Hi, the topic of this video is scalar triple product, that is a very important topic for JEE.

I will probably say this is the most important topic for JEE, more than cross product more than dot product because this combines cross product and dot product.

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It is a very important concept and I hope that this video will be able to provide you a basic understanding of scalar triple product.

The idea behind video is to first introduce scalar triple product and then do a few problems on this topic.

There will also be a followup video in which we will go for more examples of scalar triple product.

So â€œwhat is a scalar triple productâ€?

Till now we have done dot product which was a \dot b We have done cross product which was a \cross b.

Now this is a scalar triple product.

So as the name suggests â€” triple means there are three quantities: vector a, vector b, vector c â€” and it is a scalar product.

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And the output Hi, the topic of this video is scalar triple product, that is a very important topic for JEE.

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Like dot product was a scalar product, this is also a scalar product but there will be three vector quantities, a b and c. And the output would be a scalar.

Let us first discuss how we can get a scalar quantity out of three vectors.

Let us say we have vector a, and we have vector b, and we have vector c.

Now, how can I get a scalar quantity out of this?

Can I do a \dot b \dot c?

This would mean that (let me put brackets here) a \dot b is a scalar quantity. And I cannot dot a scalar with c so. Thus this not possible.

Can I do a \cross b \cross c.

This would mean that this is a vector quantity as I am taking a vector crossed with another vector - and that will give a vector output.

However, we want a scalar output so this is also not possible. We will also be discussing what kind of product is this but that is the part of the next videos.

However, can we do something like this - a \cross b \dot c. This is vector dotted with another vector, and thus gives us scalar output. This is a correct one. This is what we want to know.

Scalar tripe product is defined as a \cross b \dot c. So what does this mean? How can we then get the answer for the the scalar tripe product. The expansion would be something like this.

You can imagine that basically you have \i component and you multiply all of that with cx. So rather than doing \i \j \k separately you can just put cx cy cz in the determinant at the top.

This is an important property which i have written down. You are doing a \cross b \dot c.

Now can you see something else also which might or might not be that obvious? We know that (a \cross b) \dot c = c \dot (a \cross b). We know that because this is a dot product.

Let us call this vector d. Then d \dot c = c \dot d. So all you are saying is (a \cross b) dot c = c \dot (a \cross b) Also, there is a property of determinants that basically means that you can flip two rows, the answer would become negative of the actual answer, and thus if you flip again (the second time), then the answer would remain the same.

So if I flip (row) a and (row) c, then (row) a will come to the top and row (c) will come to the middle.

Then when I flip (row) b and (row) c, it will become a b c. In other words (a \cross b) \dot c can also be written as | ax ay az; bx by bz; cx cy cz|.

I have done this by flipping rows twice â€” flipping c row and a row, and then b row and c row.

However, if you think about this, whatever was in the \dot came at the top.

I can write this also as a \dot (b \cross c).

What I am saying is whatever is in the dot came at the top row or the single vector came at the top row.

I have written this down as (a \cross b) \dot c = a \dot (b \cross c). This is a super important property.

a \dot (b \cross c) = (a \cross b) \dot c. In other words, you can flip the sign of \dot and \cross.

The first property that we have discussed was this.

The second property that I want to discuss is (a \cross b) \dot c = a \dot (b \cross c).

Third property is a \dot (b \cross c) = (a \cross b) \dot c. And sometimes this is written as [a b c], since the position of \dot and \cross doesnâ€™t matter.

It doesnâ€™t matter you are taking dot here or cross here. It will always come out to be same.

I hope that this is making sense: we first said in a determinant and when we flipped the rows twice the value remains the same. Also because the top row belonged to the alone vector, we said a \dot (b \cross c) is same as (a \cross b) \dot c.

We also know that (a \cross b) \dot c = c \dot (a \cross b) = (c \cross a) \dot b.

Or this can also be [c a b]. Because now we are taking the associative property of dot.

Similarly, now I can write this as b \dot (c \cross a). Am flipping the dots here again, and now i am changing this as (b \cross c) \dot a i.e. changing dot and cross. And this is [b c a].

This is a very very important property.

What i am trying to say here is (a \cross b) \dot c. This seems to be a cycling property.

Cyclic property says that a should go to b, and then b should go to c, and then c should go to a.

So what is happening is [a b c]. You can always think it terms of cyclic [a b c] The cycle is always [a b c] and that means as long as you have [a b c] written in the correct cycle, then the value remains the same.

In other words, [a b c] = [c a b] = [b c a].

I just want to highlight one other thing after, [a b c] = - [a c b].

So if you flip two things, the cycle breaks.

It is not [a b c], it is [a c b] and so you have broken the cycle. Or you have reversed the cycle.

So rather than [a b c], you are now going [a c b].

Thats why you have a negative sign and you can check that so if you just flip two rows - just b and c - only once you do row exchange, then there is negative sign in front of the determinant.

Thats why there is a negative sign here.

Again you can make the cycle that [a b c] = - [b a c]. The cycle is again [a c b], not [a b c].

This [a b c] = - [c b a] This is a very very important property. I hope that you are able to now understand what we are doing.

We discussed a lot of things very quickly.

This is a scalar product: (a \cross b) \dot c â€” because (a \cross b) is a vector and then you are dotting it with (vector) c.

The second thing we discussed was that you can flip the position of \dot and \cross.

We showed through the determinant property that when you flip rows twice, you can conclude that (a \cross b) \dot c = a \dot (b \cross c).

The third property is that there is a cyclic property to the whole system.

We saw this by exchanging the \dot and \cross. And by using the property of dot product i.e.

(a \cross b) \dot c = c \dot (a \cross b).

You should take some time to digest these things because these will be used in and out.

They are very very important for the chapter of vectors.

So please please take your time to understand these things. Slowly go through this by writing these equations and then you should be able to understand this.

I donâ€™t think this a very difficult concept to grasp but for once you have to really understand it.

So two things: you can exchange the sign of \dot and \cross, and there is a cyclic property.

For evaluating (a \cross b) \dot c, you can just use the determinant.

The next thing I want to discuss â€” like all other products â€” is the physical significance.

What is the physical significance of (a \cross b) \dot c.

What basically we have to realize is that this condition a \dot (b \cross c) =0, then a b c are coplanar.

This is the first part of the significance. I want to stress on this quite a lot because you will often use this in questions.

What does this mean?

Let us say you have two vectors, b and c.

Two vectors are always coplanar. If you have two vectors, you can always pass a plane through them.

b and c are two vectors and you have passed a plane through them.

(b \cross c) is a vector which is perpendicular to both b and c. This is something we have discussed in the topic of cross product and if you canâ€™t recall it, please go and check back the notes on cross product.

Whenever we have (b \cross c), let us say n, it is perpendicular to both b and c.

What we are saying is that this n vector is perpendicular to the plane of b and c.

Because b \cross c is n, a \dot n = 0. What does a \dot n = 0 mean? This mean a and n are perpendicular.

If a and n are perpendicular, a has to lie in the plane of b and c. or a, b, c are coplanar.

What does coplanar mean? It means that if there are three vectors, you can pass a plane through them.

With two vectors, you can always pass a plane through them.

Let us say three vectors are like this (demonstration), you cannot pass a plane through them.

Only with two vectors, you can always pass a plane.

If there are three vectors, you cannot always pass a plane through them. When can you pass a plane through them? Whenever a \dot (b \cross c) = 0 or the scalar triple product = 0.

How are we explaining this?

b \cross c is a vector perpendicular to both b and c. Or is perpendicular to the plane of b and c. When a \dot n = 0, that implies a and (b \cross c) are perpendicular. Since dot product is zero whenever two vectors are perpendicular. So this forces a to lie in the plane of b and c.

This is very important and you just have to remember that whenever three vectors are coplanar, the scalar triple product is 0.

Coplanarity also means that you can represent the third vectors as linear combination of the first two vectors. This is also a way to understand coplanarity.

When can you represent a vector as a linear combination of a and b?

Let us say there are a and b, and a plane passes through them. Then basically you are saying that you are stretching one vector or compressing a vector as l and m can take any real values.

Let us say you made one vector half and other vector double, and then now you are adding them. So basically completing some kind of addition, and addition forces you to not leave that plane.

That is why these three things are coplanar.

You can also imagine that if I put c=la+mb here: if I take cross of b with b, it will become zero. Then, we will be left with (a \cross b) \dot a. Then you can flip \dot and \cross, and it will become (a \cross a) \dot b. Cross of a with a will also be zero.

You can check. You can just put this expression back and check that this always comes out to be zero.

So coplanarity means that one vector can also be written as the linear combination of other two vectors, and scalar triple product [a b c] is zero.

I hope that is giving some insight into the significance of scalar tripe product.

Dot product was the projection of a vector on the other vector.

Cross product was the vector perpendicular to both the vectors.

Scalar triple product = 0 means a condition for coplanarity.

Second thing that I want you to learn as a formula, if there is a parallelopiped â€" parallelopiped is like a 3-D parallelogram. Volume of parallelopiped is [a b c] where three sides are a vector, b vector and c vector.

There are a lot of proofs for this but I donâ€™t think you need to understand the proofs.

a vector is this entire side, c vector is this entire side, and b vector is this entire side. Then the volume of parallelopiped is [a b c].

If there is a tetrahedron, with sides a, b and c, then volume would be [a b c]/6.

If this is a little bit unclear to you, donâ€™t worry too much about this. Just go back and look at some images of tetrahedron and parallelopiped.

Just remember that whenever the side are a,b and c, then the volume of parallelopiped is [a b c] and the volume of tetrahederon is [a b c]/6.

Let us recap whatever we have discussed. This ends the property discussion of scalar triple product.

We have discussed that (a \cross b) \dot c can be written as a determinant.

We also learned that you can flip \dot and \cross. I showed this by flipping the rows twice and hence (a \cross b) \dot c = (a \cross b) \dot c.

Then we learned about the cyclic property of [a b c]. And that there will be a minus sign whenever the cycle is broken or reversed.

Then we discussed two things about physical significance.

Condition for coplanarity: one vector is a linear combination of other two vectors and that is also expressed in terms of (a \cross b) \dot c = 0. Since b \cross c is a vector perpendicular to the plane of b and c, and if a \dot n=0, a has to lie in the place of b and c.

And finally, if a,b and c are sides of parallelepiped and tetrahedron, then the volume is [a b c] and [a b c]/6.

These are the properties we have discussed. Please remember them and learn them. They should be on your fingertips whenever you see them.

Let us try to do some very quick problems.

First problem is: If a, b, c are coplanar, what is the value of [2a - b, 2b - c, 2c -a]?

We have been told that there are three vectors a,b,c that are coplanar. Then we have been asked to find the scalar triple product of vectors of the linear combination of a,b and c.

One way to solve this problem is to take a cross and then take dot, and trying to find something in terms of [a b c].

My point here is to really make you understand what this means. Think about three vectors that are coplanar: you have two vectors like this and third vector like this â€” they are in a plane.

What you are doing is that you are just taking a linear combination of these three vectors.

Can you ever go out of the plane? Can you ever create a vector which is out of the plane?

For instance, if c = la + mb, since a, b, c are coplanar.

Then 2a-b is a linear combination of a and b.

2b-c, because c = la + mb, is also a linear combination of a and b.

2c-a is also a linear combination of a and c.

So even all these vectors are just linear combination of a and b. That means that they are in the same plane. So you donâ€™t even need to do anything. You donâ€™t have to calculate.

You can just directly write that this is equal to zero.

Because a, b, c are coplanar, any linear combination of these vectors will also lie in the same plane. So these three vectors are in the same plane. So you just have to understand that since these vectors a coplanar, their linear combinations has to be coplanar, so this has to be zero since this is condition of scalar triple product.

I can bet that a lot of students will try to solve this problem. They will start with [a b c]=0 and then they will expand this by taking a cross here and dot here, and opening up the brackets and trying to solve it somehow. All you have to do is to understand that coplanarity means that that linear combination of the vectors will lie in the same plane.

I hope that this gives you some insight about scalar triple product.

The second problem that I want to do is: What is the value of \lambda such that \lambda \i + \j + \k, \i + 2 \lambda \j + \k and \i + \j + \k are coplanar.

So what is the value of \lambda such that these three vectors are coplanar.

The question is begging you to remember the condition of coplanarity that a \dot (b \cross c) = 0. Thats it.

How can we write a \dot (b \cross c) is as we have discussed a determinant. Let us call this a vector, b vector and c vector.

Then a \dot (b \cross c) = |\lambda 1 1; 1 2\lamba 1; 1 1 1 | = 0.

I hope with time you will become quick in opening up determinants.

If you open up the determinant, you will find that this is 2 \lambda ^ 2 - 3 \lambda + 1 = 0. You can check that this what it comes out.

That would give you the answer \lambda = 1, 1/2. In an objective test, this could be a multiple-choice problem.

Can we check what the answer \lambda = 1 mean? If \lambda = 1, then this becomes \i + \j + \k and then this becomes \i + \j + \k. So they are same vectors. So if they are the same vectors, that means their cross will always be zero.

So if they are the same vectors, this basically become two vectors, which will obviously be coplanar. Rather than three vectors, you have basically two vectors which are always coplanar.

If \lambda=1/2, then this become \i + \j + \k, and then these two become same. So again they become two vectors and they are always coplanar.

I hope that with this you are physically able to understand the meaning of scale triple product.

In this video, we covered very important topic of scalar triple product. We discussed its properties. We discussed 4 properties. And then we did two examples to understand the meaning of scalar triple product and how to apply it.

In the next video, we will discuss vector triple product. If you enjoyed this video, please check out the next video. Thank you!