stand.

Units: cups

INFLOWS:

Making_lemonade = MAX(Expected_lemonade_buying + Correction_in_amount_of_lemonade, 0)

DOCUMENT: The number of cups of lemonade that Howard prepares each hour.

Units: cups/hour

OUTFLOWS:

Selling = MIN(Buying_lemonade, Lemonade_ready_in_stand/DT)

DOCUMENT: The number of cups of lemonade that Howard sells each hour. Usually, it is equal to the demand for lemonade, Buying_lemonade. However, if the demand is higher than the lemonade he has ready, he will sell all the lemonade quickly, which is equal to Lemonade_ready_in_stand/DT.

Units: cups/hour

Buying_lemonade = 20 + STEP(5,1)

DOCUMENT: The hourly demand for lemonade.

Units: cups/hour

Correction_in_amount_of_lemonade = (Desired_amount_of_lemonade - Lemonade_

$$\text{ready_in_stand}) / \text{Time_to_correct_amount_of_lemonade}$$

DOCUMENT: The number of cups of lemonade that Howard prepares every hour as a result of a difference between the desired and the actual amount of lemonade.

Units: cups/hour

$$\text{Desired_amount_of_lemonade} = \text{Lemonade_coverage} * \text{Expected_lemonade_buying}$$

DOCUMENT: The number of cups of lemonade that Howard would like to have at the stand. It is equal to the demand for lemonade he expects times the Lemonade coverage.

Units: cups

$$\text{Lemonade_coverage} = 2$$

DOCUMENT: Lemonade coverage determines the number of hours worth of lemonade that Howard wants to keep ready at the stand at all times.

Units: hours

$$\text{Time_to_average_lemonade_buying} = 1$$

DOCUMENT: The time it takes Howard to recognize a permanent change in demand for lemonade from random fluctuations.

Units: hours

$$\text{Time_to_correct_amount_of_lemonade} = 1.5$$

DOCUMENT: The time in which Howard attempts to correct a difference between the desired and the actual amount of lemonade.

Units: hours

Epidemics Model

$$\text{Healthy_People}(t) = \text{Healthy_People}(t - dt) + (\text{Recovering_Rate} - \text{Catching_Illness}) * dt$$

$$\text{INIT Healthy_People} = 100 - \text{Sick_People}$$

DOCUMENT: The number of healthy people in the population.

Units: people

INFLOWS:

$$\text{Recovering_Rate} = \text{Sick_People} / \text{Duration_of_Illness}$$

DOCUMENT: The rate at which people recover from the illness.

Units: people/month

OUTFLOWS:

$$\text{Catching_Illness} = \text{Healthy_People} * \text{Probability_of_Contact_with_Sick_People} * \text{Population_Interactions} * \text{Probability_of_Catching_Illness}$$

DOCUMENT: The rate at which people catch the illness.

Units: people/month

$$\text{Sick_People}(t) = \text{Sick_People}(t - dt) + (\text{Catching_Illness} - \text{Recovering_Rate}) * dt$$

$$\text{INIT Sick_People} = 1$$

DOCUMENT: The number of sick people in the population.

Units: people

INFLOWS:

$$\text{Catching_Illness} = \text{Healthy_People} * \text{Probability_of_Contact_with_Sick_People} * \\ \text{Population_Interactions} * \text{Probability_of_Catching_Illness}$$

DOCUMENT: The rate at which people catch the illness.

Units: people/month

OUTFLOWS:

$$\text{Recovering_Rate} = \text{Sick_People} / \text{Duration_of_Illness}$$

DOCUMENT: The rate at which people recover from the illness.

Units: people/month

$$\text{Duration_of_Illness} = .5$$

DOCUMENT: The duration of the illness.

Units: months

$$\text{Population_Interactions} = 10$$

DOCUMENT: The number of interactions, or contacts, that a person has with other people during a month.

Units: (people/people)/month

$$\text{Probability_of_Catching_Illness} = .5$$

DOCUMENT: The probability that a person will become infected when in contact with a sick person.

Units: dimensionless

$$\text{Probability_of_Contact_with_Sick_People} = \text{Sick_People} / (\text{Sick_People} + \text{Healthy_People})$$

DOCUMENT: The probability that a person will meet a sick person. It is equal to the ratio of the number of sick people to the total population.

Units: dimensionless

Coffeehouse Model

$$\text{Coffee_ready}(t) = \text{Coffee_ready}(t - dt) + (\text{Making_Coffee} - \text{Selling}) * dt$$

$$\text{INIT Coffee_ready} = \text{Coffee_Coverage} * \text{Buying_Coffee}$$

DOCUMENT: The number of cups of coffee ready in the Coffeehouse.

Units: cups

INFLOWS:

$$\text{Making_Coffee} = \text{Workers} * \text{Productivity}$$

DOCUMENT: The number of cups of coffee that the Coffeehouse workers prepare each hour. It is equal to the number of workers times their average productivity.

Units: cups/hour

OUTFLOWS:

Selling = MIN(Buying_Coffee, Coffee_ready/DT)

DOCUMENT: The number of cups of coffee sold in the Coffeehouse every hour.

Usually, it is equal to the demand for coffee, Buying_Coffee. However, if the demand is higher than the coffee that is ready, the workers will sell all the coffee quickly, which is equal to Coffee_ready/DT.

Units: cups/hour

$$\text{Expected_Coffee_Buying}(t) = \text{Expected_Coffee_Buying}(t - dt) + (\text{Change_in_Buying_Expectations}) * dt$$

INIT Expected_Coffee_Buying = Buying_Coffee

DOCUMENT: The hourly demand for coffee that the workers expect.

Units: cups/hour

INFLOWS:

$$\text{Change_in_Buying_Expectations} = (\text{Buying_Coffee} - \text{Expected_Coffee_Buying}) / \text{Time_to_average_coffee_buying}$$

DOCUMENT: The rate at which the workers' expectations about demand for coffee change.

Units: (cups/hour)/hour

$$\text{Workers}(t) = \text{Workers}(t - dt) + (\text{Coming_to_Work} - \text{Going_Home}) * dt$$

INIT Workers = Desired_Workers

DOCUMENT: The number of workers currently working at the Coffeehouse.

Units: workers

INFLOWS:

$$\text{Coming_to_Work} = \text{Correction_for_Workers} + (\text{Workers} / \text{Average_Length_of_Working})$$

DOCUMENT: The rate at which workers come to work.

Units: workers/hour

OUTFLOWS:

$$\text{Going_Home} = \text{Workers} / \text{Average_Length_of_Working}$$

DOCUMENT: The rate at which the workers leave the Coffeehouse to go home and study.

Units: workers/hour

$$\text{Average_Length_of_Working} = 4$$

DOCUMENT: The number of hours that a worker spends in the Coffeehouse.

Units: hours

$$\text{Buying_Coffee} = 20 + \text{STEP}(5,3)$$

DOCUMENT: The hourly demand for coffee.

Units: cups/hour

$$\text{Coffee_Coverage} = 2$$

DOCUMENT: Coffee coverage determines the number of hours worth of coffee that Howard wants the workers to keep at the Coffeehouse at all times.

Units: hours

$$\text{Correction_for_Workers} = (\text{Desired_Workers} - \text{Workers}) / \text{Time_to_Correct_Workers}$$

DOCUMENT: The number of workers who come to the Coffeehouse as a result of a difference between the desired and the actual number of workers.

Units: workers/hour

$$\text{Correction_in_Amount_of_Coffee} = (\text{Desired_Amount_of_Coffee} - \text{Coffee_ready}) / \text{Time_to_Correct_Amount_of_Coffee}$$

DOCUMENT: The number of cups of coffee that the workers prepare every hour as a result of a difference between the desired and actual amount of coffee.

Units: cups/hour

$$\text{Desired_Amount_of_Coffee} = \text{Coffee_Coverage} * \text{Expected_Coffee_Buying}$$

DOCUMENT: The number of cups of coffee that the workers would like to have at the Coffeehouse. It is equal to the demand for coffee that they expect times the Coffee coverage.

Units: cups

$$\text{Desired_making_of_coffee} = \text{Correction_in_Amount_of_Coffee} + \text{Expected_Coffee_Buying}$$

DOCUMENT: The rate at which the workers would like to make coffee.

Units: cups/hour

$$\text{Desired_Workers} = \text{Desired_making_of_coffee} / \text{Productivity}$$

DOCUMENT: The number of workers that Howard wants to be working at the Coffeehouse.

Units: workers

$$\text{Productivity} = 20$$

DOCUMENT: The number of cups of coffee that a worker makes in an hour.

Units: cups/(worker/hour)

$$\text{Time_to_average_coffee_buying} = 2$$

DOCUMENT: The time it takes the workers to recognize a permanent change in demand for coffee from random fluctuations.

Units: hours

$$\text{Time_to_Correct_Amount_of_Coffee} = 1$$

DOCUMENT: The time in which the workers attempt to correct a difference between the desired and the actual amount of coffee.

Units: hours

Time_to_Correct_Workers = 3

DOCUMENT: The time in which Howard wants to make more workers come to work.

Units: hours

Vensim Examples: An Introduction to Sensitivity Analysis

By Aaron Diamond and Nathaniel Choge
October 2001

3.1: Lemonade Stand

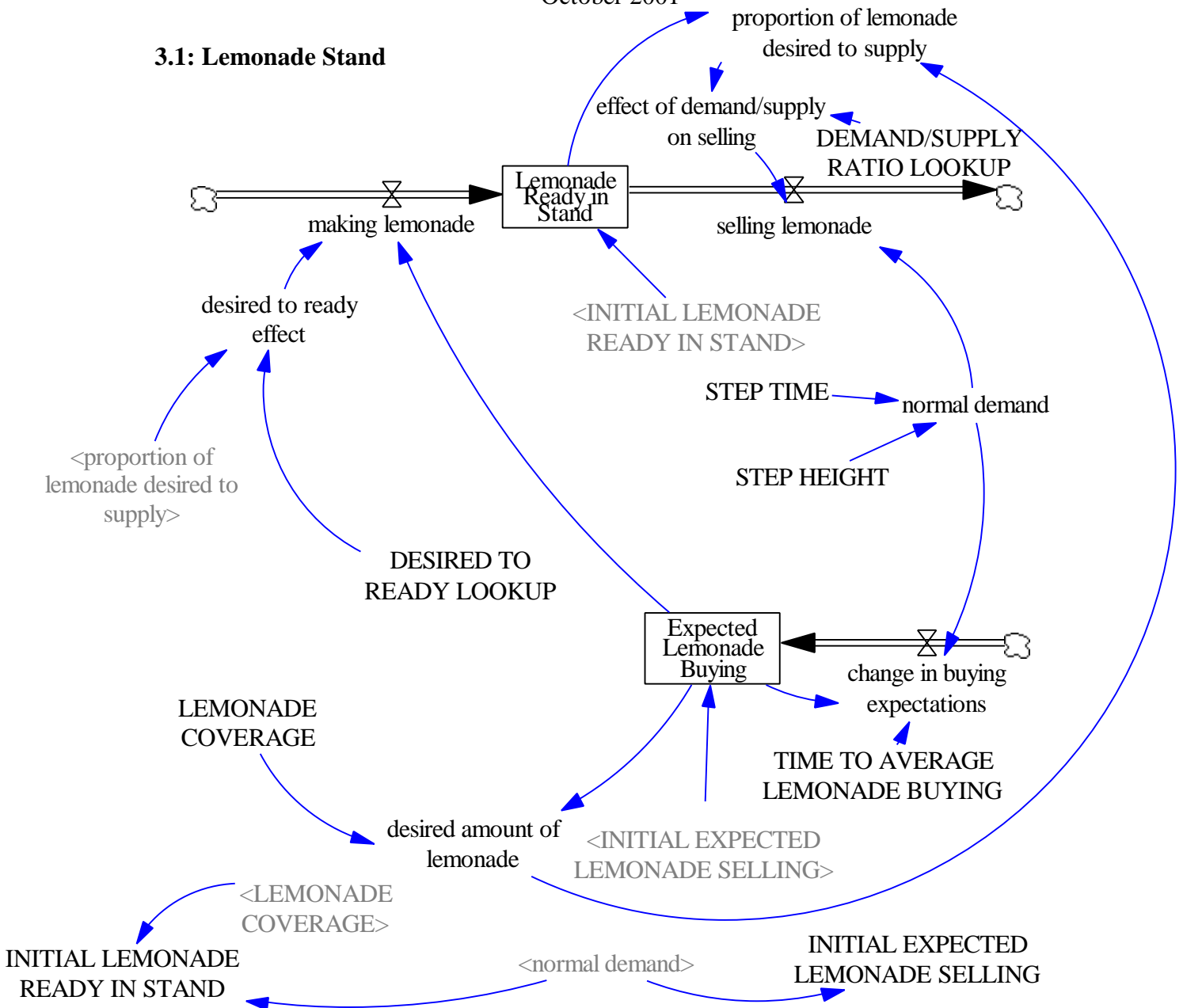


Figure 25: Vensim Equivalent of Figure 1: Model of the Lemonade stand
NOTE: Differences between the Vensim and Stella Lemonade Stand Model are described below Documentation.

Documentation for Lemonade Stand Model:

- (01) $\text{change in buying expectations} = (\text{normal demand} - \text{Expected Lemonade Buying}) / \text{TIME TO AVERAGE LEMONADE BUYING}$
 Units: (cups/hour)/hour
 The rate at which Howard's expectations about demand for lemonade change.
- (02) "DEMAND/SUPPLY RATIO LOOKUP"
 $[(0,0)-(10,10)],(0.33,3),(0.5,2),(1,1),(2,0.5),(3,0.33),(5,0.2)$
 Units: dmnl
- (03) $\text{desired amount of lemonade} = \text{LEMONADE COVERAGE} * \text{Expected Lemonade Buying}$
 Units: cups
 The number of cups of lemonade that Howard would like to have at the stand. It is equal to the demand for lemonade he expects times the LEMONADE COVERAGE.
- (04) $\text{desired to ready effect} = \text{DESIRED TO READY LOOKUP}(\text{proportion of lemonade desired to supply})$
 Units: dmnl
- (05) $\text{DESIRED TO READY LOOKUP}[(0,0)-(4,3)],(0,0.05),(1,1),(2,2),(2.5,2.4),(3,2.8),(3.5,3),(4,3)$
 Units: dmnl
- (06) "effect of demand/supply on selling"=
 "DEMAND/SUPPLY RATIO LOOKUP"(proportion of lemonade desired to supply)
 Units: dmnl
- (07) $\text{Expected Lemonade Buying} = \text{INTEG}(\text{change in buying expectations}, \text{INITIAL EXPECTED LEMONADE SELLING})$
 Units: cups/hour
 The hourly demand for lemonade that Howard expects.
- (08) $\text{FINAL TIME} = 8$
 Units: hour
 The final time for the simulation.
- (09) $\text{INITIAL EXPECTED LEMONADE SELLING} = \text{normal demand}$
 Units: cups/hour
- (10) $\text{INITIAL LEMONADE READY IN STAND} = \text{LEMONADE COVERAGE} * \text{normal demand}$

Units: cups

- (11) INITIAL TIME = 0
Units: hour
The initial time for the simulation.
- (12) LEMONADE COVERAGE=2
Units: hour
LEMONADE COVERAGE determines the number of hours worth of lemonade that Howard wants to keep ready at the stand at all times.
- (13) Lemonade Ready in Stand= INTEG (+making lemonade-selling lemonade, INITIAL LEMONADE READY IN STAND)
Units: cups
The number of cups of lemonade that Howard has ready in the stand.
- (14) making lemonade=desired to ready effect*Expected Lemonade Buying
Units: cups/hour
The number of cups of lemonade that Howard prepares each hour.
- (15) normal demand=20+STEP(STEP HEIGHT,STEP TIME)
Units: cups/hour
The hourly demand for lemonade.
- (16) proportion of lemonade desired to supply=desired amount of lemonade/Lemonade Ready in Stand
Units: dmnl
Calculates the proportion of demand for lemonade (units cups/hour) to supply of lemonade (divided by TIME TO SELL CUP OF LEMONADE to achieve units cups/hour).
- (17) SAVEPER = TIME STEP
Units: hour
The frequency with which output is stored.
- (18) selling lemonade="effect of demand/supply on selling"*normal demand
Units: cups/hour
- (19) STEP HEIGHT=5
Units: dmnl
- (20) STEP TIME=1
Units: dmnl

- (21) TIME STEP = 0.0625
Units: hour
The time step for the simulation.
- (22) TIME TO AVERAGE LEMONADE BUYING=1
Units: hour
The time it takes Howard to recognize a permanent change in demand for lemonade from random fluctuations.

Differences Between the Vensim and Stella Lemonade Stand Model:

1. Below the large model is a smaller model that shows the calculation of the INITIAL stock variables. The model is split to avoid confusion and emphasize the dynamic parts of the system.
2. The variable “buying lemonade” in the Stella version has been changed to “normal demand” in the Vensim paper to more explicitly show that this variable captures the demand for lemonade. The “STEP HEIGHT” and “STEP TIME” are constants that are separately entered into the “normal demand” equation and are the inputs to the STEP function in the “buying lemonade” equation.
3. To avoid poor modeling practices, the Vensim model does not use MIN/MAX functions.
 - a. **making lemonade**-Instead of inputting the equation “MAX(Expected Lemonade Buying + correction in amount of lemonade, 0),” we have constructed an effect variable, called “desired to ready effect” that takes the proportion of the “lemonade desired” to the “Lemonade Ready in Stand” (called “proportion of lemonade desired to supply”) and outputs the result of the “DESIRED TO READY LOOKUP”. The flow “making lemonade” is computed by multiplying the effect by “Expected Lemonade Buying”. The lookup is constructed so that when the output of “proportion of lemonade desired to supply” is multiplied by “Expected Lemonade Buying,” the result is equivalent to adding “Expected Lemonade Buying” and “correction in amount of lemonade”. The lookup curve is linear until about 2.5 times the current level of lemonade is desired. We assume that the stand is not able to more than triple their production rate at any time, so the curve approaches 3, and for lemonade demands larger than about 3.5 times the current level of lemonade, the stand will only be able to make lemonade at 3 times current expectations, and the lookup is function remains flat at 3. The lookup is shown below:



Figure 26: "CORRECTION/EXPECTED BUYING LOOKUP"

- b. **selling lemonade-** Instead of inputting the equation "MIN(NORMAL DEMAND, Lemonade Ready in Stand/DT), we compute the ratio of the "desired amount of lemonade" to the "Lemonade Ready in Stand" to obtain a value of demand/supply (called "proportion of lemonade desired to supply"). We know that if the demand is greater than the supply of lemonade (i.e., if the proportion is greater than 1) then the sales of lemonade will slow, since much of the stand's efforts will be toward making lemonade. If the proportion is smaller than 1, however, then the stand's efforts will be toward selling lemonade, and their "selling" rate will increase to a maximum value. The "DEMAND/SUPPLY RATIO LOOKUP" captures this effect, and is shown below:

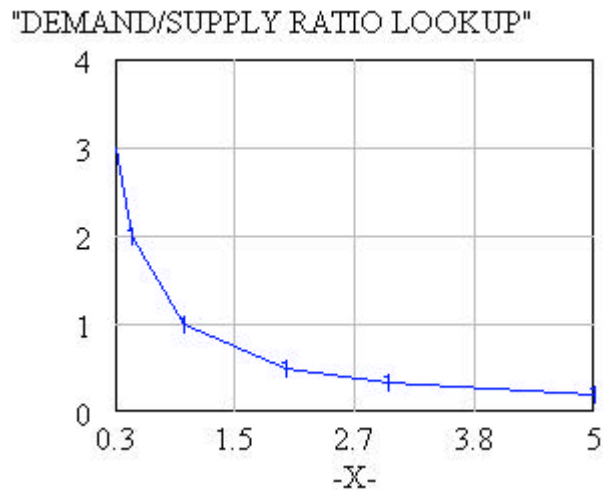
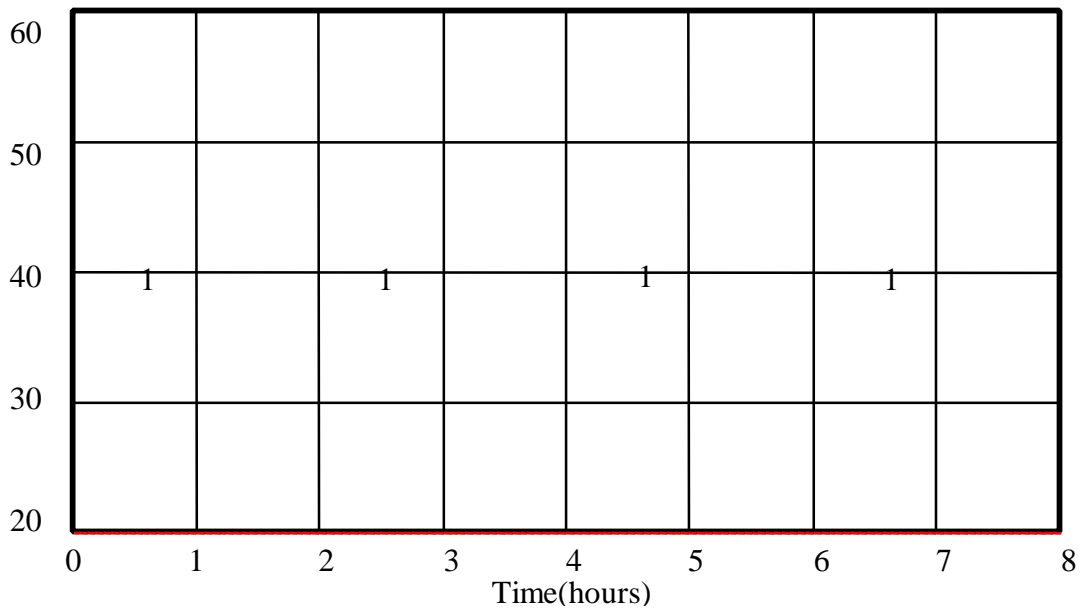


Figure 27: DEMAND/SUPPLY RATIO LOOKUP

The appropriate output of the lookup is computed by the "effect of demand/supply on selling" variable, which is multiplied by the "NORMAL DEMAND" to compute "selling."

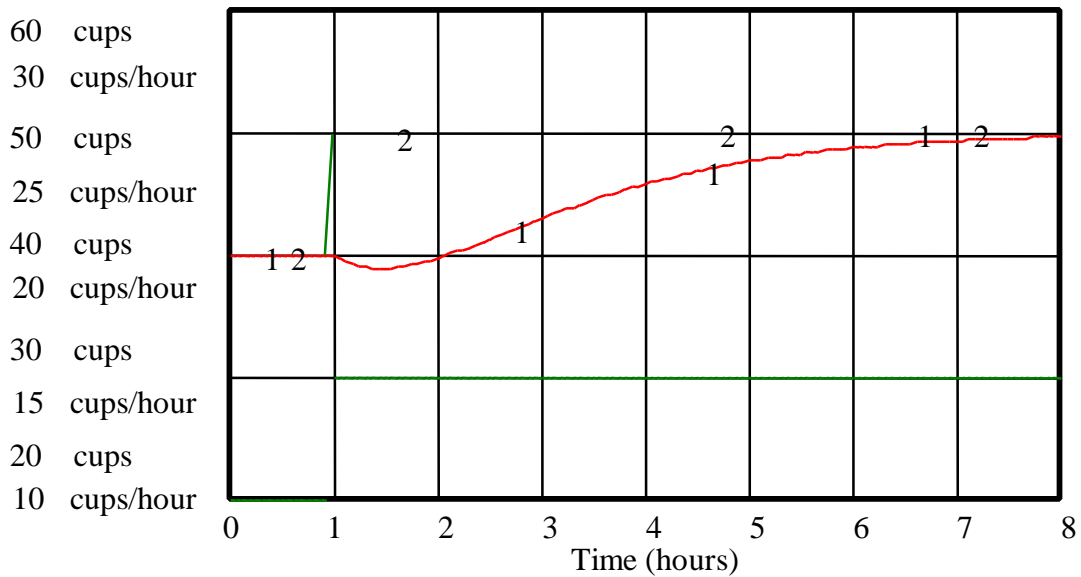
Graph of Base Run of Lemonade Stand Model



Lemonade Ready in Stand : — 1 — 1 — 1 — cups

Figure 28: Vensim Equivalent of Figure 2: Base run of the Lemonade Stand model

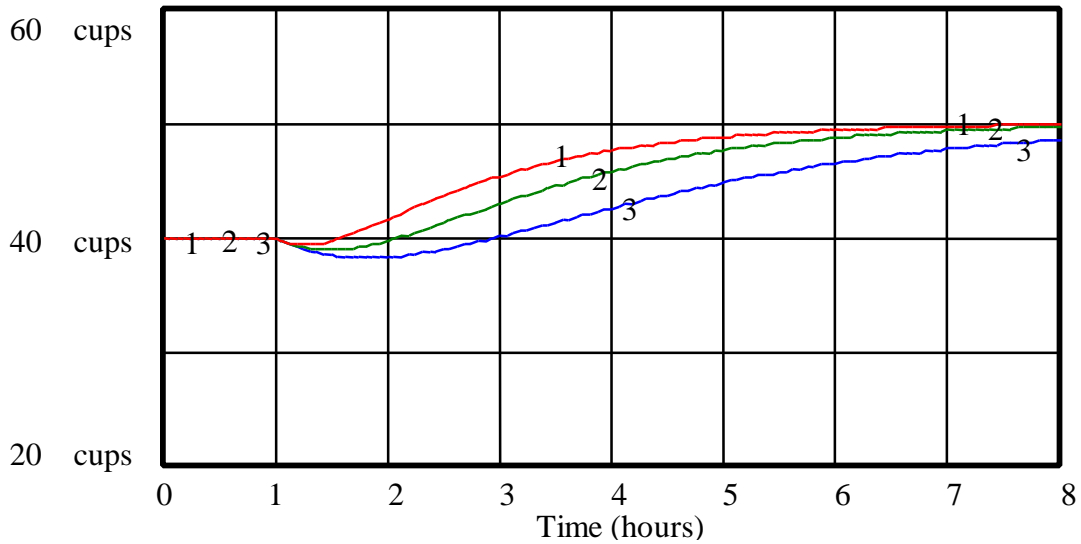
Graph of Response to a STEP Increase in BUYING LEMONADE



Lemonade Ready in Stand : — 1 — 1 — 1 — cups
 buying lemonade : — 2 — 2 — 2 — cups/hour

Figure 29: Vensim Equivalent of Figure 3: The system responding to a step increase in “buying lemonade.”

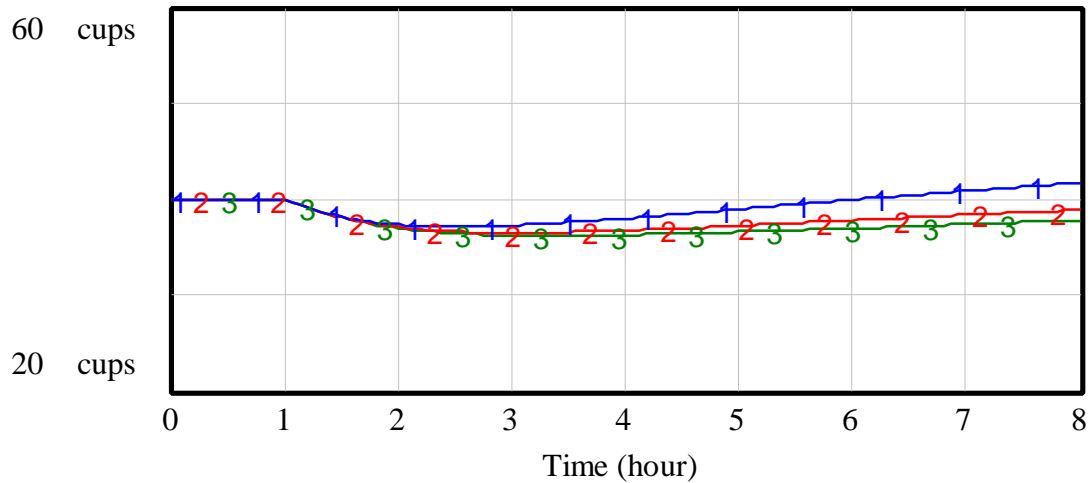
Graph of Response to Changes in TIME TO AVERAGE LEMONADE BUYING



Lemonade Ready in Stand : .5 hours — 1 — cups
 Lemonade Ready in Stand : 1 hour — 2 — cups
 Lemonade Ready in Stand : 1.5 hours — 3 — cups

Figure 30: Vensim Equivalent of Figure 4: The effect of changes in “TIME TO AVERAGE LEMONADE BUYING”

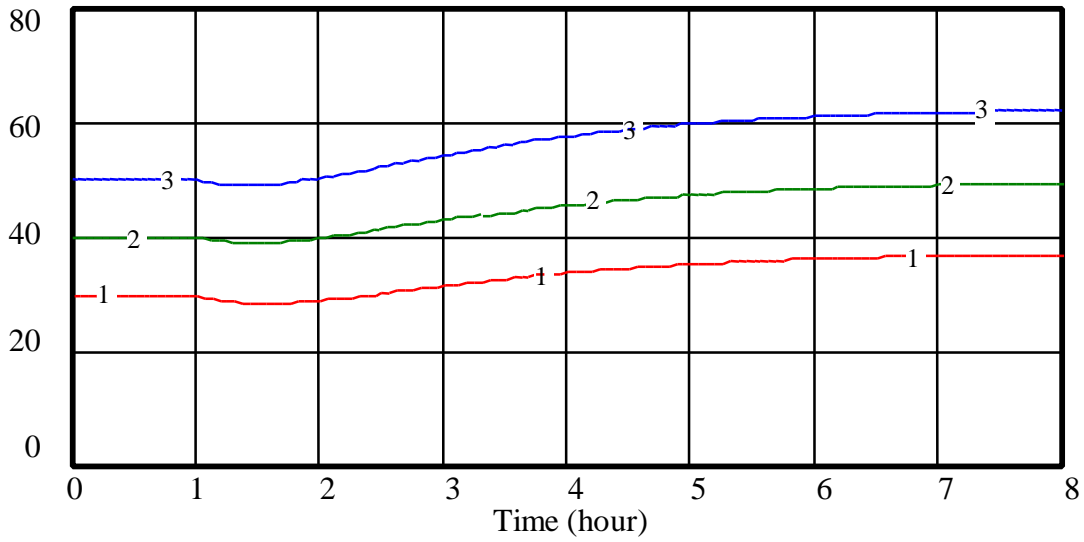
Graph of Response to Extreme Changes in TIME TO AVERAGE LEMONADE BUYING



Lemonade Ready in Stand : 10 hr — 1 — cups
 Lemonade Ready in Stand : 20 hr — 2 — cups
 Lemonade Ready in Stand : 30 hr — 3 — cups

Figure 31: Extra graph. Effect of extreme changes to “TIME TO AVERAGE LEMONADE BUYING”

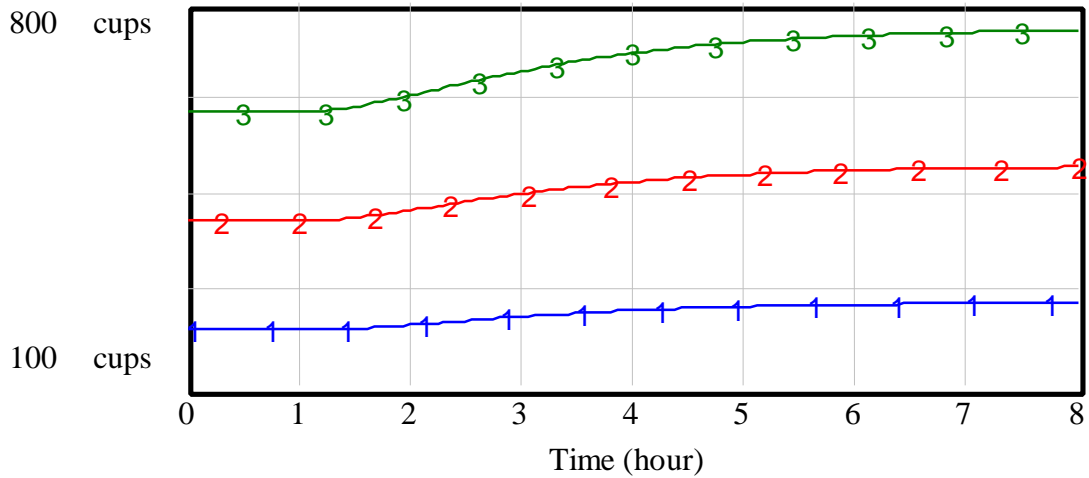
Graph of Response to Changes in LEMONADE COVERAGE



Lemonade Ready in Stand : 1.5 hour — 1 — cups
 Lemonade Ready in Stand : 2 hour — 2 — cups
 Lemonade Ready in Stand : 2.5 hour — 3 — cups

Figure 32: Vensim Equivalent of Figure 6: The effect of changes in ‘LEMONADE COVERAGE’

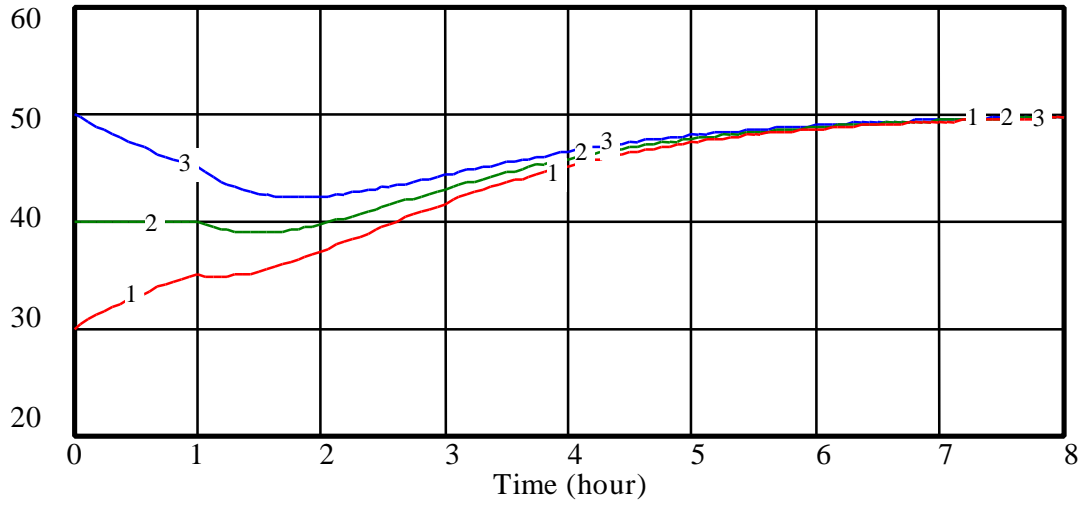
Graph of Response to Extreme Changes in LEMONADE COVERAGE



Lemonade Ready in Stand : 10 hr — 1 — cups
 Lemonade Ready in Stand : 20 hr — 2 — cups
 Lemonade Ready in Stand : 30 hr — 3 — cups

Figure 33: Extra Graph. The effect of extreme changes in ‘LEMONADE COVERAGE’

Graph of Response to Changes in INITIAL LEMONADE READY IN STAND



Lemonade Ready in Stand : 30 cups —1— cups
 Lemonade Ready in Stand : 40 cups —2— cups
 Lemonade Ready in Stand : 50 cups —3— cups

Figure 34: Vensim Equivalent of Figure 7: The effect of changes in “INITIAL LEMONADE READY IN STAND”

3.2 Exploration 2: Epidemics

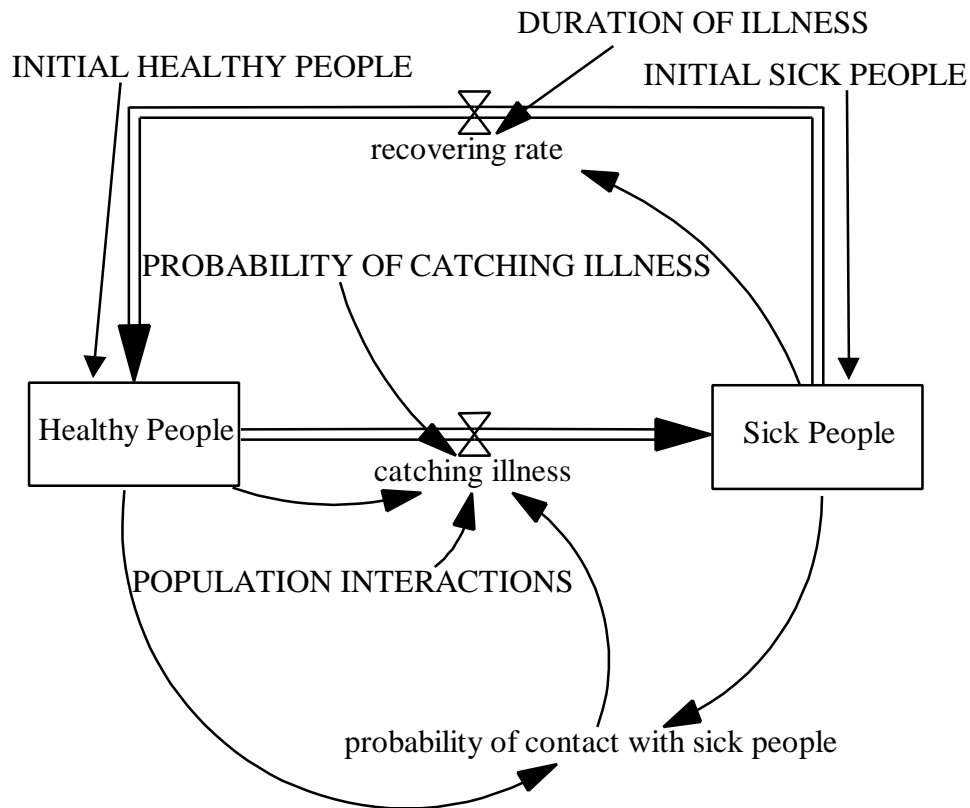


Figure 35: Vensim Equivalent of Figure 8: The Epidemics Model

Documentation for Epidemics Model

- (01) catching illness=Healthy People*PROBABILITY OF CATCHING ILLNESS
*probability of contact with sick people*POPULATION INTERACTIONS
Units: people/Month
The rate at which people catch the illness.
- (02) DURATION OF ILLNESS=0.5
Units: Month
The duration of the illness.
- (03) FINAL TIME =6
Units: Month
The final time for the simulation.
- (04) Healthy People= INTEG (recovering rate-catching illness, INITIAL HEALTHY PEOPLE)
Units: people
The number of healthy people in the population.
- (05) INITIAL HEALTHY PEOPLE=99
Units: people
- (06) INITIAL SICK PEOPLE=1
Units: people
- (07) INITIAL TIME = 0
Units: Month
The initial time for the simulation.
- (08) POPULATION INTERACTIONS=10
Units: (people/people)/Month
The number of interactions, or contacts, that a person has with other people during a month.
- (09) PROBABILITY OF CATCHING ILLNESS=0.5
Units: dmnl
The probability that a person will become infected when in contact with a sick person.
- (10) probability of contact with sick people=Sick People/(Sick People+Healthy People)
Units: dmnl
The probability that a person will meet a sick person. It is equal to the ratio of the number of sick people to the total population.

- (11) recovering rate=Sick People/DURATION OF ILLNESS
Units: people/Month
The rate at which people recover from the illness.
- (12) SAVEPER = TIME STEP
Units: Month
The frequency with which output is stored.
- (13) Sick People= INTEG (catching illness-recovering rate, INITIAL SICK PEOPLE)
Units: people
The number of sick people in the population.
- (14) TIME STEP = .0625
Units: Month
The time step for the simulation.

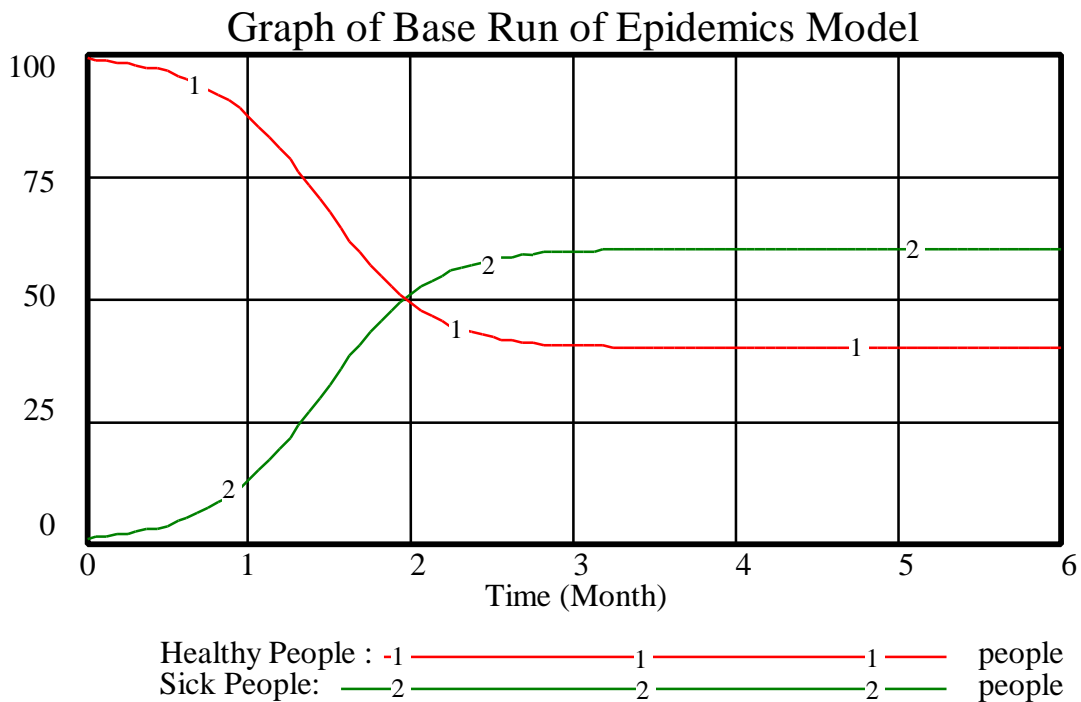
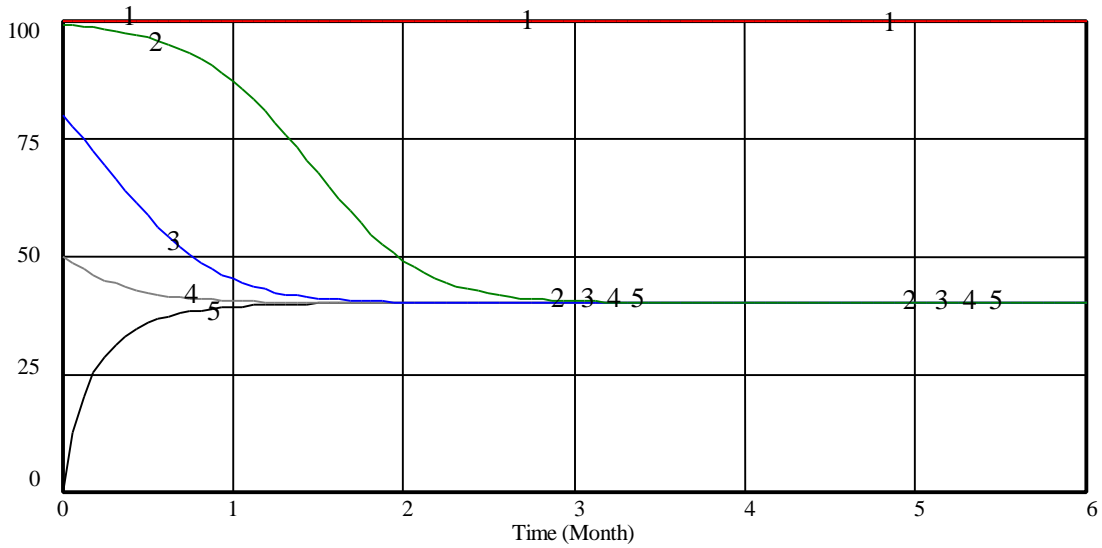
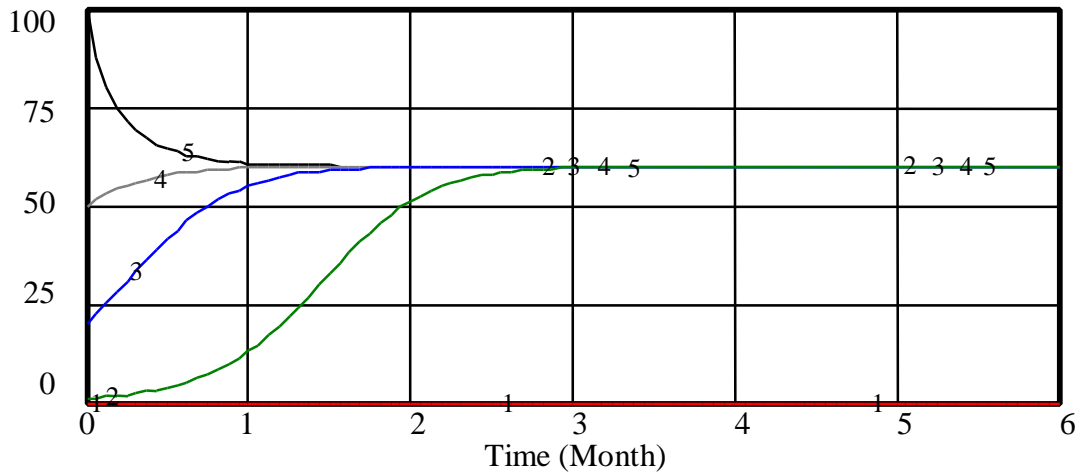


Figure 36: Vensim Equivalent of Figure 9: Base run of the Epidemics Model

Graphs of Response to Changes in INITIAL SICK PEOPLE



Healthy People : 0 people 1
 Healthy People : 1 person 2
 Healthy People : 20 people 3
 Healthy People : 50 people 4
 Healthy People : 100 people 5



Sick People : 0 people 1
 Sick People : 1 person 2
 Sick People : 20 people 3
 Sick People : 50 people 4
 Sick People : 100 people 5

Figure 37: Vensim equivalent of Figure 10: The effect of changes in the initial value of Sick People

Graphs of Response to Changes in INITIAL SICK PEOPLE

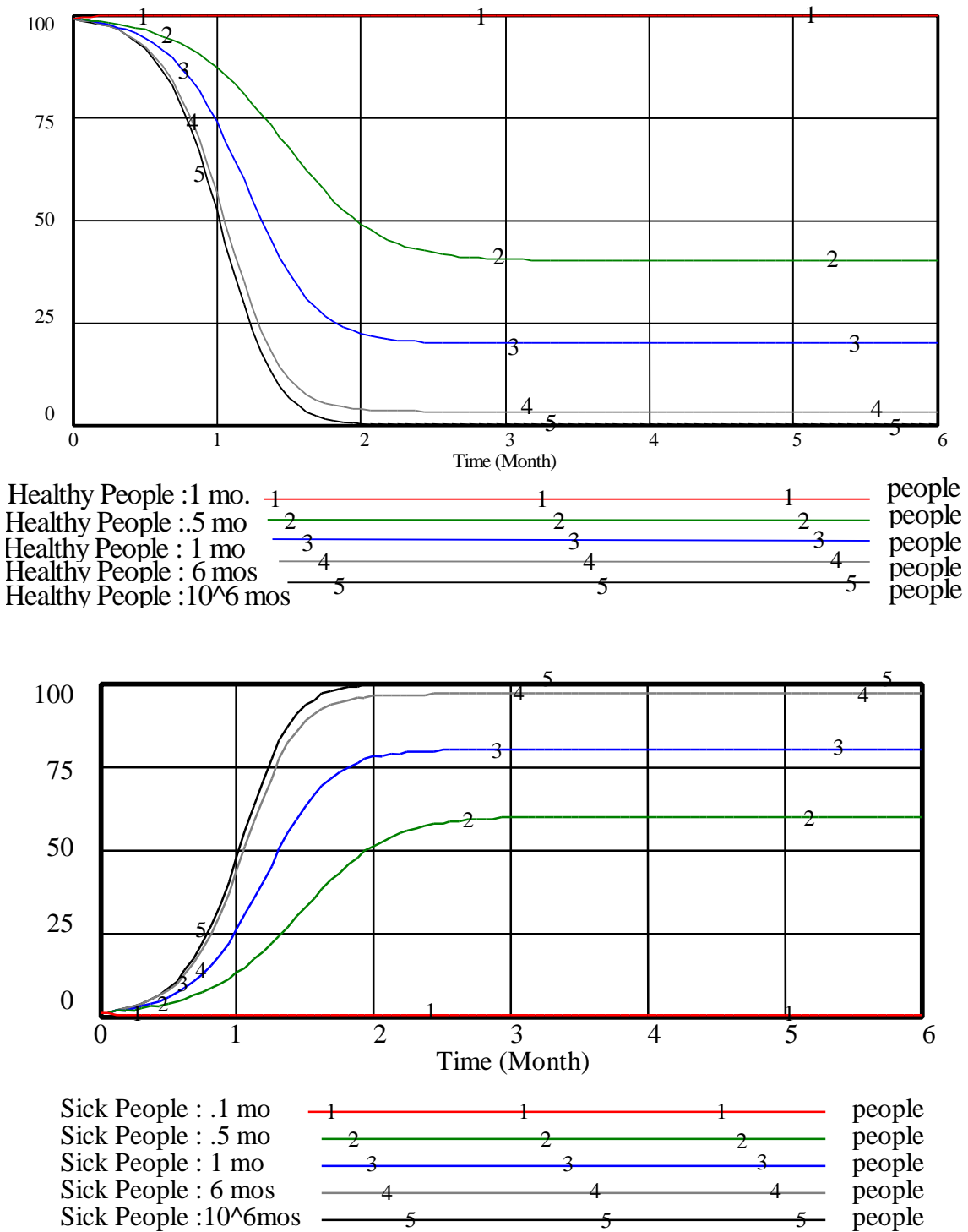
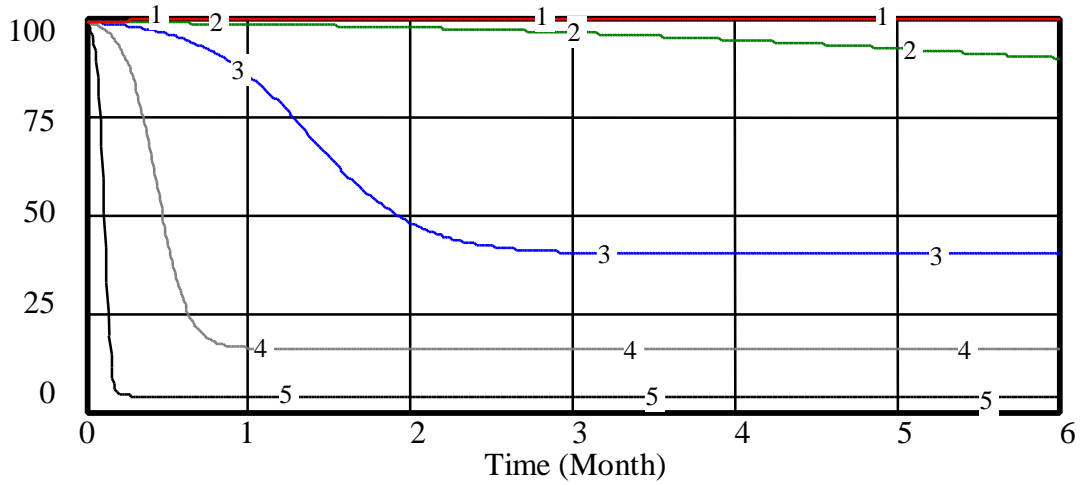
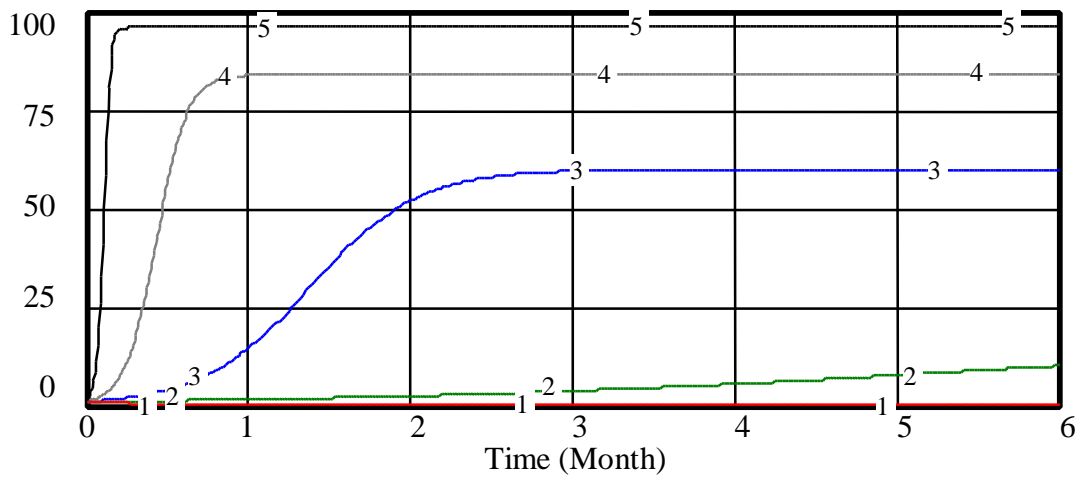


Figure 38: Vensim equivalent of Figure 11: The effect of changes in “DURATION OF ILLNESS”

Graphs of Response to Changes in POPULATION INTERACTIONS



Healthy People : 1 int	—1—	—1—	—1—	—1—	people
Healthy People : 5 ints	—2—	—2—	—2—	—2—	people
Healthy People : 10 ints	—3—	—3—	—3—	—3—	people
Healthy People : 25 ints	—4—	—4—	—4—	—4—	people
Healthy People : 99 ints	—5—	—5—	—5—	—5—	people



Sick People : 1 int	—1—	—1—	—1—	—1—	people
Sick People : 5 ints	—2—	—2—	—2—	—2—	people
Sick People : 10 ints	—3—	—3—	—3—	—3—	people
Sick People : 25 ints	—4—	—4—	—4—	—4—	people
Sick People : 99 ints	—5—	—5—	—5—	—5—	people

Figure 39: Vensim Equivalent to Figure 12: The effect of changes in ‘POPULATION INTERACTIONS’

4. Independent Exploration: Coffeehouse

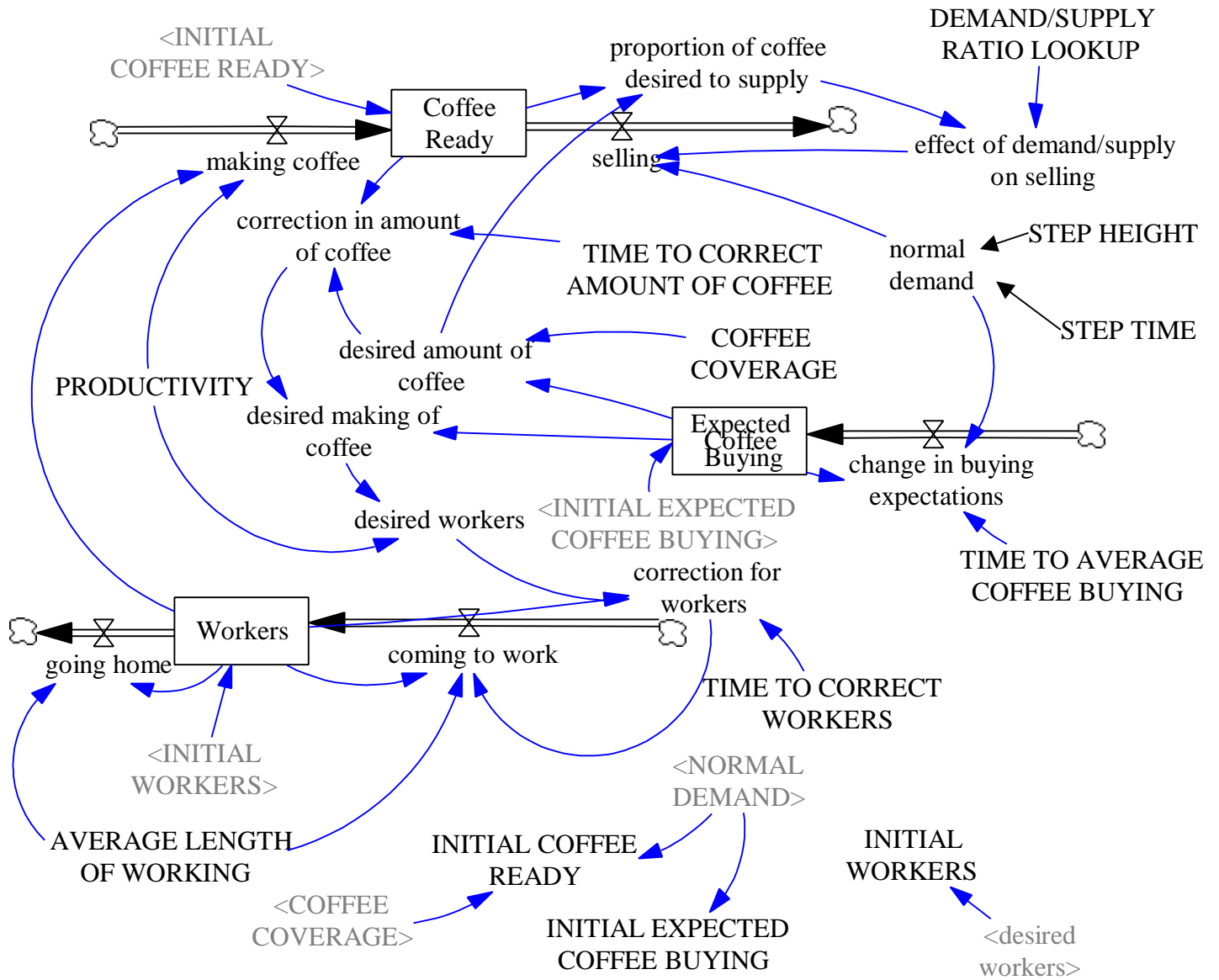


Figure 40: Vensim equivalent of Figure 13: The Coffeehouse Model

NOTE: Differences between the Vensim and Stella Coffeehouse Model are described below Documentation.

Documentation for the Coffeehouse Model

- (01) AVERAGE LENGTH OF WORKING=4
 Units: hour
 The number of hours that a worker spends in the Coffeehouse.
- (02) change in buying expectations=(NORMAL DEMAND-Expected Coffee Buying)/TIME TO AVERAGE COFFEE BUYING
 Units: (cups/hour)/hour
 The rate at which the workers' expectations about demand for coffee change.
- (03) COFFEE COVERAGE=2
 Units: hour
 Coffee coverage determines the number of hours worth of coffee that Howard wants the workers to keep at the Coffeehouse at all times.
- (04) Coffee Ready= INTEG (making coffee-selling, INITIAL COFFEE READY)
 Units: cups
 The number of cups of coffee ready in the Coffeehouse.
- (05) coming to work=
 correction for workers+(Workers/AVERAGE LENGTH OF WORKING)
 Units: workers/hour
 The rate at which workers come to work.
- (06) correction for workers=
 (desired workers-Workers)/TIME TO CORRECT WORKERS
 Units: workers/hour
 The number of workers who come to the Coffeehouse as a result of a difference between the desired and the actual number of workers.
- (07) correction in amount of coffee=(desired amount of coffee-Coffee Ready)/TIME TO CORRECT AMOUNT OF COFFEE
 Units: cups/hour
 The number of cups of coffee that the workers prepare every hour as a result of a difference between that desired and actual amount of coffee.
- (08) DEMAND/SUPPLY RATIO LOOKUP([(0,0)-(10,10)],(0.33,3),(0.5,2),(1,1),(2,0.5),(3,0.33),(5,0.2))
 Units: dmnl

- (09) desired amount of coffee=COFFEE COVERAGE*Expected Coffee Buying
Units: cups
The number of cups of coffee that the workers would like to have at the Coffeehouse. It is equal to the demand for coffee that they expect times the COFFEE COVERAGE.
- (10) desired making of coffee=correction in amount of coffee+Expected Coffee Buying
Units: cups/hour
The rate at which the workers would like to make coffee.
- (11) desired workers=desired making of coffee/PRODUCTIVITY
Units: workers
The number of workers that Howard wants to be working at the Coffeehouse.
- (12) "effect of demand/supply on selling"=
"DEMAND/SUPPLY RATIO LOOKUP"(proportion of coffee desired to supply)
Units: dmnl
- (13) Expected Coffee Buying= INTEG (change in buying expectations, INITIAL EXPECTED COFFEE BUYING)
Units: cups/hour
The hourly demand for coffee that the workers expect.
- (14) FINAL TIME = 48
Units: hour
The final time for the simulation.
- (15) going home=Workers/AVERAGE LENGTH OF WORKING
Units: workers/hour
The rate at which the workers leave the Coffeehouse to go home and study.
- (16) INITIAL COFFEE READY=COFFEE COVERAGE*NORMAL DEMAND
Units: cups
- (17) INITIAL EXPECTED COFFEE BUYING=NORMAL DEMAND
Units: cups/hour
- (18) INITIAL TIME = 0
Units: hour
The initial time for the simulation.
- (19) INITIAL WORKERS=desired workers
Units: workers

- (20) $\text{making coffee} = \text{Workers} * \text{PRODUCTIVITY}$
Units: cups/hour
The number of cups of coffee that the coffhouse workers prepare each hour. It is equal to the number of workers times their average productivity.
- (21) $\text{NORMAL DEMAND} = 20 + \text{STEP}(\text{STEP HEIGHT}, \text{STEP TIME})$
Units: cups/hour
The hourly demand for coffee.
- (22) $\text{PRODUCTIVITY} = 20$
Units: (cups/hour)/workers
The number of cups of coffee that a worker makes in an hour.
- (23) $\text{proportion of coffee desired to supply} = \text{desired amount of coffee} / \text{Coffee Ready}$
Units: dmnl
- (24) $\text{SAVEPER} = \text{TIME STEP}$
Units: hour
The frequency with which output is stored.
- (25) $\text{selling} = \text{NORMAL DEMAND} * \text{"effect of demand/supply on selling"}$
Units: cups/hour
- (26) $\text{STEP HEIGHT} = 5$
Units: dmnl
- (27) $\text{STEP TIME} = 3$
Units: dmnl
- (28) $\text{TIME STEP} = 0.125$
Units: hour
The time step for the simulation.
- (29) $\text{TIME TO AVERAGE COFFEE BUYING} = 2$
Units: hour
The time it takes the workers to recognize a permanent change in demand for coffee from random fluctuations.
- (30) $\text{TIME TO CORRECT AMOUNT OF COFFEE} = 1$
Units: hour
The time in which the workers attempt to correct a difference between the desired and the actual amount of coffee.

- (31) TIME TO CORRECT WORKERS=3
Units: hour
The time in which Howard wants to make more workers come to work.
- (32) Workers= INTEG (coming to work-going home,
INITIAL WORKERS)
Units: workers
The number of workers currently working at the Coffeehouse.

Differences Between the Vensim and Stella Coffeehouse Model:

1. Below the large model is a smaller model that shows the calculation of the INITIAL stock variables. The model is split to avoid confusion and emphasize the dynamic parts of the system.
2. The variable “Buying Coffee” in the Stella version has been changed to “normal demand” in the Vensim paper to more explicitly show that this constant captures the demand for coffee. The “STEP HEIGHT” and “STEP TIME” are constants that are separately entered into the “normal demand” equation.
3. To avoid poor modeling practices, the Vensim model does not use MIN/MAX functions.

selling lemonade- Instead of inputting the equation “MIN(NORMAL DEMAND, Coffee Ready/DT), we compute the ratio of the “desired amount of coffee” to the “Coffee Ready” to obtain a value of demand/supply (called “proportion of coffee desired to supply”). We know that if the demand is greater than the supply of coffee (i.e., if the proportion is greater than 1) then the sales of coffee will slow, since much of the Coffeehouse’s efforts will be toward making coffee. If the proportion is smaller than 1, however, then the coffeehouse’s efforts will be toward selling coffee, and their “selling” rate will increase to a maximum value. The “DEMAND/SUPPLY RATIO LOOKUP” captures this effect, and is shown below:

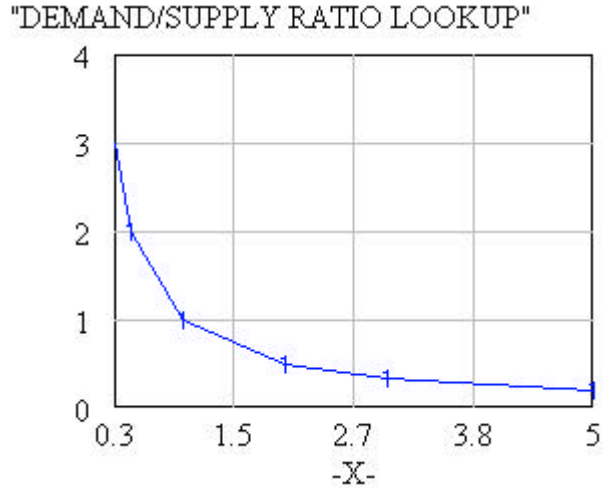


Figure 41: DEMAND?SUPPLY RATIO LOOKUP

The appropriate output of the lookup is computed by the “effect of demand/supply on selling” variable, which is multiplied by the “NORMAL DEMAND” to compute “selling.”

Graph of Base Run of Coffeehouse Model

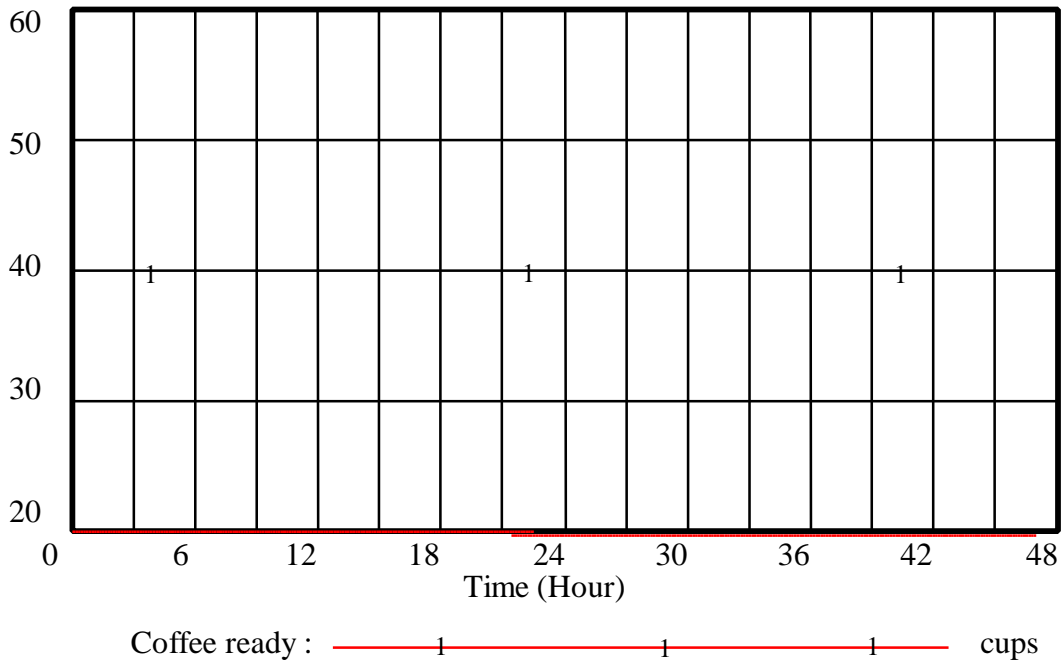


Figure 42: Vensim equivalent of Figure 14: Base run of the Coffeeshouse model

Graph of Response to STEP Increase in BUYING COFFEE

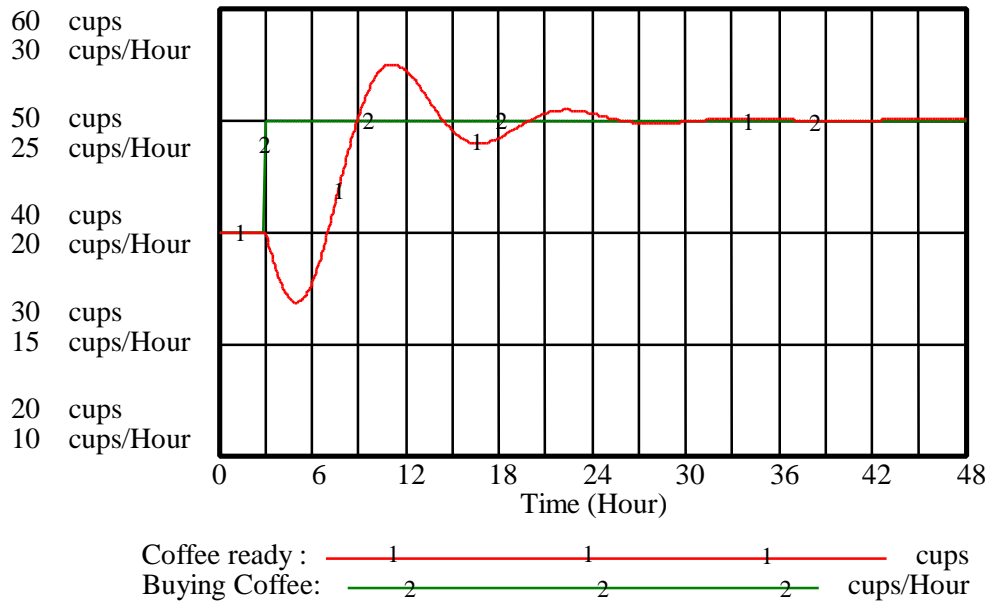


Figure 43: Vensim equivalent of Figure 15: The system responding to a step increase in “BUYING COFFEE”

Graph of Response to Changes in INITIAL COFFEE READY

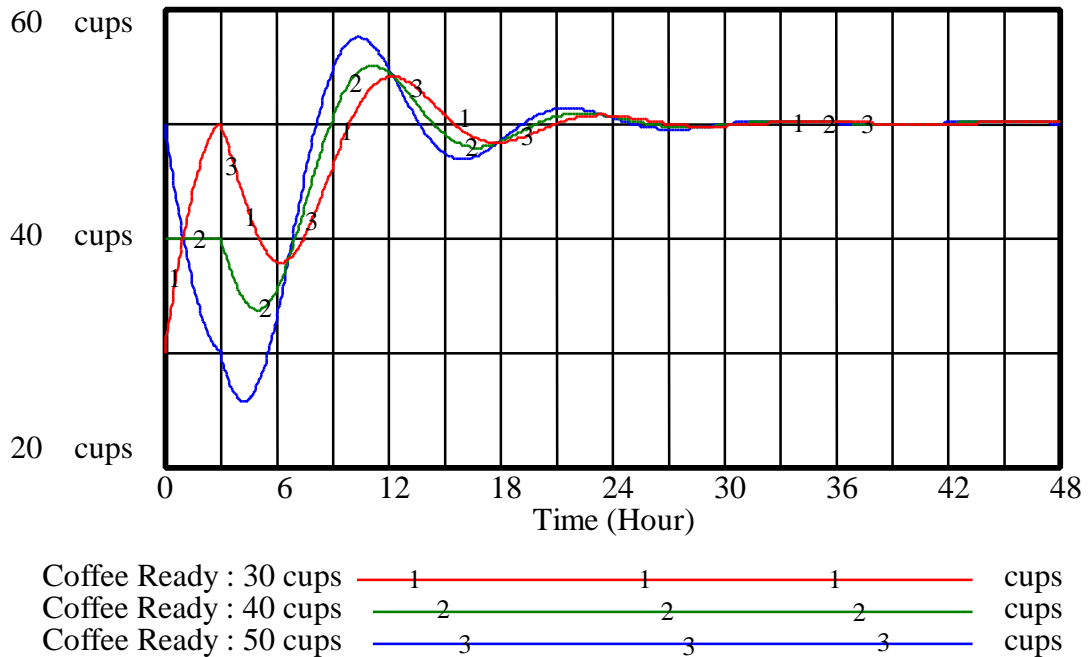


Figure 44: Vensim equivalent of Figure 16: The effect of changes in “INITIAL COFFEE READY”

Graph of Response to Changes in TIME TO AVERAGE COFFEE BUYING

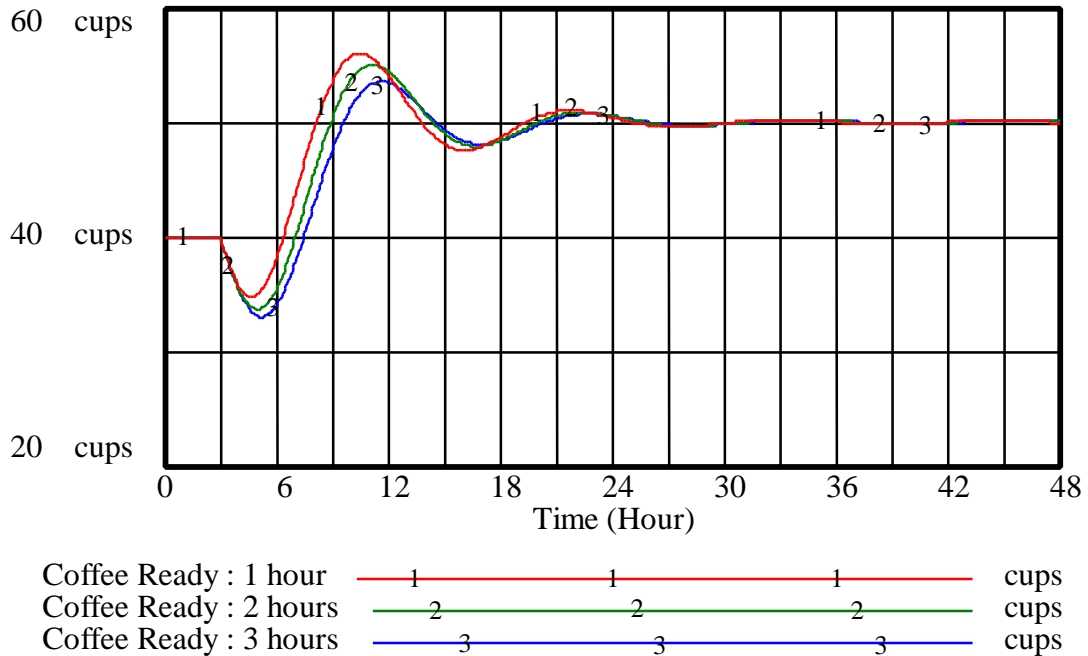


Figure 45: Vensim equivalent of Figure 17: The effects of changes in “TIME TO AVERAGE COFFEE BUYING”

Graph of Response to Changes in TIME TO CORRECT AMOUNT OF COFFEE

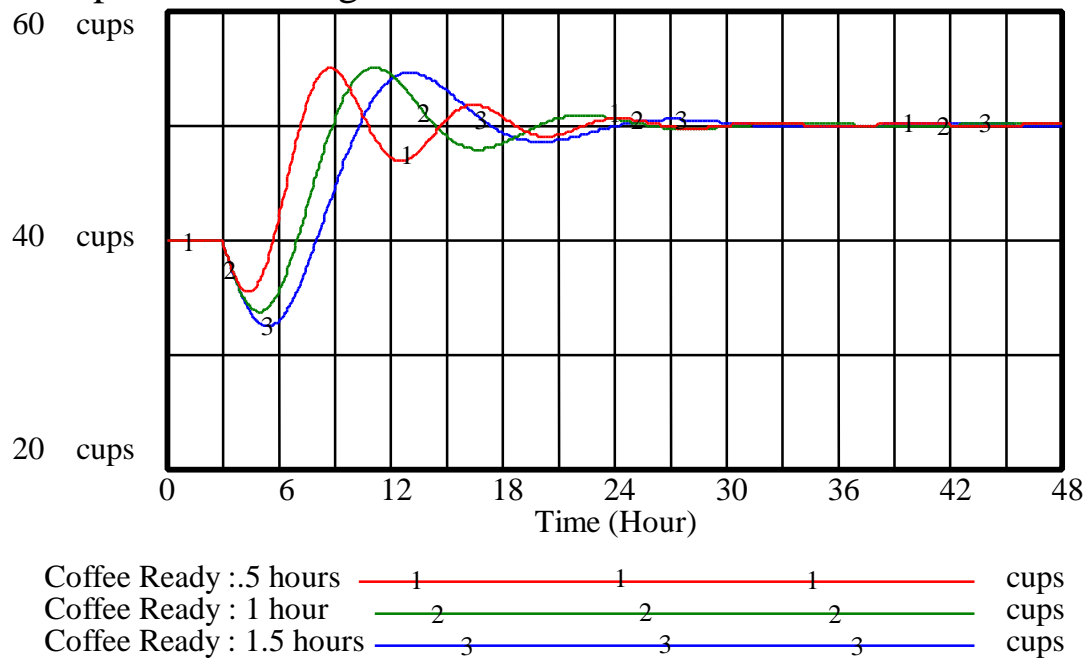


Figure 46: Vensim equivalent of Figure 18: The effect of changes in the “TIME TO CORRECT AMOUNT OF COFFEE”

Graph of Response to Changes in COFFEE COVERAGE

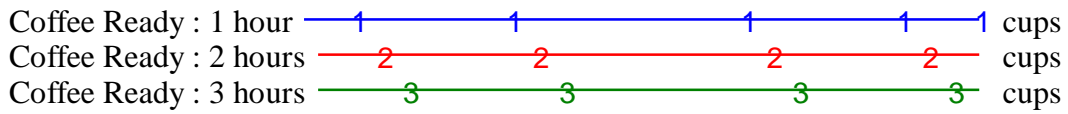
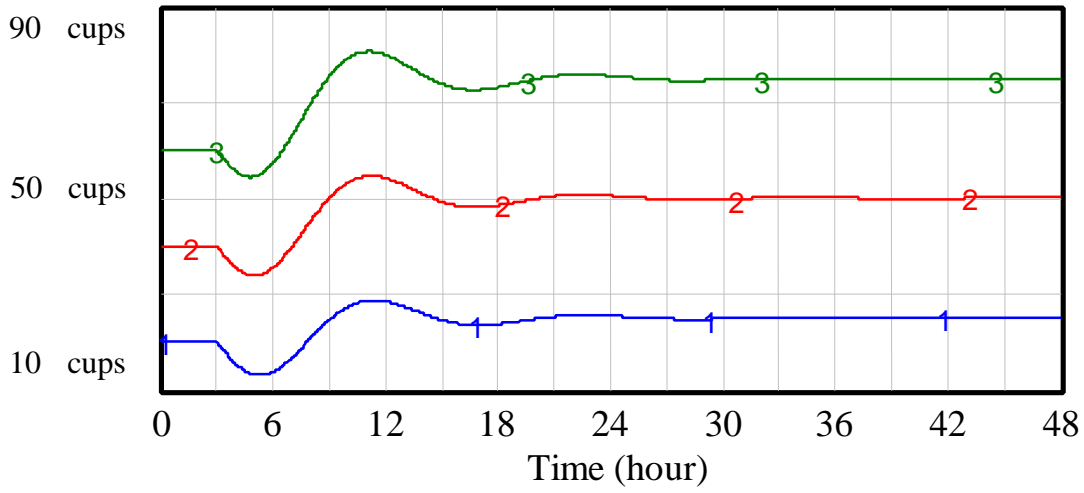


Figure 47: Vensim equivalent of Figure 19: The effect of changes in “COFFEE COVERAGE”

Graph of Response to Changes in TIME TO CORRECT WORKERS

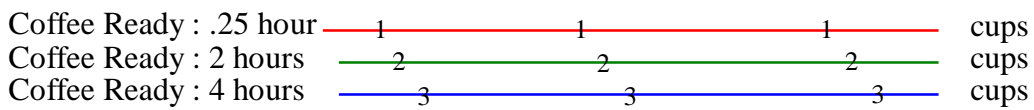
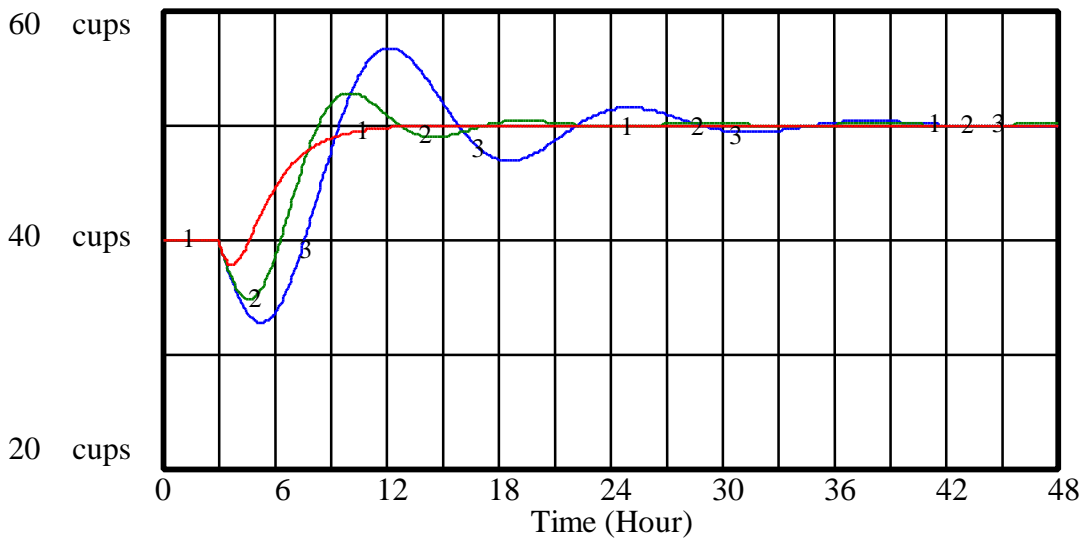


Figure 48: Vensim equivalent of Figure 20: The effect of changes in “TIME TO CORRECT WORKERS”

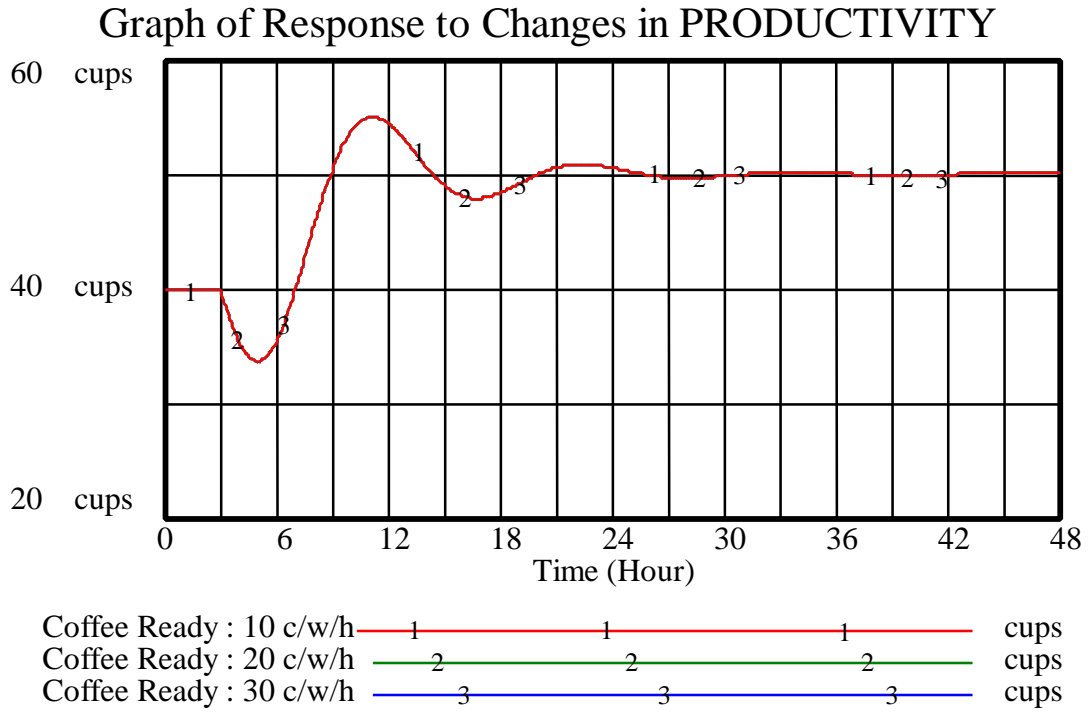


Figure 49: Vensim equivalent of Figure 21: The effect of changing “PRODUCTIVITY”

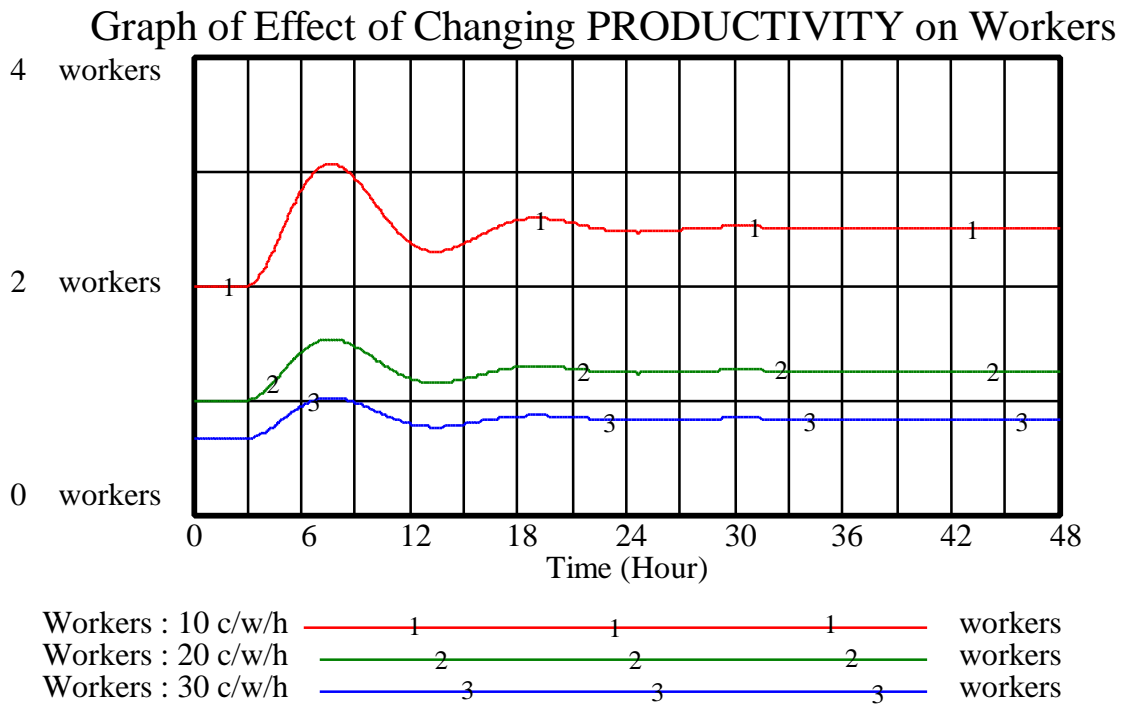


Figure 50: Vensim equivalent of Figure 22: The effect of changing “PRODUCTIVITY” on the number of “Workers”

Graph of Response to Changes in AVERAGE LENGTH OF WORKING

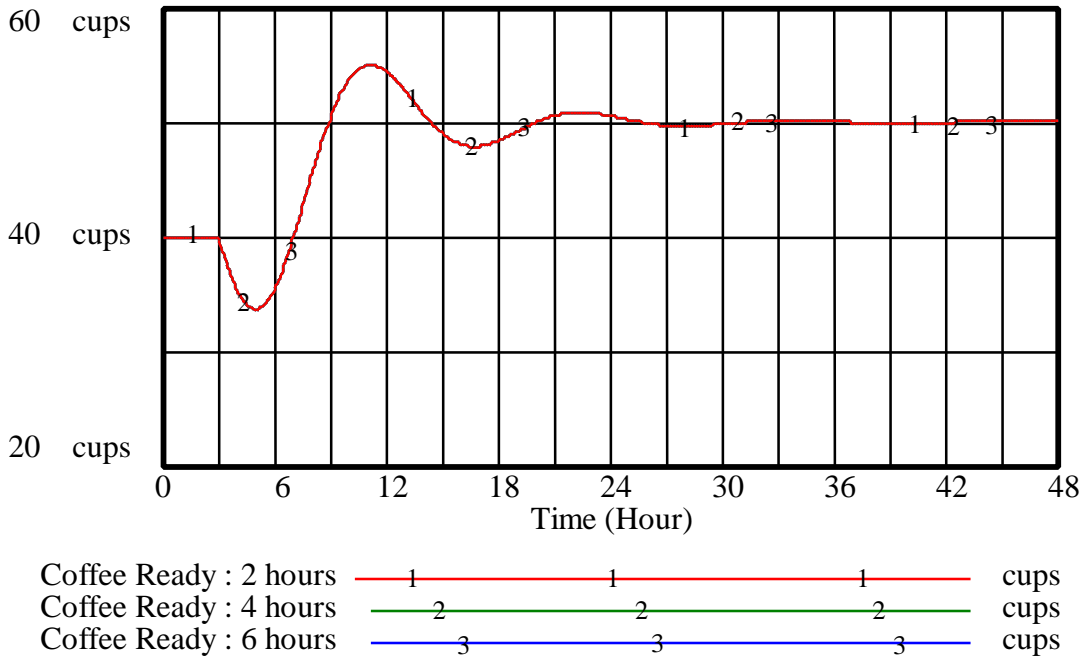


Figure 51: Vensim equivalent of Figure 23: The effect of changes in “AVERAGE LENGTH OF WORKING”

Graph of Effect of Changes in AVERAGE LENGTH OF WORKING on Workers

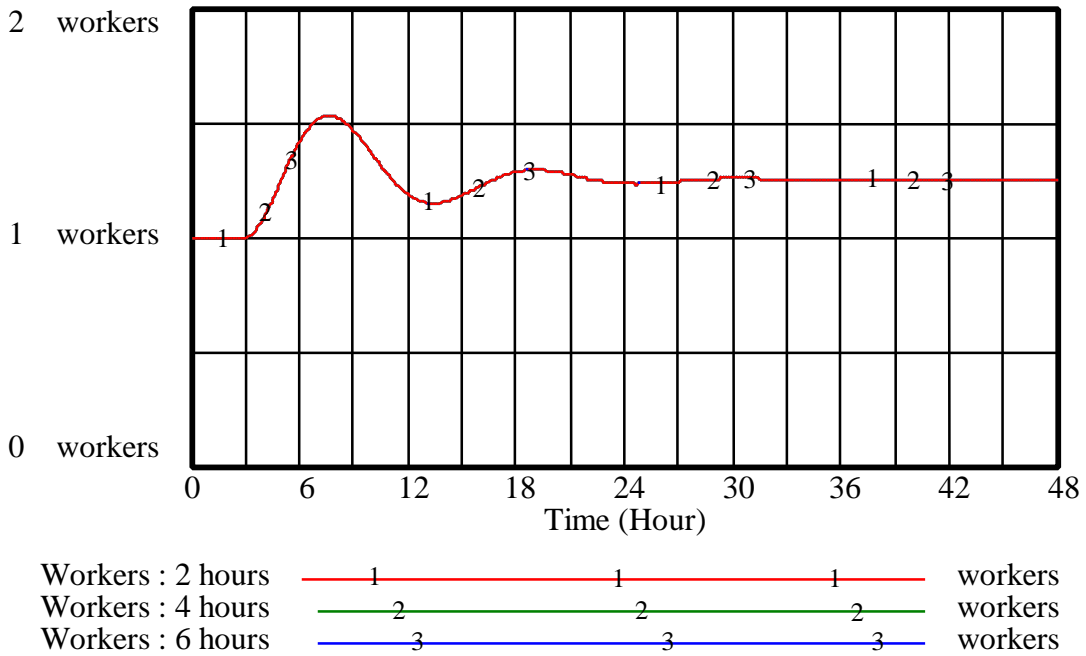


Figure 52: Vensim equivalent of Figure 24: The effect of changes in “AVERAGE LENGTH OF WORKING” on the number of “Workers”