We have observed in the simple example from the previous clip that when the Markov chain initially starts in state one, the probability that it finds itself in state one after a long period of time converges to a constant value, in our case, 2/7.

In addition, if the Markov chain initially starts in state two, the probability that it finds itself in state one after a long period of time also converges to the same constant value, 2/7.

Are these two properties of long term convergence and of vanishing effect of the initial state over the long term convergence always true?

Mathematically, we are asking the question, is rij of n pi j when n goes to infinity?

The answer is that for nice Markov chains, this will be true, but this is not always true.

Consider the first question.

Does rij(n) always converge to something as n goes to infinity?

Look at the following simple Markov chain.

When in state two, you will never be in state two at the next transition.

You will end up next in either state one or state three.

However, no matter where you end up, you're sure that the next transition will bring you back to state two, either here or from here.

In other words, for n odd, r22 of n will always be 0, and for n even, r22 of n will always be 1.

And so r22 of n will never converge.

It will always alternate between 1 or 0.

Convergence has failed.

That chain has a periodic structure, and we will see in the next lecture that if periodicity is absent from a chain, then we don't have a problem with convergence.

Consider now the second question dealing with a vanishing importance of initial states when convergence occurs.

For this, consider the following Markov chain.

If you start in state one, there is no way you can escape.

You are certain to stay there forever.

So r11 of n will always be 1.

On the other hand, if you start in state three, there is no way you will ever reach state one.

So r31 of n will be 0.

The initial state of where you started does matter in this example, and its influence never vanishes in the long run.

The second nice property has failed here.

And here, this has to do with the Markov structure where some states are not accessible from some other states, and we will address this in the final portion of this lecture.

Finally, let us calculate r21 of n for large n.

So you start in state two, and you ask yourself, what is the probability that I will end up in state one after n steps for n large?

Well, when you start in two, you may stay in two for a while by doing this kind of transition and this transition and this transition.

But eventually, with probability one, you will escape.

Either you will go to state one, or you will escape to state three.

And in that case, you will never go back to two.

If you are in one, you will never go back here to two, and from three, you will never go back to two.

Because of the symmetry between these probabilities here-- 0.3 on this side and 0.3 on this side-- when you do escape state two, you are equally likely to escape toward one or toward three.

So what you have is that r21 of n will be 1/2.