25 Feedback

Solutions to Recommended Problems

S25.1

(a)

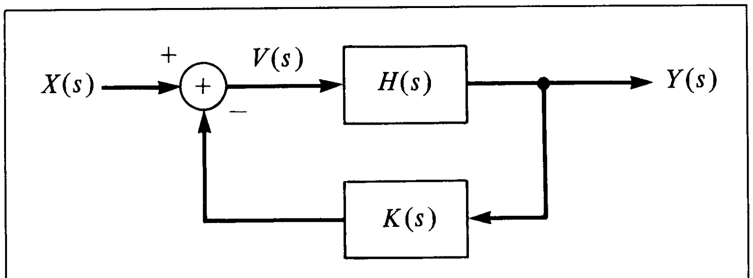

Figure S25.1-1

We have

$$V(s) = X(s) - Y(s)K(s) \qquad \text{(S25.1-1)}$$

and

$$Y(s) = V(s)H(s) \qquad \text{(S25.1-2)}$$

From eq. (S25.1-2),

$$V(s) = \frac{Y(s)}{H(s)} \qquad \text{(S25.1-3)}$$

Substituting eq. (S25.1-3) into eq. (S25.1-1), we have

$$\frac{Y(s)}{H(s)} = X(s) - Y(s)K(s),$$

$$Y(s)[1 + H(s)K(s)] = H(s)X(s),$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + H(s)K(s)}$$

Similarly,

$$\frac{Y(z)}{X(z)} = \frac{H(z)}{1 + H(z)K(z)}$$

(b) $Q(s) = \dfrac{H(s)}{1 + KH(s)}, \qquad Q(z) = \dfrac{H(z)}{1 + KH(z)}$

For $H(s) = 2/(s - 2)$ and $H(z) = 2/(z - 2)$,

$$Q(s) = \frac{2}{(s - 2) + 2K} = \frac{2}{s - 2(1 - K)}$$

$$Q(z) = \frac{2}{(z - 2) + 2K} = \frac{2}{z - 2(1 - K)}$$

For $K = 0$,

$$Q(s) = \frac{2}{s - 2} \quad \text{and} \quad Q(z) = \frac{2}{z - 2},$$

as shown in Figures S25.1-2 and S25.1-3, respectively.

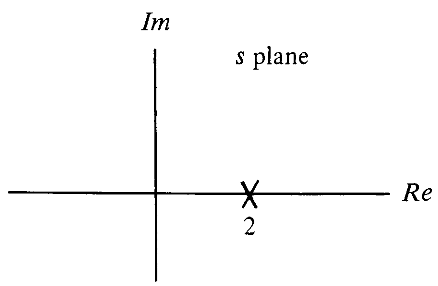

Figure S25.1-2

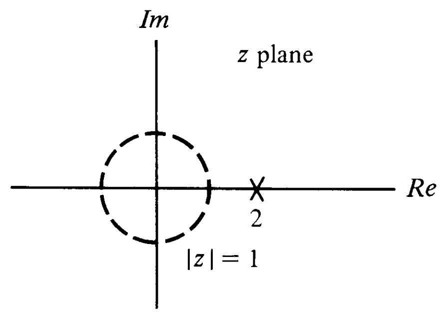

Figure S25.1-3

For $K = -1$,

$$Q(s) = \frac{2}{s-4} \quad \text{and} \quad Q(z) = \frac{2}{z-4},$$

as shown in Figures S25.1-4 and S25.1-5, respectively.

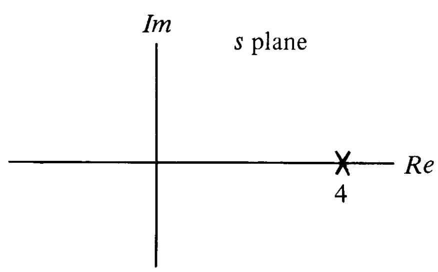

Figure S25.1-4

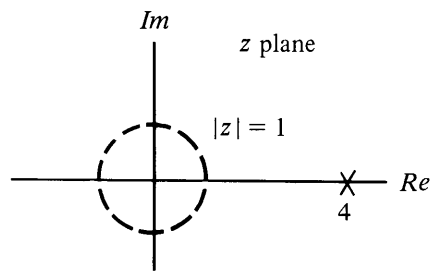

Figure S25.1-5

For $K = 1$,

$$Q(s) = \frac{2}{s} \quad \text{and} \quad Q(z) = \frac{2}{z},$$

as shown in Figures S25.1-6 and S25.1-7, respectively.

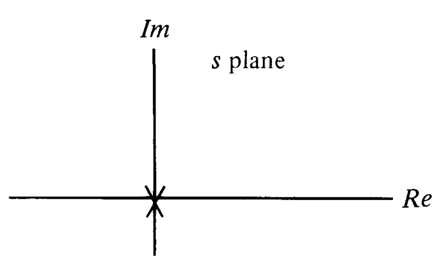

Figure S25.1-6

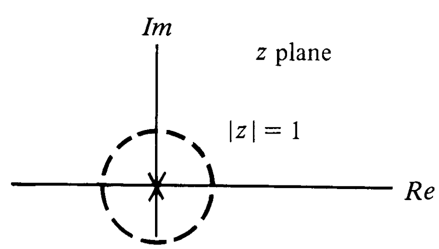

Figure S25.1-7

(c) $Q(s) = \dfrac{2}{s - 2(1-K)}$

The pole is located at $s = 2(1 - K)$, as shown in Figure S25.1-8.

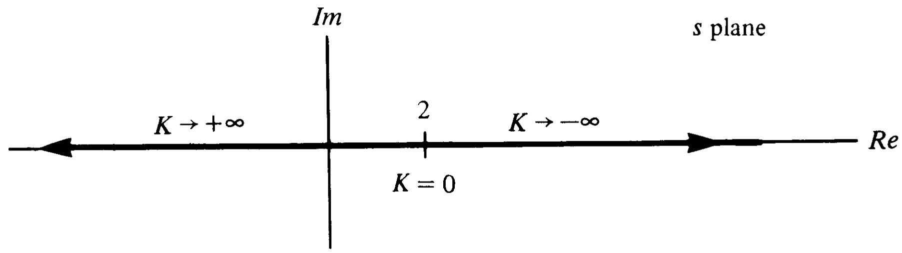

Figure S25.1-8

Hence, the locus of the pole is the line $Re\{s\} = 0$. Similarly, for

$$Q(z) = \frac{2}{z - 2(1 - K)},$$

the locus of the pole is also the line $Re\{z\} = 0$, shown in Figure S25.1-9.

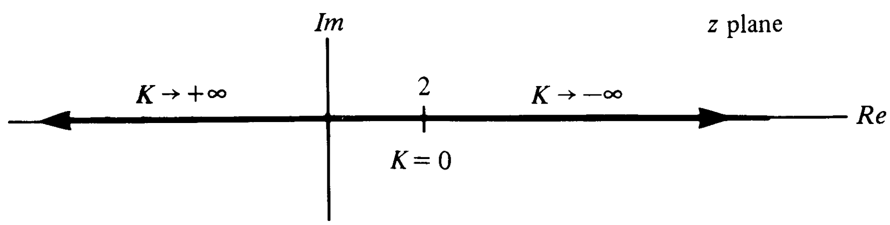

Figure S25.1-9

The root location decreases as K moves to infinity and increases as K moves to negative infinity.

(d) $Q(s) = \dfrac{2}{s - 2(1 - K)}$

The system is stable for $2(1 - K) < 0$, or $K > 1$.

$$Q(z) = \frac{2}{z - 2(1 - K)}$$

The system is stable for $-1 < 2(1 - K) < 1$, or $\frac{1}{2} < K < \frac{3}{2}$.

S25.2

We use Problem P25.1.

(a) (i) $$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s)H(s)}$$

(ii)
$$\begin{aligned}
E(s) &= X(s) - R(s) \\
&= X(s) - Y(s)G(s) \\
&= X(s) - E(s)H(s)G(s), \\
E(s)[1 + H(s)G(s)] &= X(s), \\
\frac{E(s)}{X(s)} &= \frac{1}{1 + H(s)G(s)}
\end{aligned}$$

(iii) $\dfrac{Y(s)}{E(s)} = H(s)$

(iv) $\dfrac{Y(s)}{R(s)} = \dfrac{1}{G(s)}$

(b) $W(z) = X(z)\dfrac{H_1(z)}{1 + G(z)H_1(z)}$,

$Y(z) = W(z) + X(z)H_0(z)$,

$Y(z) = \dfrac{X(z)H_1(z)}{1 + G(z)H_1(z)} + X(z)H_0(z)$

Thus,

$$\frac{Y(z)}{X(z)} = \frac{H_1(z)}{1 + G(z)H_1(z)} + H_0(z)$$

(c) $\dfrac{Y(s)}{W(s)} = \dfrac{H_1(s)}{1 + G_1(s)H(s)}$, as shown in Figure S25.2.

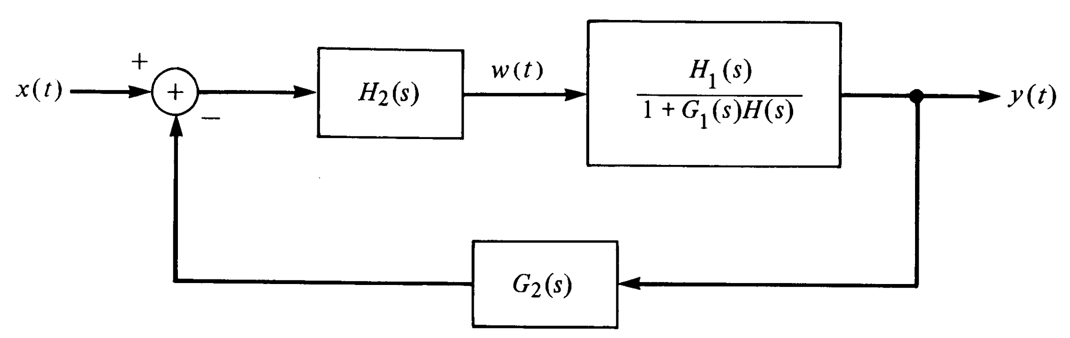

Figure S25.2

$$\frac{Y(s)}{X(s)} = \frac{\dfrac{H_1(s)H_2(s)}{1 + G_1(s)H_1(s)}}{1 + \dfrac{G_2(s)H_1(s)H_2(s)}{1 + G_1(s)H_1(s)}}$$

$$= \frac{H_1(s)H_2(s)}{1 + G_1(s)H_1(s) + G_2(s)H_1(s)H_2(s)}$$

S25.3

(a)

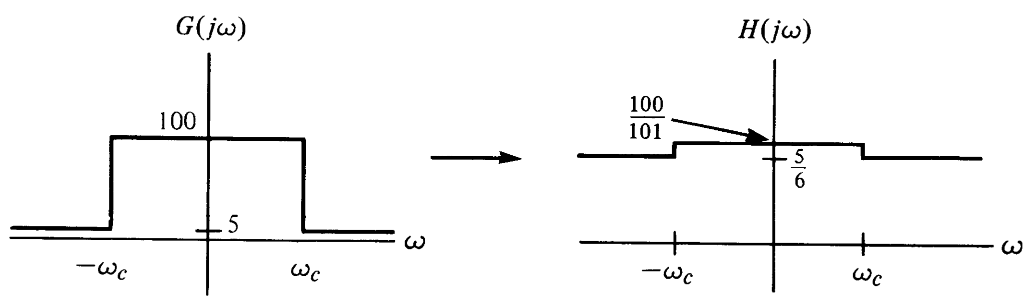

Figure S25.3-1

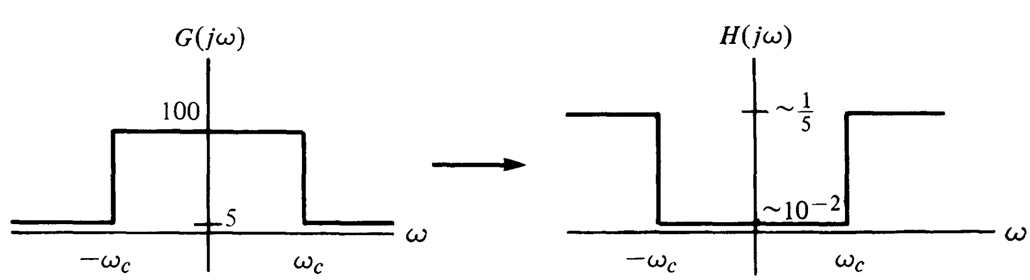

Figure S25.3-2

(b) From the frequency response in part (a), clearly system 1 tends to make the response more constant and system 2 tends to resemble the inverse of $G(j\omega)$.

S25.4

For the system in Figure S25.4-1, we denote the closed-loop system function by

$$V = \frac{H}{1 + GH}$$

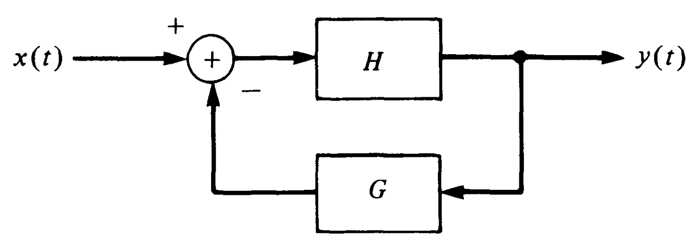

Figure S25.4-1

(a) $$V(s) = \frac{\dfrac{1}{(s+1)(s+3)}}{1 + \dfrac{1}{(s+1)(s+3)}} = \frac{1}{(s+1)(s+3)+1}$$

$$= \frac{1}{s^2 + 4s + 4} = \frac{1}{(s+2)^2}$$

Therefore,

$$v(t) = te^{-2t}u(t)$$

(b) $$V(s) = \frac{\dfrac{1}{s+3}}{1 + \left(\dfrac{1}{s+3}\right)(s+1)} = \frac{1}{(s+3)+(s+1)}$$

$$= \frac{1}{2s+4} = \frac{1}{2}\frac{1}{s+2}$$

In this case,

$$v(t) = \tfrac{1}{2}e^{-2t}u(t)$$

(c) The system function $G(s) = e^{-s/3}$ corresponds to a delay of $\frac{1}{3}$, i.e., the feedback system of Figure P25.4(a) becomes that shown in Figure S25.4-2.

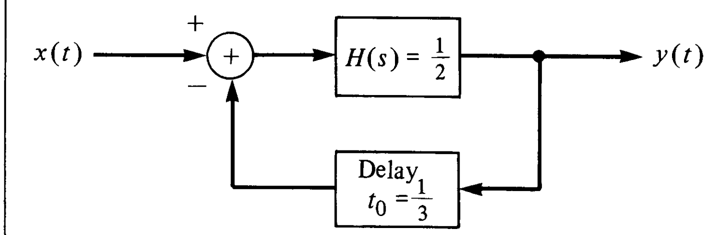

Figure S25.4-2

We can now recursively obtain the impulse response by inspection. With $x(t) = \delta(t)$,

$$y(t) = \tfrac{1}{2}\delta(t) - \tfrac{1}{2}[\tfrac{1}{2}\delta(t - \tfrac{1}{3})] + \tfrac{1}{2}[\tfrac{1}{4}\delta(t - \tfrac{2}{3})] - \cdots$$
$$= \frac{1}{2}\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n \delta\left(t - \frac{n}{3}\right)$$

(d) $$V(z) = \frac{\dfrac{z^{-1}}{1 - \frac{1}{2}z^{-1}}}{1 + \left(\dfrac{z^{-1}}{1 - \frac{1}{2}z^{-1}}\right)\left(\dfrac{2}{3} - \dfrac{1}{6}z^{-1}\right)}$$
$$= \frac{z^{-1}}{(1 - \frac{1}{2}z^{-1}) + (\frac{2}{3}z^{-1} - \frac{1}{6}z^{-2})}$$
$$= \frac{z^{-1}}{1 + \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}$$
$$= \frac{z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 + \frac{1}{2}z^{-1})}$$
$$= \frac{\frac{6}{5}}{1 - \frac{1}{3}z^{-1}} - \frac{\frac{6}{5}}{1 + \frac{1}{2}z^{-1}}$$

Therefore,

$$v[n] = \tfrac{6}{5}[(\tfrac{1}{3})^n u[n] - (-\tfrac{1}{2})^n u[n]]$$

(e) $$V(z) = \frac{H(z)}{1 + H(z)G(z)} = \frac{\frac{2}{3} - \frac{1}{6}z^{-1}}{1 + \left(\dfrac{2}{3} - \dfrac{1}{6}z^{-1}\right)\left(\dfrac{z^{-1}}{1 - \frac{1}{2}z^{-1}}\right)}$$
$$= \frac{(\frac{2}{3} - \frac{1}{6}z^{-1})(1 - \frac{1}{2}z^{-1})}{(1 - \frac{1}{2}z^{-1}) + (\frac{2}{3} - \frac{1}{6}z^{-1})z^{-1}}$$
$$= \frac{\frac{2}{3} - \frac{2}{3}z^{-1} + \frac{1}{12}z^{-2}}{1 + \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}}$$

Thus,

$$v[n] = \tfrac{2}{3}\tilde{v}[n + 1] - \tfrac{2}{3}\tilde{v}[n] + \tfrac{1}{12}\tilde{v}[n - 1],$$

where $\tilde{v}[n]$ is $v[n]$ in part (d).

Solutions to Optional Problems

S25.5

$$y(t) = K_2 w(t) + K_1 K_2 v(t) \qquad \text{(S25.5-1)}$$

By taking the transform of eq. (S25.5-1), we have

$$Y(s) = K_2 W(s) + K_1 K_2 V(s)$$

Also

$$V(s) = X(s) + \frac{s}{s+\alpha} Y(s)$$

Therefore,

$$Y(s) = K_2 W(s) + K_1 K_2 \left[X(s) + \frac{s}{s+\alpha} Y(s) \right],$$

$$Y(s)\left(1 - \frac{K_1 K_2 s}{s+\alpha}\right) = K_2 W(s) + K_1 K_2 X(s),$$

and

$$Y(s) = \frac{K_2 W(s) + K_1 K_2 X(s)}{1 - \dfrac{K_1 K_2 s}{s+\alpha}} = \frac{(s+\alpha)[K_2 W(s) + K_1 K_2 X(s)]}{(1 - K_1 K_2)s + \alpha}$$

S25.6

(a) The system function of the system given in Figure P25.6 must be determined first. So we write down the difference equation

$$y[n] = x[n] + y[n-1] + 4y[n-2]$$

Taking the z-transform of the equation, we have

$$Y(z)(1 - z^{-1} - 4z^{-2}) = X(z), \qquad \text{or} \qquad H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - z^{-1} - 4z^{-2}}$$

The poles of this system are located at

$$z^2 - z - 4 = 0, \qquad \text{or} \qquad z = \frac{1}{2} \pm \frac{\sqrt{17}}{2}$$

Since $|z| > 1$ for at least one pole the system is unstable.

(b) With closed-loop feedback, the difference equation is

$$y[n] = x_e[n] - Ky[n-1] + y[n-1] + 4y[n-2]$$

Thus,

$$H(z) = \frac{z^2}{z^2 + (K-1)z - 4}$$

The poles are now located at

$$z = \frac{-(K-1) \pm \sqrt{(K-1)^2 + 16}}{2}$$

Note that the roots are purely real because the term inside the square root is always positive. For $z = 1$,

$$1 + \frac{K}{2} - \frac{1}{2} = \pm \frac{\sqrt{(K-1)^2 + 16}}{2},$$
$$K + 1 = \pm \sqrt{(K-1)^2 + 16}$$

Thus,

$$K^2 + 2K + 1 = K^2 - 2K + 17,$$
$$4K = 16, \quad \text{or} \quad K = 4$$

We can also calculate z_2:

$$z_2 = -4$$

Similarly, $z_1 = -1$, $z_2 = 4$ for $K = -2$. Observe the root locus in Figure S25.6-1.

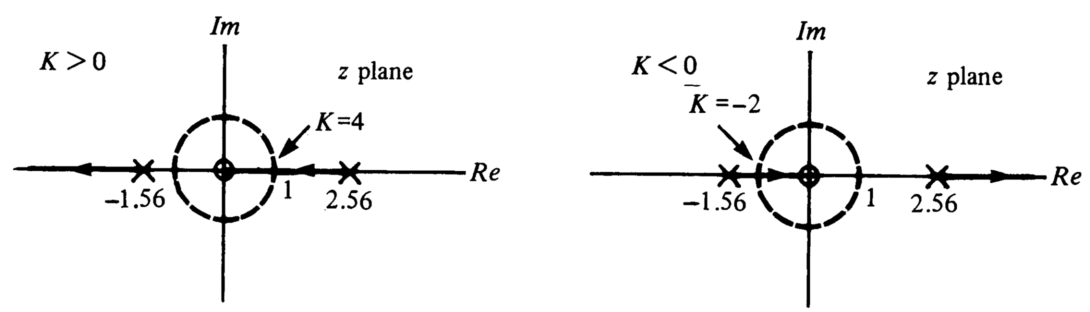

Figure S25.6-1

Observe that if one of the poles is inside $|z| \leq 1$, the other is outside. Hence, the system is unstable for all values of K.

(c) The difference equation can be written as

$$y[n] = x_e[n] + y[n-1] + (4-K)y[n-2]$$

Therefore,

$$H(z) = \frac{z^2}{z^2 - z + (K-4)}$$

In this case, the poles are located at

$$z = \frac{1}{2} \pm \frac{\sqrt{17 - 4K}}{2}$$

For a stable system, we want

$$|z| < 1,$$
$$|z| = \left| \frac{1}{2} \pm \frac{\sqrt{17-4K}}{2} \right|$$

If we set $17 - 4K > 0$, then

$$\left| \frac{1}{2} \pm \frac{\sqrt{17-4K}}{2} \right| < 1,$$

or

$$\pm \frac{\sqrt{17-4K}}{2} < \frac{1}{2},$$
$$17 - 4K < 1,$$
$$K > 4$$

Now suppose $17 - 4K < 0$. Then

$$\left| \tfrac{1}{2} \pm j\sqrt{|\tfrac{17}{4} - K|} \right| < 1 \quad \text{or} \quad \tfrac{17}{4} - K > -\tfrac{3}{4},$$
$$-K > -\tfrac{20}{4},$$
$$K < 5$$

Thus, for K in the range $4 < K < 5$, we have a stable system. The root locus is shown in Figure S25.6-2.

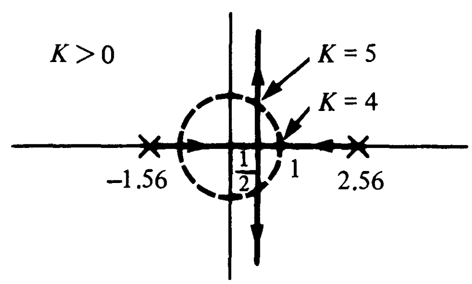

Figure S25.6-2

S25.7

(a) The dc gain of the amplifier is $|H(0)| = |G|$.

(b) $h(t) = Gae^{-at}u(t)$. Therefore, the time constant is $1/a$.

(c) $|H(j\omega_c)|^2 = \dfrac{G^2a^2}{a^2 + \omega_c^2} = \dfrac{1}{2}G^2$

Thus $\omega_c = \pm a$. Hence the bandwidth is a.

(d) The closed-loop transfer function is

$$V(s) = \frac{\dfrac{Ga}{s+a}}{1 + \dfrac{KGa}{s+a}} = \frac{Ga}{(1+KG)a + s}$$

From part (a), the time constant is

$$\frac{1}{(1+KG)a}$$

From part (c), the bandwidth is $(1 + KG)a$. From part (a), the dc gain is

$$\left|\frac{G}{1+KG}\right|$$

(e) We require $(GK + 1)a = 2a$. Hence, $K = 1/G$. So the bandwidth becomes $2a$. The time constant is $1/(2a)$, and $|H(0)| = |G/2|$, the dc gain.