PROFESSOR: So now that we've done a very simple calculation of C of t, the concentration of virions in the air per volume, which is coming from just a mass balance in a well-mixed room, let's zoom in, now, and think about transmission between the infected person, or an infected person and then a susceptible person who is not yet infected.

And the transmission is occurring by breathing the infected droplets in, and then the virus has to get out of those droplets and interact with the host tissues.

So if we let beta be the transmission rate, so this is basically new infection per time, so basically, it's the rate at which another person may become infected.

Then how will we write this?

Well, we could write it as-- so we're, now, forgetting about the infected person.

We care about their breathing only because it's producing this concentration C that we just talked about.

But now we're going to focus on the susceptible person.

The susceptible person is now breathing in at the same flow rate Qb because the breathing in and breathing out are the same.

And so Qb is the volume per time around, let's say, 0.5 meters cubed per hour with which they're sampling the air.

And the air comes in, and then C of t is the concentration of virions per volume.

So this combination now tells me how many virions per time they're actually taking in.

There'll be a quantity Ci, which we mentioned earlier, which is the infectivity, and that's the probability that an individual virion actually causes this person to get sick and to become infected themselves.

So this is the infectivity of an individual virion.

And then on top of that, we also want to put in-- I'll just change the color to highlight it-- the mask factor that we talked about earlier.

So that's the transmission or penetration probability for the droplets of interest through the mask.

These quantities, many of them will depend on size, and we'll come to that, size of the droplet, but just as a rough approximation, this is the starting point.

So this is the number of, basically, new infections per time, and there is a useful notion in epidemiology, which is that of the infection quantum.

So transmission rates are often written as infection quanta per time, and that is the rate at which a person, which is susceptible, will get infected.

What we have not yet captured is if you have a finite number of people in the room, when someone gets infected, they can't get infected again.

So the numbers of susceptible people is changing.

So we have to model the progression of the disease in the room, which we have not done yet.

So that's why the beta is not the number of infected people cause eventually you run out of people to infect.

So we have to account for that later.

But a useful way to think of it is that beta is sort of the rate at which this person is sending infection quanta over here.

Those quanta may not actually lead to an infection because they might already have been infected.

But if they're susceptible, that tells you the rate at which that person would become infected.

So that's the notion of infection quanta is essentially defined by the transmission rate beta.

Now, this infectivity is something we'll come back to.

We will actually go through the calculation for SARS-CoV-2, but it's been estimated before to be at about 2% from the original SARS virus in 2003.

And in fact, I will argue that it's greater than 10% for SARS-CoV-2.

And we'll do that by analyzing spreading data with the model.

And, of course, that helps to explain why SARS-CoV-2 has led to the COVID-19 pandemic, and SARS-CoV-1 was not able to spread as much.

So now we have here our transmission rate.

And we can ask ourselves, this is a transmission rate, which is time-dependent, but what if now we calculate the steady-state transmission rate?

So the transient would be when the infected person first enters the room.

The concentration is changing in time in the air, but eventually, there's kind of a steady-state where there's a balance of the production and then the flow rate through the room of refreshing the air with outdoor air.

And in steady-state, we have the transmission rate is going to go to a constant value, which I'll call beta bar, and that is given by the steady-state concentrations.

So here I'll just write it-- I'll rewrite this expression here Qb Ci Pm and then the steady-state C, which is the production rate P divided by the outdoor airflow rate Q.

And another way we can write that is that remember Q we can write as lambda a times V. So this is Qb, and in fact, let me-- well, here, I'll write it one more time.

That's my Ci Pm capital P over lambda A V.

Now, recall that the P we had written as, that's the production rate, also depends on Qb, that's the rate at which the infected person is exhaling infected air.

So that was Qb times nd, the number of droplets per volume, Vd, the volume of liquid in a droplet, so this nd Vd's the volume fraction of liquid.

And then we needed Cv, which was the concentration of virions in the liquid or in the fluid.

And there's also a factor of Pm if that person is wearing a mask.

So we put all this together, we get an important result here, which is that the steady-state transmission rate can be written, when I plug-in here, as Qb squared times Pm squared.

So the mask factor comes in twice because if they are wearing masks, there's two masks.

You have a mask at the source.

You also mask at the target, and the fluid has to go through both of those filters.

So that's one reason, as we'll see, that masks can be, actually, very effective.

And then we'll lump all the parameters in something I'll call Cq, which I'll come back to, and then we'll leave lambda a V in the denominator.

So this is the main transmission rate where I've defined this important parameter Cq which has all the information about the specific disease, and what is it?

It's everything else is left.

It's nd Vd, so that has to do with respiration, so the distribution of droplet sizes and the size of the droplets is something that's coming from the type of respiration that the infected person is engaging in.

Cv is their viral load, so it has to do with the progression of their disease and how many viruses or virions are found.

And then we have Ci, which is this infectivity, the probability that any one of those virions will actually infect this susceptible person if it manages to get in there and diffuse out of the droplets.

So coming back to this notion of infection quanta, while beta is an infection quanta per time, which are being kind of transmitted from one infected person to one susceptible person, the way I've written it here is I've reexpressed it as infection quanta per volume of air exhaled.

So while C is the concentration of virions in the background, there is this Cq, which is essentially the infection quanta that are being released, and the factor of CvCi is actually what connects those two.

In fact, sometimes we write Cq little cq CvCi, this is infection quanta per liquid volume in a drop.

So from the mucus or material that's being released, there is a certain concentration of infection quanta, which is the physical concentration of the virions Cv times the probability that if they were to be exposed to the susceptible person's cells, that they would actually infect those cells and cause a transmission of the infection.

So that's another important quantity, and this here is really the primary sort of lumped or combined disease and physiological parameter in the model.

So what's nice about separating this way is that Qb is something which has to do with people's activity, it's how fast they're breathing, and that's something we know very easily whether they're at rest or they're exercising.

Pm is also known if we know the kinds of masks people are wearing.

There's various studies of transmission factors and filtration efficiencies of masks.

And so we can put reasonable estimates there.

And then here we see the importance of lambda a, which is the air exchange rate.

So how quickly is fresh air coming in the room?

That's a physical parameter of the room, has nothing to do with the disease.

And V, of course, is a geometrical parameter, the room, which is the volume.

And so all the disease aspects are kind of lumped into the Cq.

So if I want to apply this to actual spreading of COVID-19, I have all these parameters that I know that come from the physics and fluid mechanics of the room.

And then, I have a single parameter Cq that I need to obtain from an understanding of disease transmission and looking at spreading events in indoor situations.