Now that we have in our hands the PMF of the random variable N tau, which is the number of arrivals during an interval of length tau, we can go ahead and try to calculate the mean and variance of this quantity.

Regarding the mean, we could use just the definition of the expected value and then carry out of this calculation, which is not too hard.

And in the end, we would obtain an answer equal to lambda times tau.

This is indeed the correct formula for the expected value.

But let us understand why this formula should be true without doing any calculation.

Remember that the random variable, the number of arrivals in the Poisson process, is well approximated by a binomial random variable with those particular parameters n and p in the limit when delta goes to zero.

And this works through a discretization argument.

Therefore, the expected value of N tau should be approximately equal to the expected value of that we get from the Bernoulli processes, that is the expected value of the binomial random variable.

And the expected value of a binomial random variable is n times p.

And n times p evaluates approximately to lambda times tau.

The second equality here is approximate because we're neglecting this order of delta squared term.

Now, these approximations are increasingly exact as we let delta go to 0.

And when we take the limit as delta goes to 0, we see that the expected value of the number of arrivals in the Poisson process will be equal to lambda tau.

For the variance, we can follow a similar argument.

First do a binomial approximation and use the formula for the variance of a binomial random variable.

And then, when delta is small, this number p is small.

And it's negligible compared to 1.

n times p is approximately lambda [tau].

And so we obtain this expression This expression here is the correct one.

If we were to use the formal definition of the variance and carry out the calculations using the PMF, this is what we would obtain, except that it would be somewhat tedious.

The formulas that we have derived, at least the first one, is quite intuitive and has a natural form.

The expected number of arrivals is proportional to tau.

If we double the length of the time interval for interest, we expect to see twice as many arrivals.

This formula also reinforces the interpretation of lambda as an arrival rate.

Since lambda is the expected number of arrivals divided by the length of time, it is, really, the expected number of arrivals per unit time.

So it's natural to call lambda the arrival rate, or the intensity of the arrival process.

Finally, regarding the variance, it is a curious fact that the variance turns out to be exactly the same as the expected value.

And this is a special property of the Poisson PMF.