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PROFESSOR: So in order to use our results for the analysis of the well-mixed room into the safety guideline, let me just summarize the results for the general case of transient build-up of aerosols in the room and the associated transmission.

So here is the result that we derived earlier, which is that the transmission rate as a function of time is an integral over all the droplet sizes.

And then you have here the mask filtration factor, which depends on size P_m .

You have the breathing rate Q_b that comes in squared because there's one person breathing out, another person breathing in.

You got the volume of the room.

You have here the relaxation rate for the concentration of aerosol in the room, which here is given by four factors for ventilation, sedimentation, filtration, and deactivation.

And then for the production of aerosols, there's this $n_q(r)$, which is the number of exhaled infection quanta per volume, per air volume leaving the breath per radius because it's still resolved by the different droplet radii.

And this has several contributions.

It has $n_d(r)$, which is the distribution of droplet sizes.

$V_d(r)$ is the volume of each droplet.

Depends on the respiratory activity.

C_v is the viral load, which we are typically assuming is near the maximum when we're concerned about controlling spreading.

And $C_i(r)$ is the infectivity per virion, which we have discussed before also may have a size dependence and is most likely higher in the aerosol droplets.

So that's the general solution.

And the safety guideline we just discussed has in it the time average beta, so beta with the two brackets, which is the integral in time of beta divided by the time tau.

So you break that into two parts.

So we integrate this here.

The one here is-- it gives you a steady state term, and the interval is shown right here.

And that's basically the-- ultimately the average that remains, but initially when the infected person first walks in the room, there's a time to build up the concentration, which only lowers the transmission rate.

So the average transmission rate is always less than the steady state.

You're approaching the steady state from below because you need the time to build up those droplets.

And so the DELTA beta here, which is that correction, takes the following form.

What you can do is bring the interval over time and switch places with the integration over r and do the time integral inside the integral.

And so that allows you to get-- instead of λ_c here, you get λ_c squared, and you get the following expression for the $\Delta \beta$.

It may not be obvious looking at it, but if you take a look at τ going to 0, this expression leads to just $\bar{\beta}$.

So $\Delta \beta$ of 0 is $\bar{\beta}$, and that's because if you take this exponential here and you go to small times, you can linearize that and find its $\lambda_c t$.

So it factors-- cancels one factor of λ_c , one factor of τ , and you end up with just a single λ_c as above.

So what that means is that the average β , which we're plotting here, as a function of the time τ starts out at 0.

It ramps up and then eventually approaches a steady state, and here's the full solution.

So all the information that we've talked about before in terms of filtration, sedimentation, other phenomena in the well-mixed room are all included in this framework and can then be put into the safety guideline to derive a general safety guideline that has all of the physics that we want in there and allows you to define a safe occupancy for a room.