We have said that the Bernoulli process is the simplest stochastic processes there is.

But what is a stochastic process anyway?

A stochastic process can be thought of as a sequence of random variables.

Now, how is this different from what we have doing before, where we have dealt with multiple random variables?

Well, one difference is that here we're talking about an infinite sequence of random variables.

And that complicates things to a certain extent.

Now, what does it take to describe a stochastic process?

We should specify the properties of each one of those random variables.

For example, we might be interested in the mean, variance, or PMF of those random variables.

For the case of the Bernoulli process, this would be easy to do.

We know what the expected value is.

We have a formula for the variance.

And we have a fairly simple PMF.

There's probability p that X is equal to 1 and probability 1 minus p that X equals to 0.

But this is not enough.

We also need to know how the different random variables are related to each other.

And this is done by specifying, directly or indirectly, the joint distribution, the joint PMF or PDF, of the random variables involved.

And because we have an infinite number of random variables, it's not enough to do this, let's say, for the first n of them.

We need to be able to specify this joint distribution no matter what the number n is.

For the case of the Bernoulli process, we have specified this joint PMF in an indirect way, because we have said that the random variables are independent of each other.

So the joint factors as a product of the marginals.

And we already know what the marginals are.

So we do, indeed, have a specification of the joint PMF, and we have that for all values of n.

Of course, for more complicated stochastic processes, this calculation might be somewhat more difficult.

Now, there is a second view of a stochastic process which rests on the following.

It's not just a collection of random variables, but they are a collection that's indexed by an index that keeps increasing.

And quite often, we think of this index as corresponding to time.

And so we have a mental picture that involves a process that keeps evolving in time.

What is this picture?

This picture is best developed if we think in terms of the sample space.

Although we have an infinite sequence of random variables, we are dealing with a single experiment.

And that single experiment runs in time.

And when we carry out the experiment, we might to get an outcome such as the following.

For the Bernoulli process, we might get a 0, 0, 1, 0, 1, 1, 0, and so on.

And we continue.

So an infinite sequence of that kind is one possible outcome of this infinitely long experiment, one particular outcome of the stochastic process.

If we carry out the process once more, we might get a different outcome.

For example, we might get a 0, 1, 1, 0, 0, 0, 1, 1, and so on, and continuing.

And in general, any time function of this kind is one possible outcome of the experiment.

Overall, the sample space that we're dealing with is the set of all infinite sequences of 0s and 1s.

This point of view emphasizes the fact that we have a phenomenon that evolves over time and can be used to answer questions that have to do with the long-term evolution of this process.

Here's one particular kind of question we might want one answer.

What is the probability that all of the Xi's turn out to be 1?

Notice that this is an event that involves all of the Xi's not just a finite number of them.

So this is not a probability that we can calculate right away by using this joint pmf.

We need to do a little more work.

What is the work that we want to do?

Instead of calculating this quantity, we will calculate a somewhat related quantity.

Let us look at the event that the first n results were equal to 1.

How is this event related to this event?

Well, this event here implies that this event has happened.

So this is a smaller event.

This is more difficult to obtain than this one.

And this gives us an inequality for the probabilities that go this way.

Now, we know that this probability is equal to p to the n.

And this inequality here is true for all n.

No matter how large n we take, this quantity is smaller than that.

But now, since p has been assumed to be less than 1, when we take n larger and larger, this number becomes arbitrarily small.

So this quantity is less than or equal to an arbitrarily small number.

So this quantity can only be equal to 0.

And this is a simple example of how we calculate properties of the stochastic process as it evolves over the infinite time horizon and how we can sometimes calculate them using these so-called finite dimensional joint probabilities that tell us what the process is doing over a finite amount of time.

Throughout, we will sometimes view stochastic processes in this manner, in terms of probability distributions.

But sometimes we will also want to reason in terms of the behavior of the stochastic process as a time function, as a process that evolves in time.